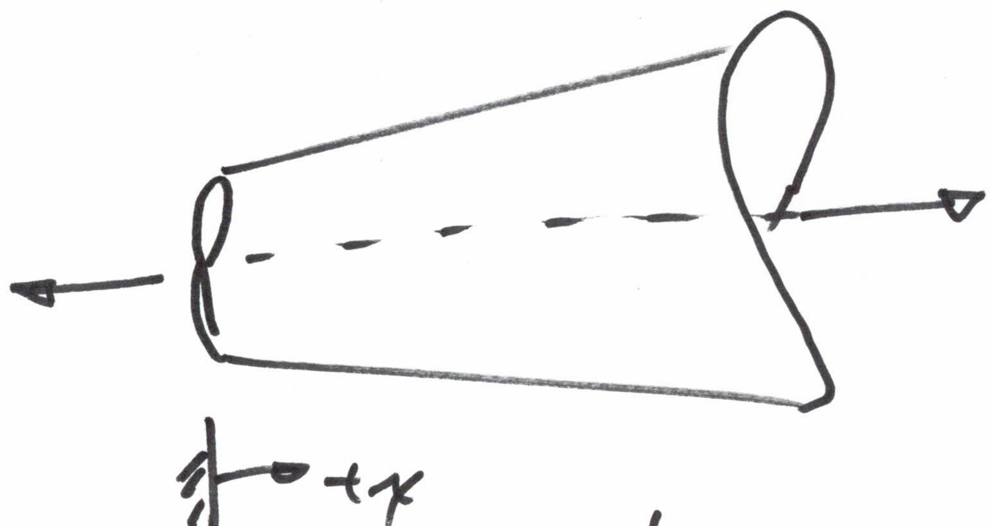
III General Axial Deformation

- not constant x-section
- determinate & indeterminate

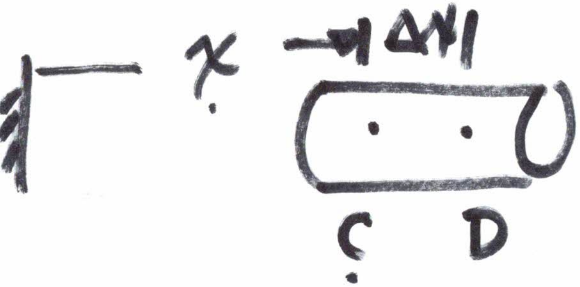
A. Axial Deformation

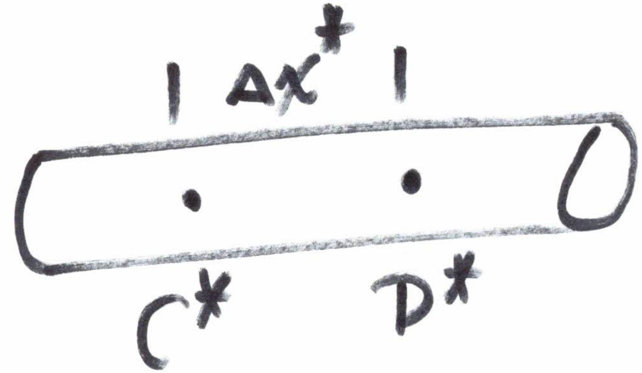


Resultant force acts through centroid (no bending).

Bar could have changing x-section, material properties, or load along x-axis.
Any of these would make the strain distribution non-uniform.

1. Strain at a point





$$\epsilon_{CD} = \frac{\Delta x^* - \Delta x}{\Delta x}$$

$\Delta x^* - \Delta x$ is the relative change in ~~displacement~~ position between C & D

$$\Delta x^* - \Delta x = u(D) - u(C)$$
$$= u(x + \Delta x) - u(x)$$

u_C - displacement of C

u_D - displacement of D

$u(x=C)$ $\quad u(x=D)$

$u(C)$ $\quad u(D)$

$$\epsilon_{CD} = \frac{u(x+\Delta x) - u(x)}{\Delta x}$$

at a point: $\displaystyle \epsilon_{CD \atop \Delta x \to 0} = \lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x}$

$$\epsilon_{pt} = \frac{du}{dx}$$ u = displacement equation

if we want u

$$du = \epsilon \, dx$$

$$u = \underbrace{\int_0^L \epsilon \, dx}_{\text{material stretching}} + \underbrace{\text{Const}}_{\text{Rigid Body motion}}$$

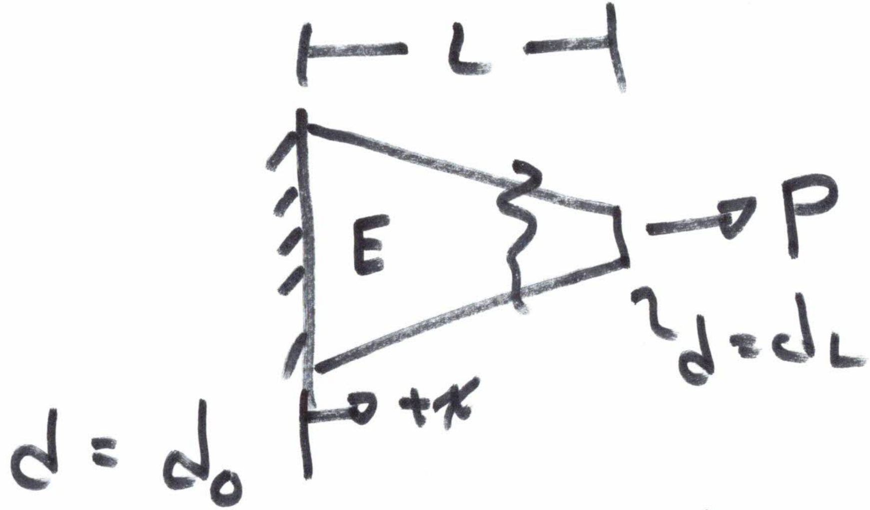
$$\Delta L = e = \int_0^L \epsilon \, dx \qquad \sigma = E\epsilon$$

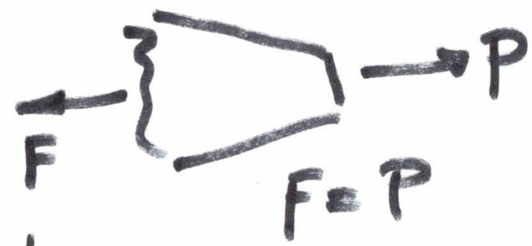
$$\Delta L = e = \int_0^L \frac{\sigma}{E} \, dx \qquad \sigma = \frac{F}{A}$$

$$\Delta L = e = \int_0^L \frac{F}{AE} \, dx$$

Find change in length

Round bar, linear taper





$$e = \frac{F}{E}\int_0^L \frac{1}{A}\,dx \qquad A = \frac{\pi}{4}d^2$$

need an equation

$y = mx + b$

$d = mx + b$

$x = 0,\ d = d_0 \Rightarrow b = d_0$

$x = L,\ d = d_L \Rightarrow d_L = mL + d_0$

$$m = \frac{d_L - d_0}{L}$$

$$e = \frac{F}{E} \int_0^L \frac{4}{\pi} (mx+b) \, dx$$

let $U = mx + b \quad dU = m \, dx$

$$dx = \frac{1}{m} dU$$

$$e = \frac{F}{E} \int_0^L \frac{4}{\pi} \frac{1}{m} U^{-2} \, dU$$

$$e = -\frac{F}{E} \cancel{\int_0^L} \frac{4}{\pi} \frac{1}{m} U^{-1} \Big]_0^L$$

$$e = -\frac{F}{E} \frac{4}{\pi} \frac{1}{m} \left[mx+b\right]^{-1} \Big]_0^L$$

2. constant F, A, over L

$$e = \int_0^L \frac{F}{AE}\, dx$$

$$e = \frac{F}{AE} \int_0^L dx$$

$$\boxed{e = \frac{FL}{AE}}$$

constant F, A, E over L

$$\Delta L = \epsilon L = \frac{\sigma}{E} L = \frac{FL}{AE}$$