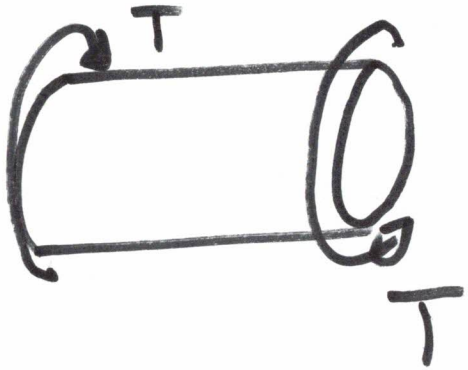
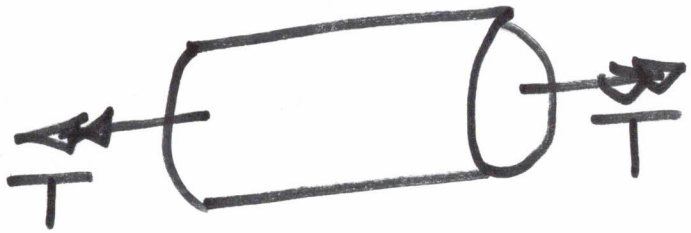
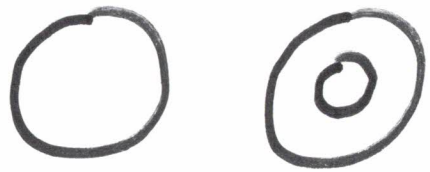
IV Torsion





double-headed arrow notation

In AEM 250, we will study solid & hollow circular sections (only)

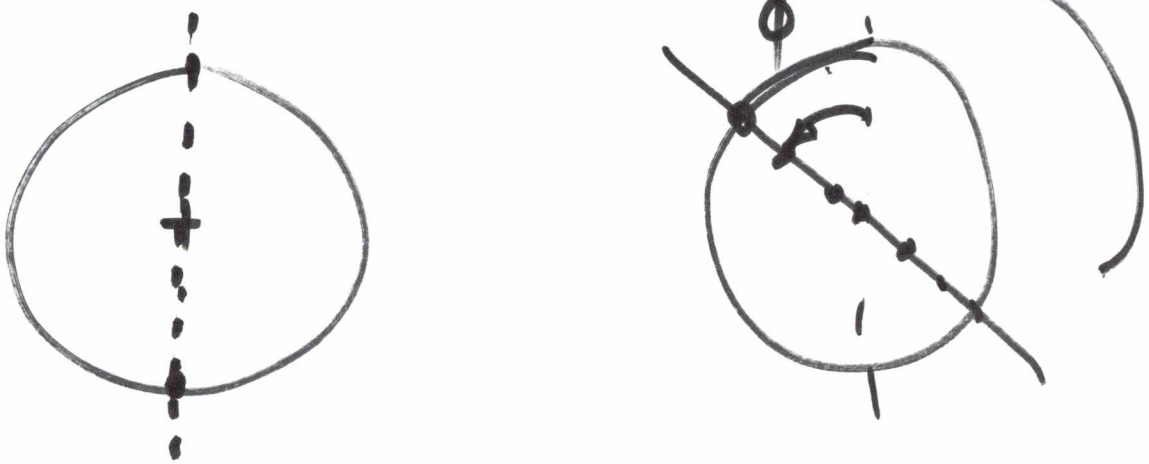


(□ 工 ⊏ ○)

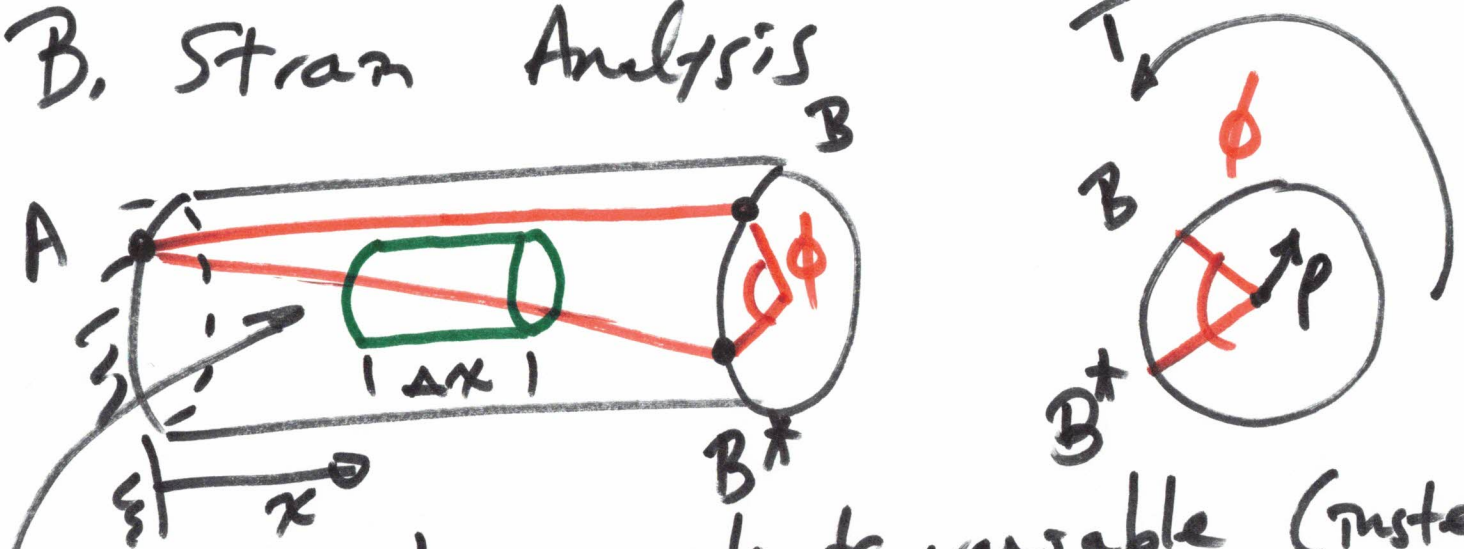
↑ other classes

A. Observations of Elastic Deformation (Round bars)

1. plane cross-section remain plane after loading
2. bar does not get longer (inextensible)
3. all points on a cross-section move through same angle of twist

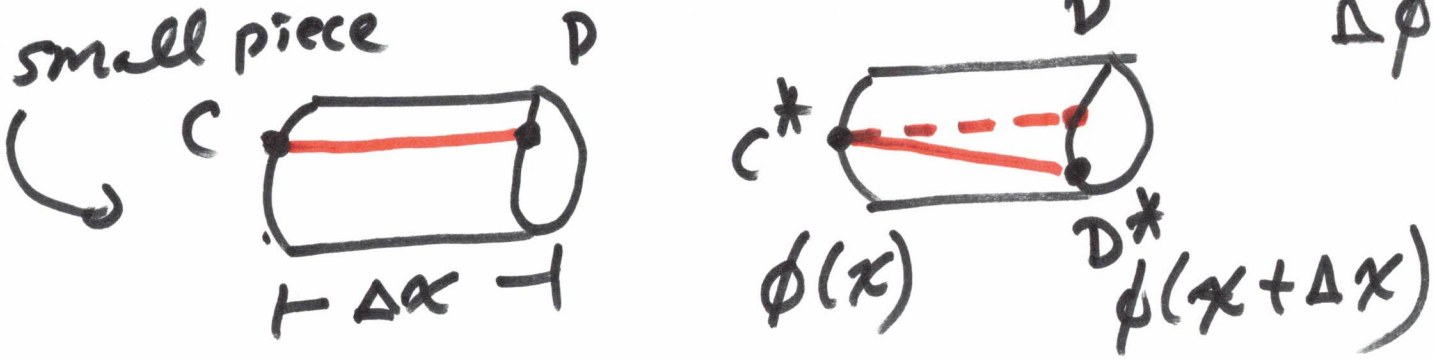


B. Strain Analysis



ρ - polar coordinate variable (instead of r in other cases)

small piece



$\Delta\phi = \phi(x+\Delta x) - \phi(x)$

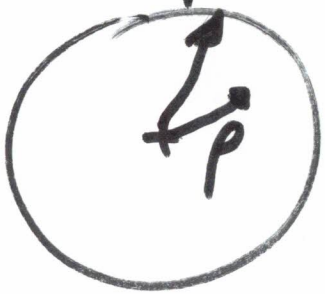
γ = change in angle from $\pi/2$ RAD

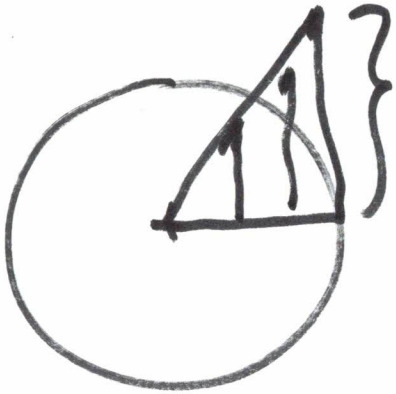
$$\tan\gamma = \frac{D'D^*}{\Delta x} = \frac{\rho\,\Delta\phi}{\Delta x} \approx \gamma$$

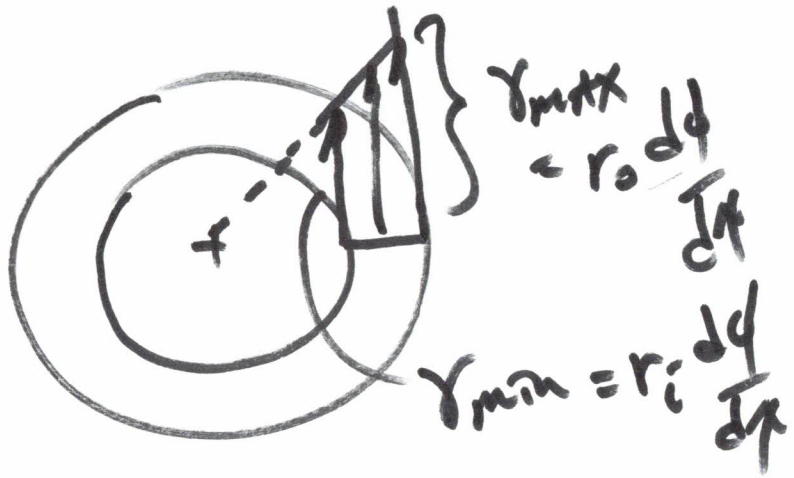
$$\gamma \approx \rho \underbrace{\frac{\Delta\phi}{\Delta x}} = \rho \underbrace{\frac{d\phi}{dx}}$$

twist rate

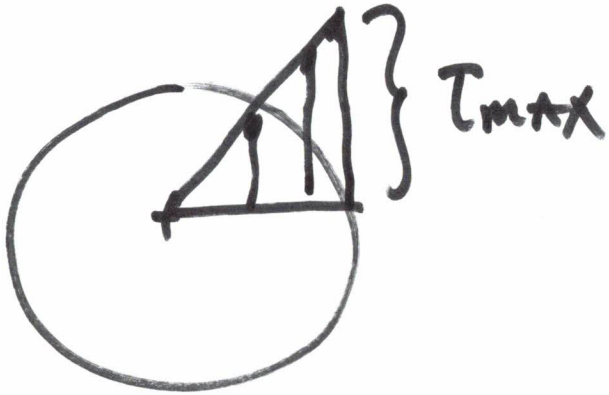
$\rho = r_o$

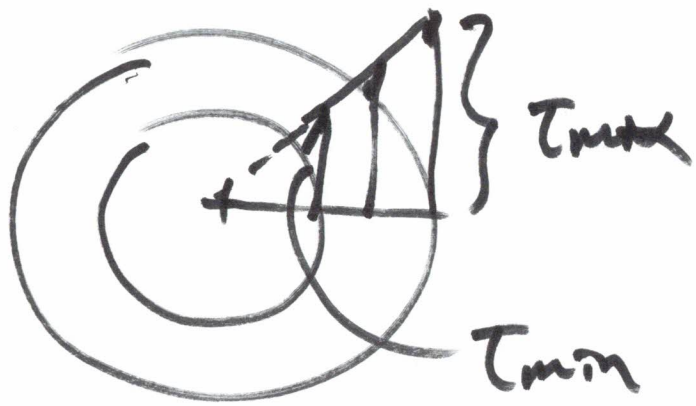




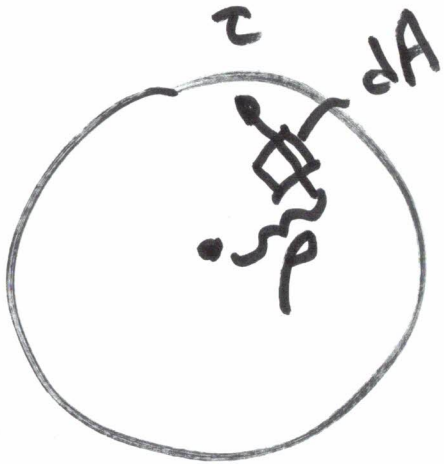


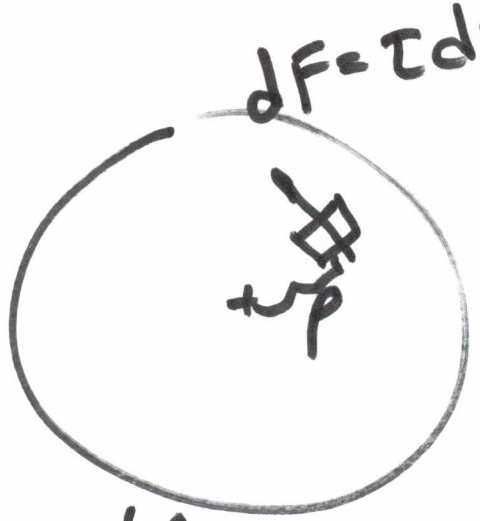
C. Hooke's Law $\tau = G\gamma = G\rho\frac{d\phi}{dx}$





D. Equilibrium





$$dT = \rho dF = \rho \tau dA$$

$$dT = \rho G \rho \frac{d\phi}{dx} dA$$

$$T = \int G \rho^2 \frac{d\phi}{dx} dA$$

$$T = G \frac{d\phi}{dx} \underbrace{\int \rho^2 dA}_{I_p}$$

I_p – polar moment of inertia

$$T = G \frac{d\phi}{dx} I_p$$

$$I_p = \frac{\pi}{32} d^4$$

$$I_p = \frac{\pi}{32} \left[d_o^4 - d_i^4 \right]$$

$$T = G \frac{d\phi}{dx} I_p \qquad \tau = \rho G \frac{d\phi}{dx}$$

$$\frac{d\phi}{dx} = \frac{T}{G I_p} \Rightarrow \tau = \frac{\rho G T}{G I_p}$$

$$\boxed{\tau = \frac{T\rho}{I_p}}$$

$$\tau_{MAX} = \frac{T\, d/2}{I_p} \quad \text{or} \quad \tau_{MAX} = \frac{T\, d_o/2}{I_p}$$

E. Angle of twist

$$\frac{d\phi}{dx} = \frac{T}{G I_p} \qquad d\phi = \frac{T}{G I_p} dx$$

$$\phi = \int_0^L \frac{T}{G I_p} \, dx$$

If T, G, I_p constant

$$\boxed{\phi = \frac{TL}{I_p G}}$$

(RADIANS)

$$e = \frac{FL}{AE}$$