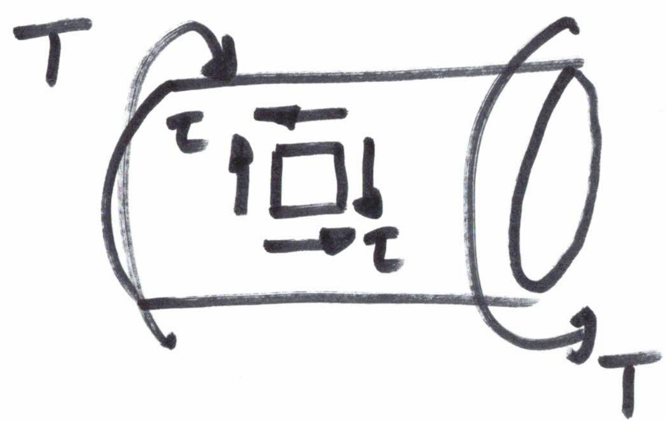
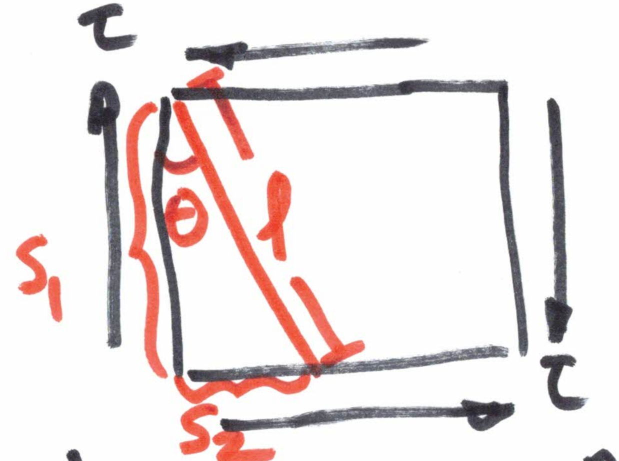
F. Stress State from Torsion

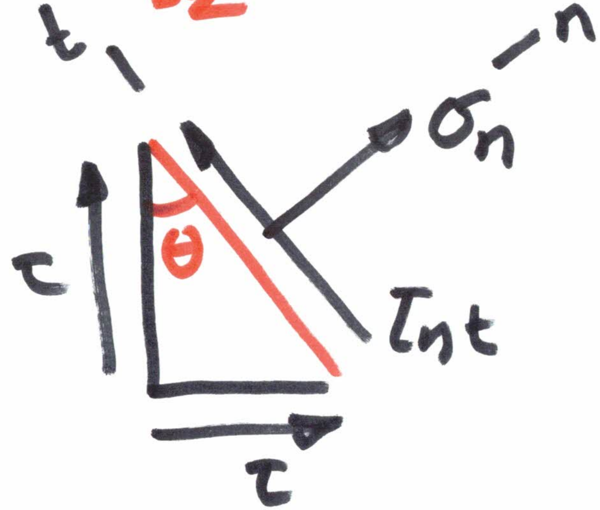


on outside of bar

$$\tau = \frac{T d/2}{I_p}$$

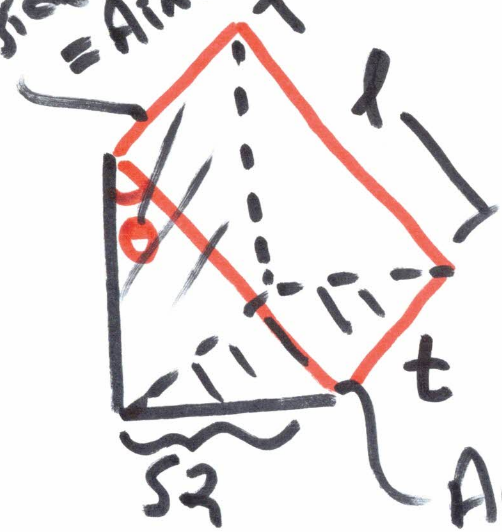


$\cos\theta = \frac{s_1}{\ell}$ $\sin\theta = \frac{s_2}{\ell}$



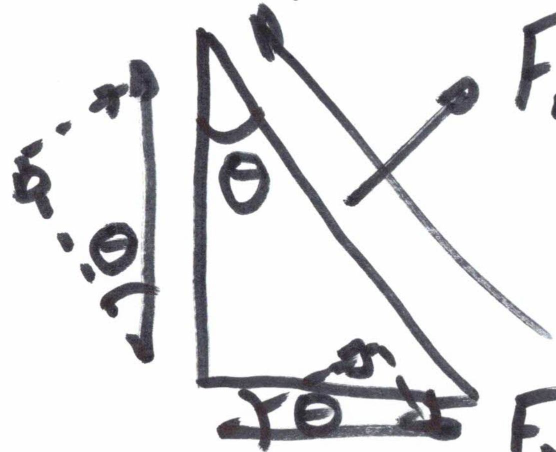
$A_{side} = s_1 t = A_{inc} \cos\theta$

$A_{inc} = t \cdot \ell$



$A_{bott} = s_2 t = \ell \sin\theta \cdot t = A_{inc} \sin\theta$

$F_y = \tau A_{inc} \cos\theta$ $\quad V = \tau_{nt} A_{inc}$



$F_n = \sigma_n A_{inc}$

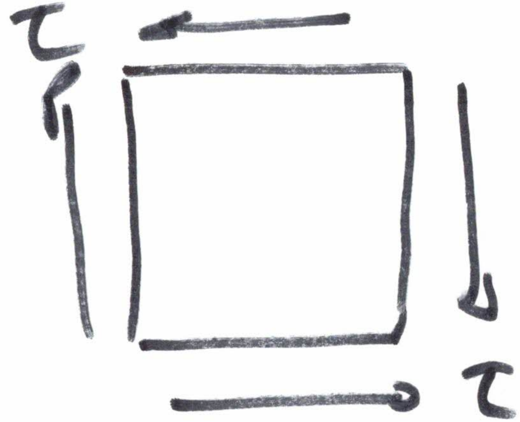
$F_x = \tau A_{inc} \sin\theta$

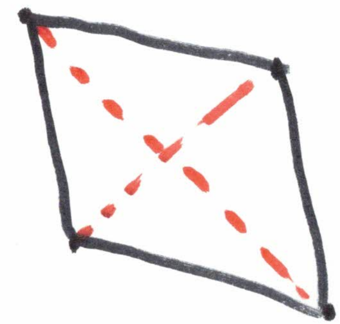
$\sum F_n \nearrow^{+}$ $\quad \sigma_n A_{inc} + \tau A_{inc} \cos\theta \sin\theta + \tau A_{inc} \sin\theta \cos\theta = 0$

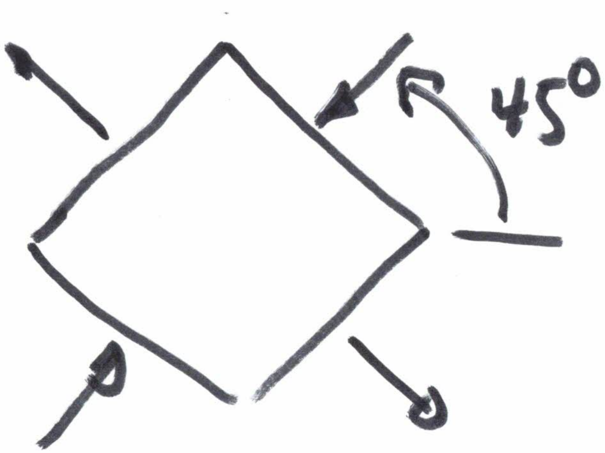
$\sum F_t \nwarrow^{+}$ $\quad \tau_{nt} A_{inc} + (\tau A_{inc} \cos\theta)\cos\theta - (\tau A_{inc} \sin\theta)\sin\theta = 0$

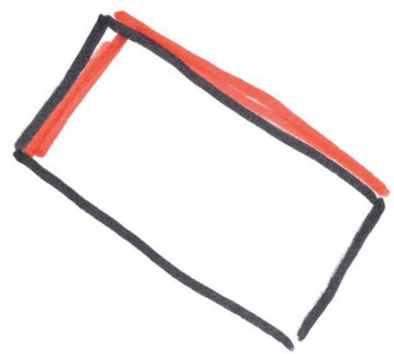
$\sigma_n = -2\tau A_{inc} \sin\theta \cos\theta = \underline{-\tau \sin 2\theta}$

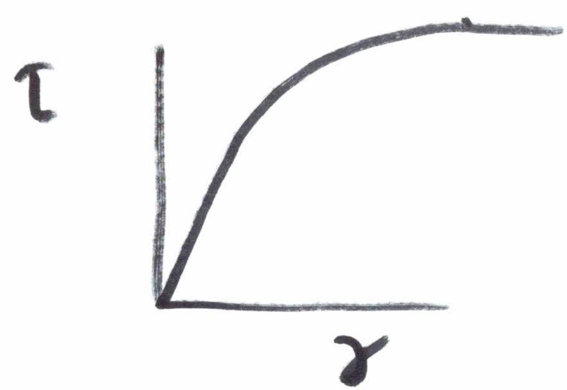
$\tau_{nt} = -\tau \cos^2\theta + \tau \sin^2\theta = \underline{-\tau \cos 2\theta}$


τ
τ




45°




τ
γ

Material behavior

- ductile materials tend to fail due to shear stresses (τ)
- brittle materials tend to fail due to normal stresses (σ)

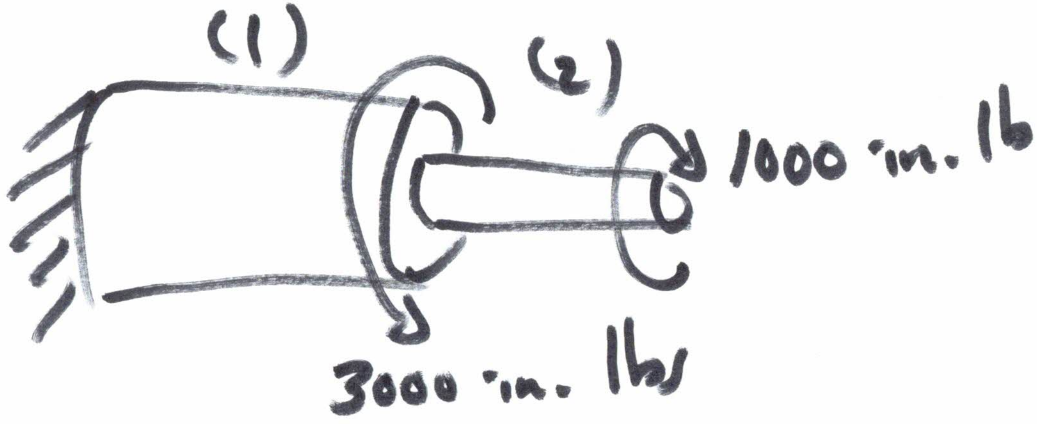
G. Torsion of systems

$$\phi = \int_0^L \frac{T}{I_p G}\,dx = \frac{T}{I_p G}\int_0^L dx$$

$$\boxed{\phi = \frac{TL}{I_p G}}$$ if T, G, I_p are constant

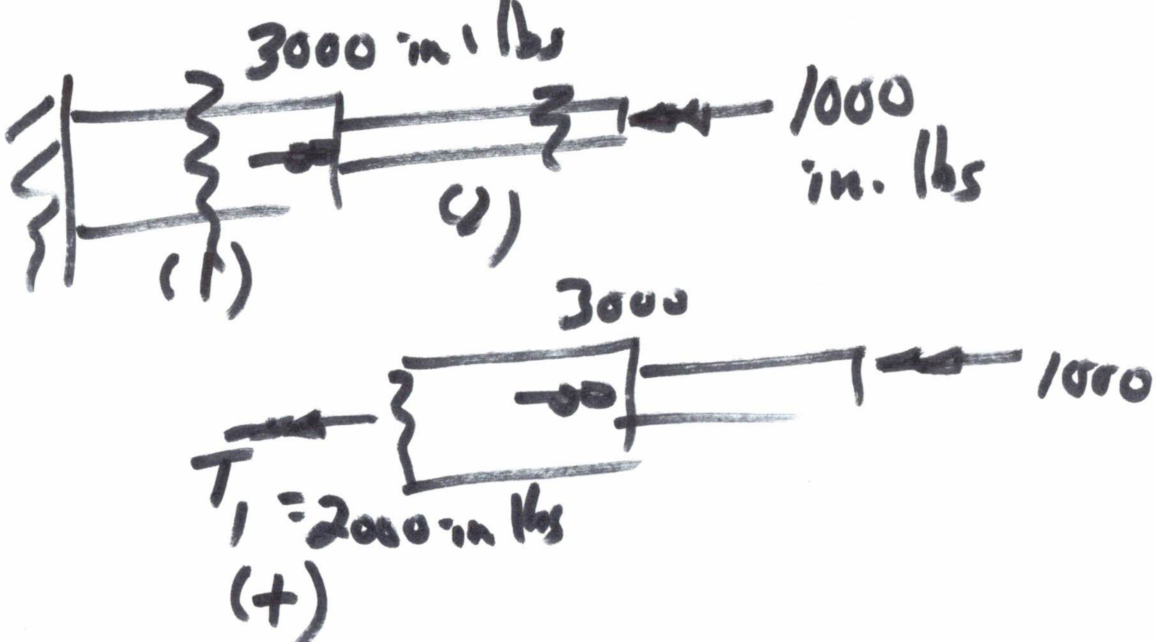
(RADIANS

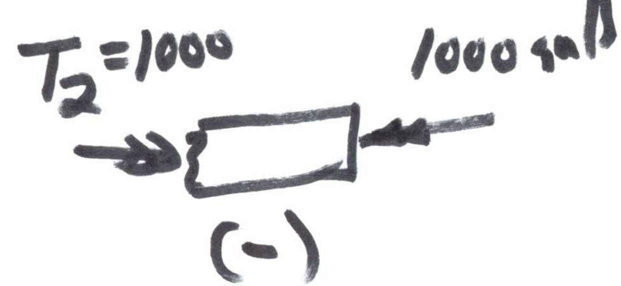
$$e = \frac{FL}{AE}$$



$$\phi_{end} = \phi_1 + \phi_2$$

$$\phi_{end} = \frac{T_1 L_1}{I_{p1} G_1} + \frac{T_2 L_2}{I_{p2} G_2}$$





$$\phi_{end} = \frac{2000\, L_1}{I_{p1}\, G_1} - \frac{1000\,(L_2)}{I_{p2}\, G_2}$$

$$\tau_{max_1} = \frac{T_1\, d_1/2}{I_{p1}} = \frac{2000\, d_1/2}{\pi/32\, d_1^4}$$

$$\tau_{max_2} = \frac{1000\, d_2/2}{\pi/32\, d_2^4}$$