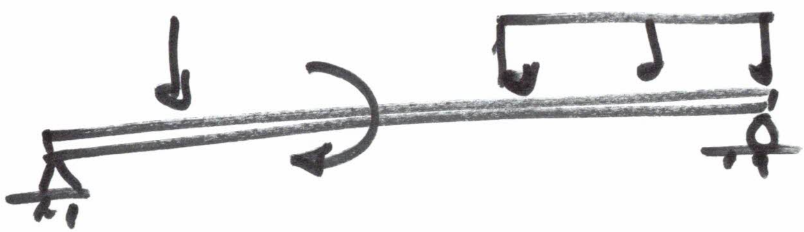
V Equilibrium of Beams

Beam – long, slender, supports transverse loads



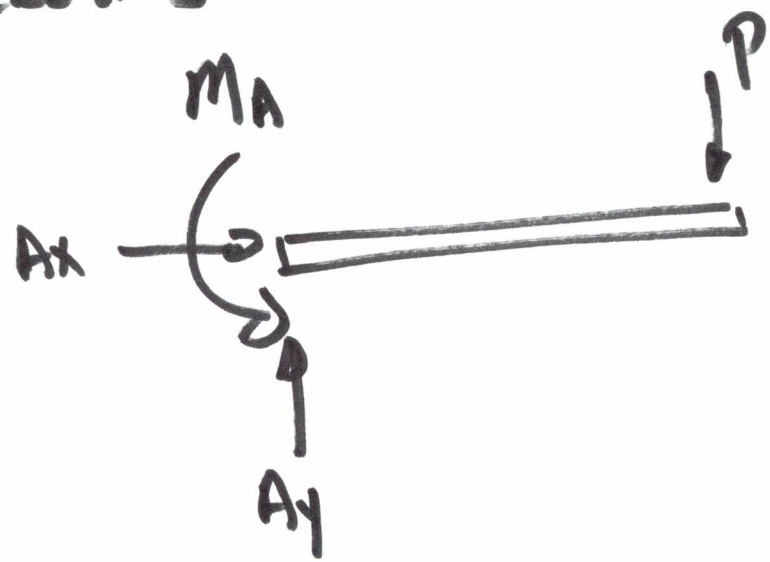
A. Review of Statics

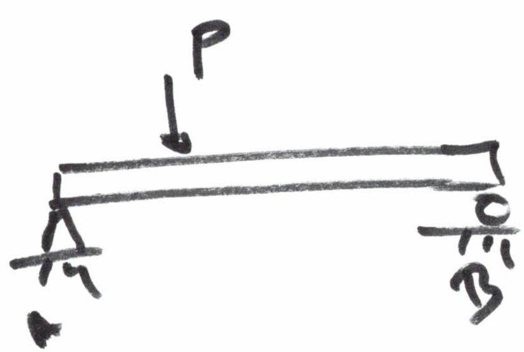
1. FBD's & Reactions (2-D Planar Statics)

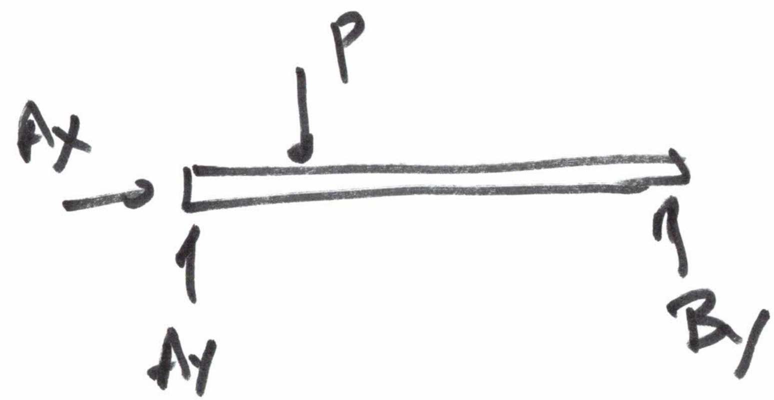
$\sum M_A = 0$

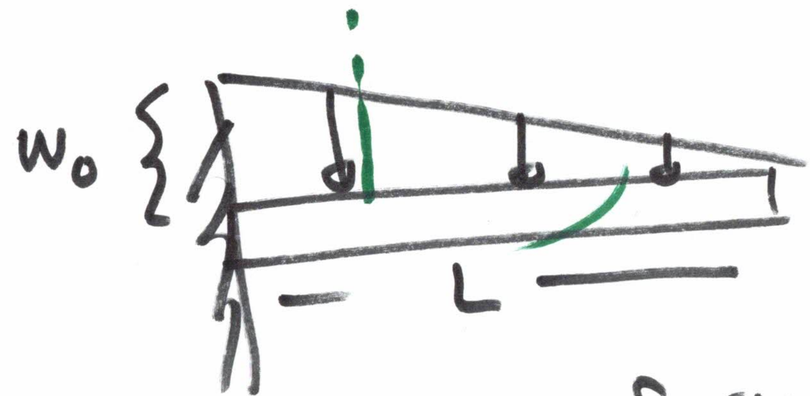
$\sum F_y = 0$

$\sum F_x = 0$

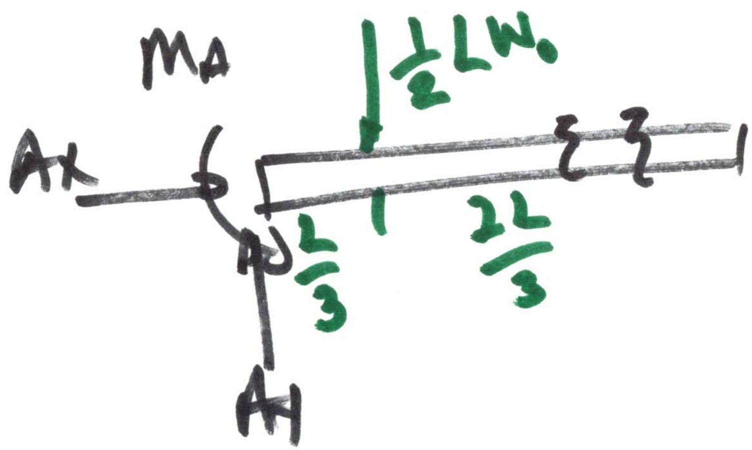




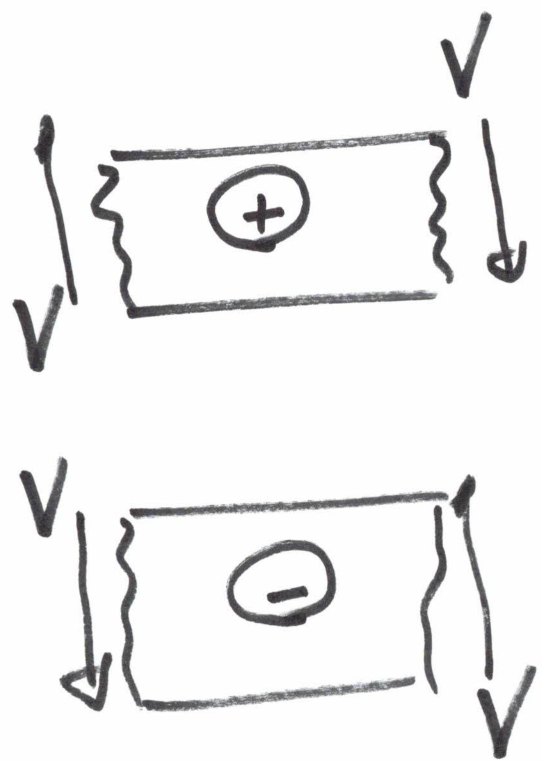


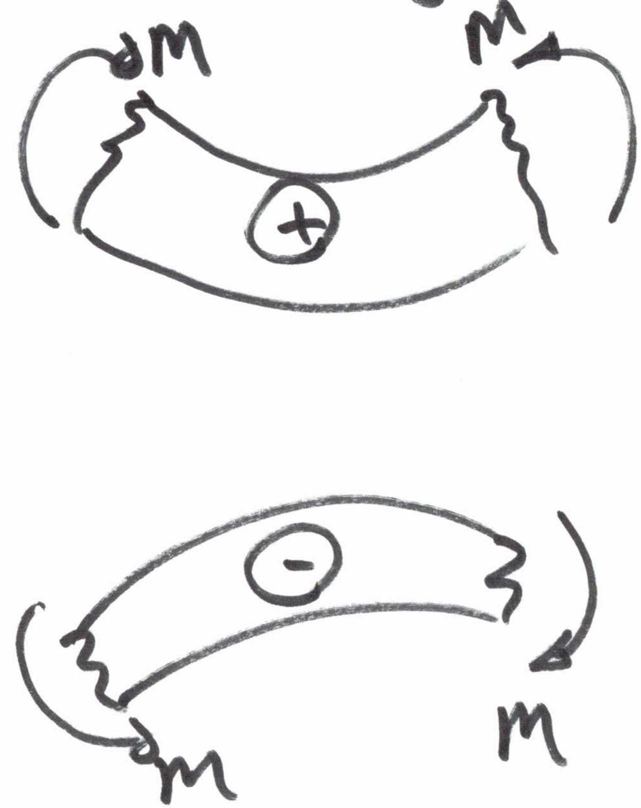


For purposes of finding RXNS, replace the distributed load with a force equal to the area of the distribution, located at the centroid of the distribution.

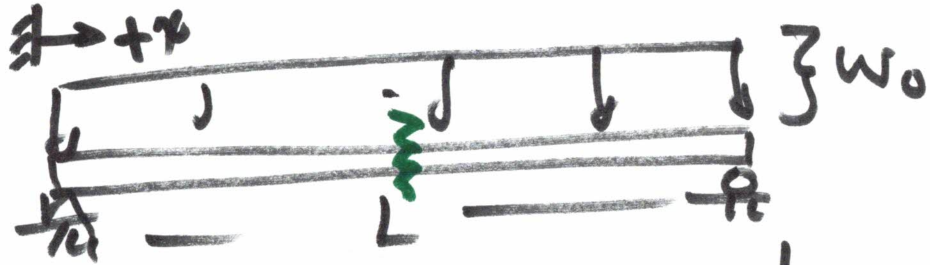


2. Internal Shear force & Bending Moment



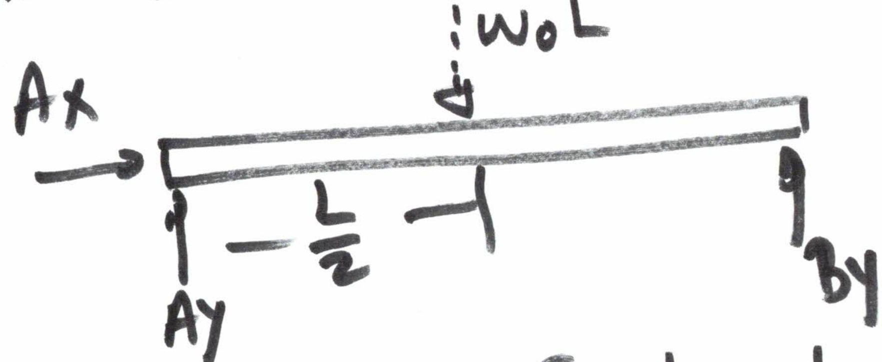


EXAMPLE:



Find V & M @ mid-span of beam

① Find Beam RXNS:



$\overrightarrow{\Sigma F_x}$: $A_x = 0$

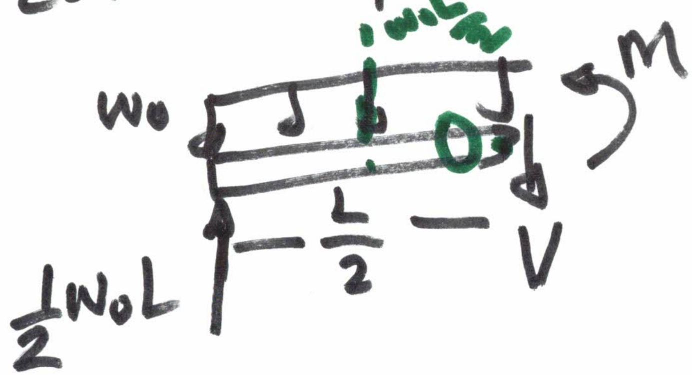
$\Sigma F_y \uparrow +$: $A_y + B_y - w_0 L = 0$

$\circlearrowleft + \Sigma M_A$: $B_y \cdot L - w_0 L \frac{L}{2} = 0$

$B_y = \frac{w_0 L}{2}$

$A_y = \frac{w_0 L}{2}$

② cut at point of interest

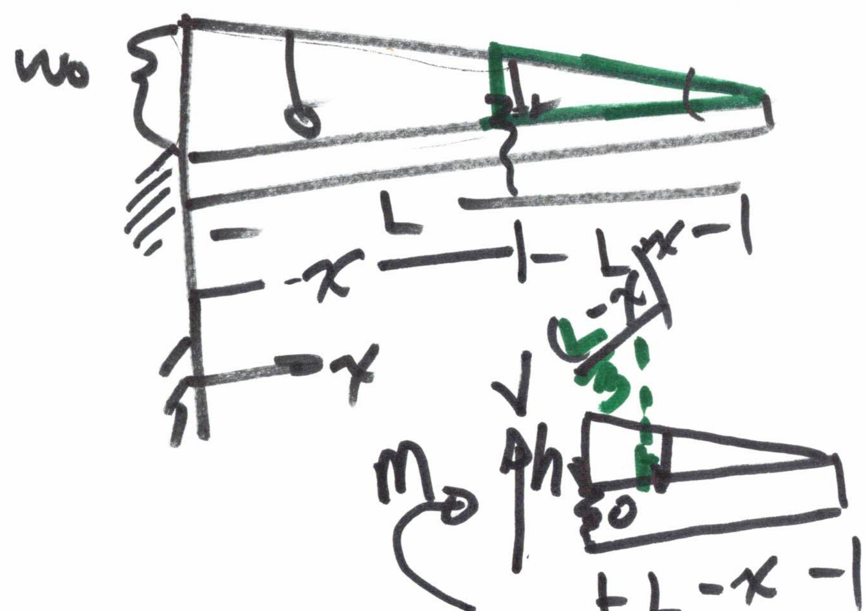


$\Sigma F_y \uparrow^+$ $\quad \frac{1}{2}w_0 L - w_0 \frac{L}{2} - V = 0 \qquad V = 0$

$\circlearrowleft^+ \Sigma M_0$ $\quad +M - \frac{1}{2}w_0 L \cdot \frac{L}{2} + \frac{w_0 L}{2}\frac{L}{4} = 0$

$$M = \frac{1}{8} w_0 L^2$$

EXAMPLE:



Find $V(x)$ & $M(x)$

$$\frac{h}{L-x} = \frac{w_0}{L}$$

$$h = \frac{w_0 (L-x)}{L}$$

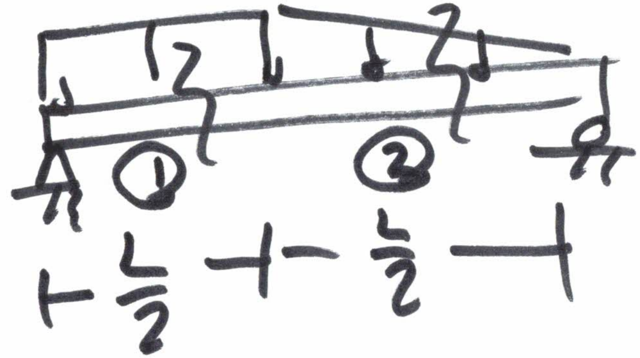
$\Sigma F_y \uparrow^{+}$ $\quad V - \frac{1}{2}(L-x)\frac{w_0(L-x)}{L} = 0$

$$V = \frac{1}{2}\frac{w_0}{L}(L-x)^2$$

$^{+}\circlearrowleft \Sigma M_o$ $\quad -m - \frac{1}{2}(L-x)\frac{w_0(L-x)}{L}\left(\frac{L-x}{3}\right) = 0$

$$m = -\frac{1}{6}\frac{w_0}{L}(L-x)^3$$

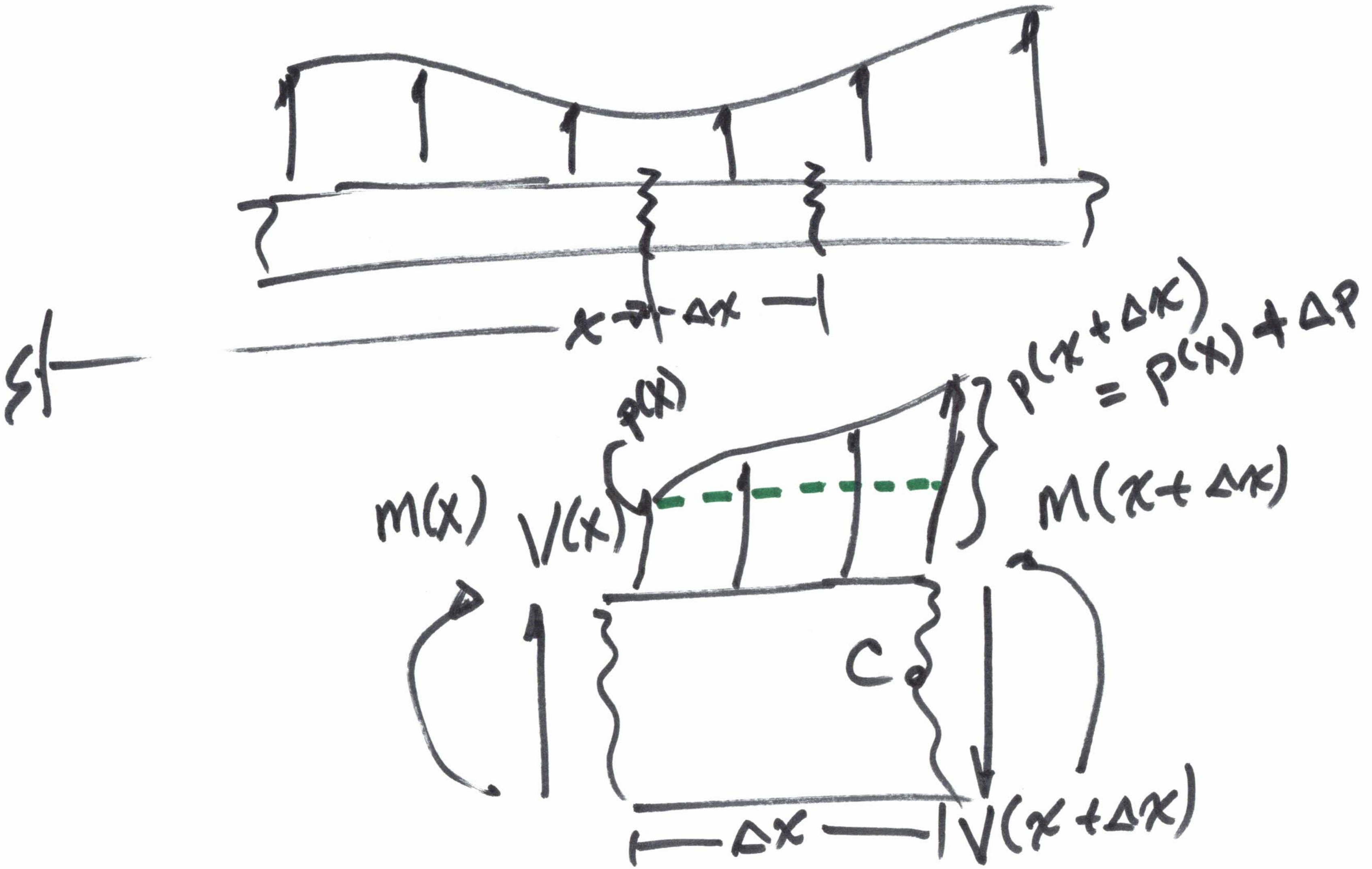
Note: $V = \frac{dm}{dx}$



Find $V_1(x)$ $M_1(x)$

& $V_2(x)$ $M_2(x)$

B. Load - Shear - Moment Relationship



1. load- shear

$$\Sigma F_y \uparrow^{+} \quad V(x) - V(x+\Delta x) + p(x)\Delta x + c_1 (\Delta p)(\Delta x) = 0$$

$$V(x+\Delta x) - V(x) = p(x)\Delta x + c_1 \Delta p \Delta x$$

$$\lim_{\Delta x \to 0} \left[\frac{V(x+\Delta x) - V(x)}{\Delta x} = p(x) + c_1 \Delta p \right]$$

$$\frac{dV}{dx} = p(x) \qquad \text{usually } V(x) = \int p(x)\,dx$$

2 Shear-Moment

$$\overset{+}{\circlearrowright} \Sigma M_c \quad M(x+\Delta x) - M(x) - V(x)\Delta x - p(x)\Delta x \cdot \frac{\Delta x}{2} - C_2 \Delta p \,\Delta x^2 = 0$$

$$\lim_{\Delta x \to 0} \cancel{m(x)} \left[\frac{M(x+\Delta x) - M(x)}{\Delta x} = \frac{V(x)}{\cancel{\Delta x}} + p(x)\frac{\Delta x}{2} + C_2 \Delta p \Delta x \right]$$

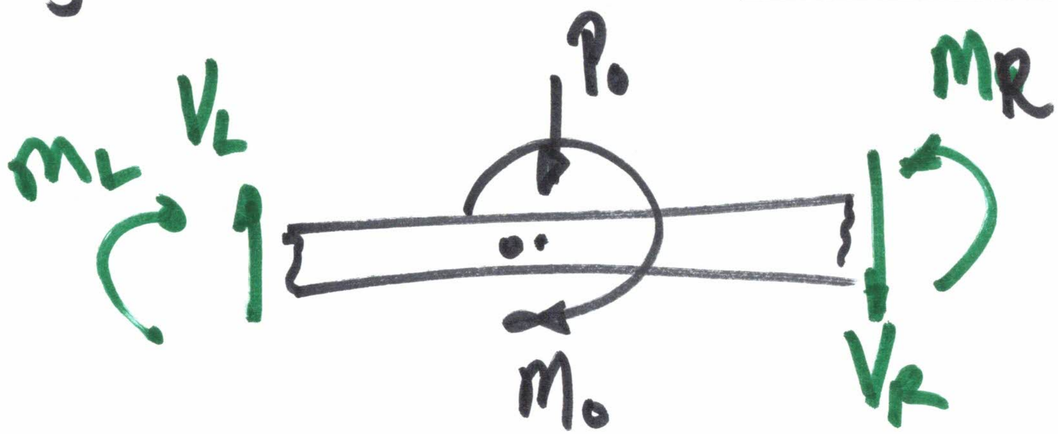
$$\frac{dM}{dx} = V(x)$$

usually use $\quad M = \int V(x)\, dx$

$$V = \int p(x)\,dx + C_{onst_1}$$

$$M = \int V(x)\,dx + Const_2$$

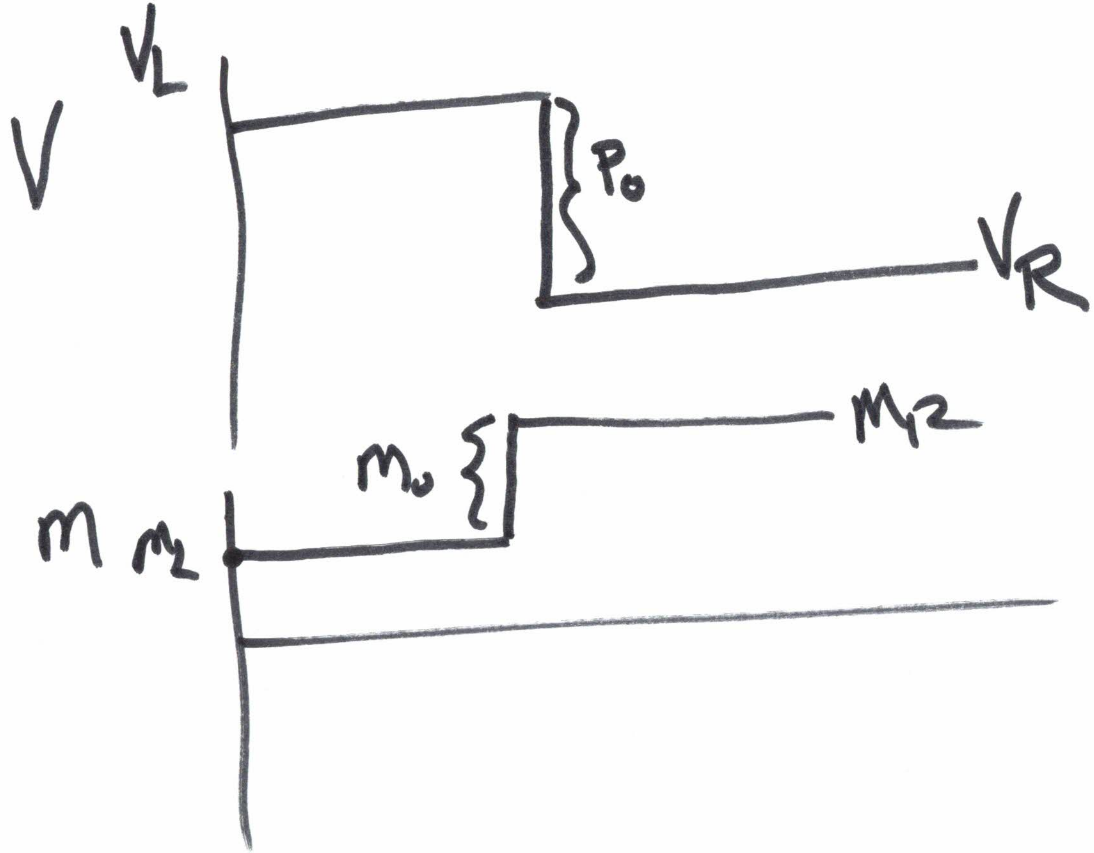
Concentrated forces & moments cause "jumps" in the equation



$\curvearrowleft + \sum M_0$

$+M_R - M_0 - M_L = 0$

$M_R = M_L + M_0$


V
V_L
p_0
V_R
m_0
m_R
m
m_L