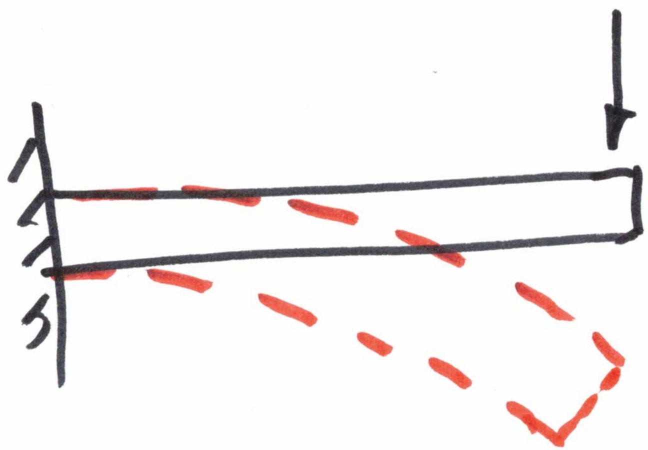
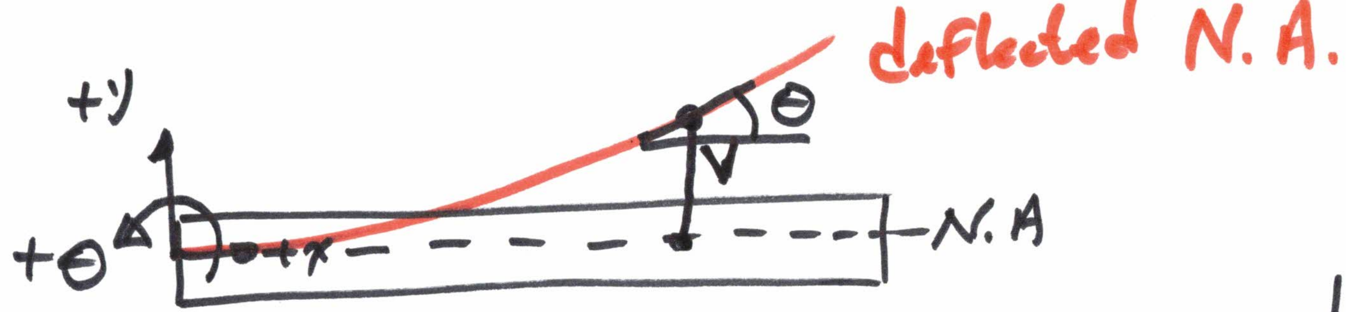
VII Deflection of Beams



Beams deflect due to loads. Our reference will be the neutral axis.

We will be able to calculate the <u>deflection</u> and <u>slope</u> of the beam anywhere along its length.



v – deflection   positive upward

θ – slope   CCW positive

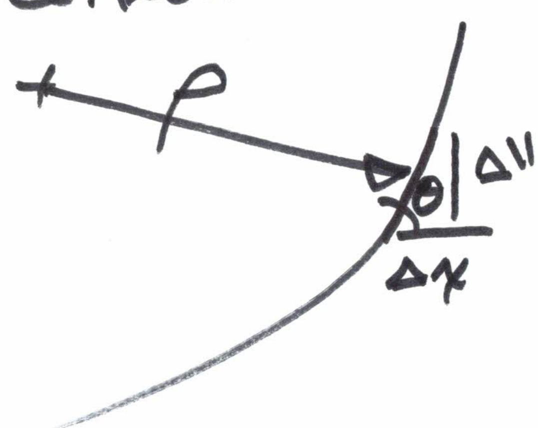
A. Deflection Curve Relation

$$M = \frac{EI}{\rho}$$

ρ ← radius of curvature

1. Slope & deflection

$$\tan\theta = \frac{\Delta v}{\Delta x}$$



$$\Theta = \tan^{-1}\frac{\Delta v}{\Delta x} = \tan^{-1}\frac{dv}{dx}$$

if $\frac{dv}{dx} << 1$ then $\Theta = \frac{dv}{dx} = v'$

2. curvature relation from Calculus

$$\frac{1}{\rho} = \frac{\frac{d^2v}{dx^2}}{\left[1 + \left(\frac{dv}{dx}\right)^2\right]^{3/2}} \approx \frac{d^2v}{dx^2} = v''$$

~~if~~ if $\frac{dv}{dx}$ is small

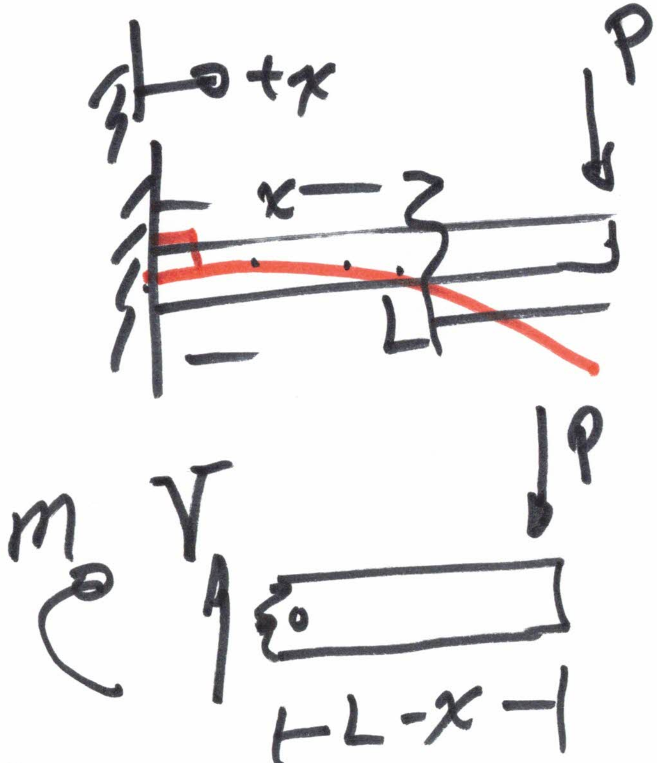
3. moment curvature again

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{d^2v}{dx^2} = v''$$

$$EI\,v'' = M$$

moment equation

Integrate to get $V' = \theta$
& V



$$EIV'' = M$$

$\circlearrowleft + \Sigma M_o$ $\quad -M - P(L-x) = 0$

$$M = -P(L-x)$$

$$M = -PL + Px$$

Assume EI is Constant

$$EIV'' = M = -PL + Px$$

$$EIV' = -PLx + \frac{Px^2}{2} + C_1$$

$$EIV = -PL\frac{x^2}{2} + \frac{Px^3}{6} + C_1 x + C_2$$

Boundary Conditions (BC's)

$$X=0,\ V=0 \Rightarrow C_2=0$$

$$X=0,\ V'=0 \Rightarrow C_1=0$$

$$EIV' = EI\theta = -PLx + \frac{Px^2}{2}$$

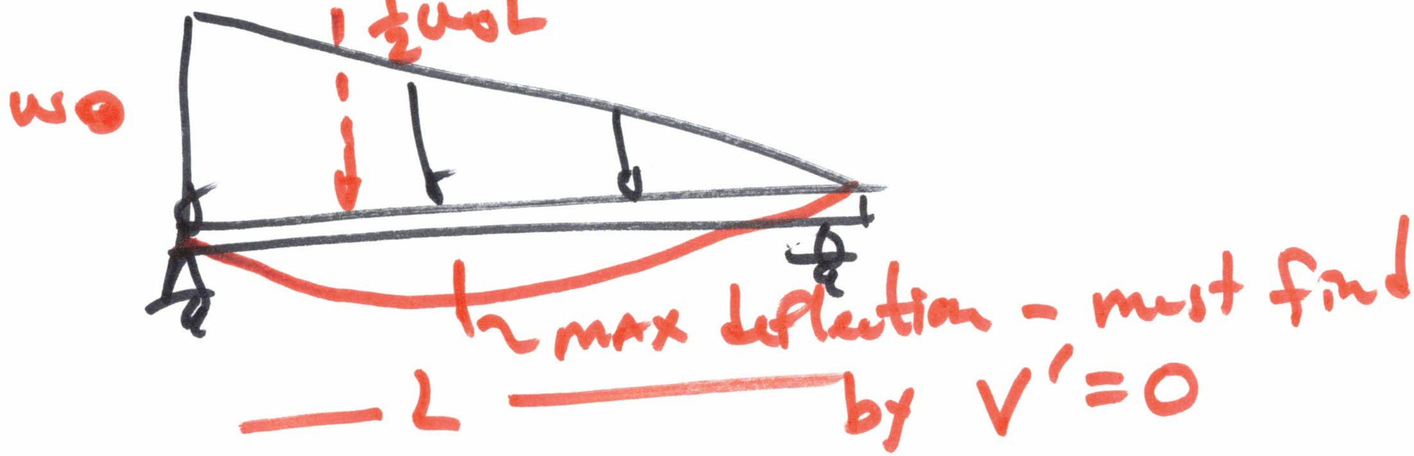
$$EIV = \frac{-PLx^2}{2} + \frac{Px^3}{6}$$

In this case V_{MAX} is @ $X=L$

$$EIV_{MAX} = \frac{-PL\,L^3}{2} + \frac{PL^3}{6} = \left[\frac{-3}{6}+\frac{1}{6}\right]PL^3$$

$$EIV_{MAX} = -\frac{1}{3}PL^3$$

$$V_{MAX} = \frac{-PL^3}{3EI}$$



w_0
$\frac{1}{2}w_0L$
max deflection - must find by $V'=0$
L

$$EIV'' = M = \frac{1}{2} w_0 L x - w_0 \frac{x^2}{2}$$

$$EIV' = \frac{1}{2} w_0 L \frac{x^2}{2} - \frac{w_0 x^3}{6} + C_1$$

$$EIV = \frac{1}{2} w_0 L \frac{x^3}{6} - \frac{w_0 x^4}{24} + C_1 x + C_2$$

BC's $X = 0,\ V = 0$

$X = L,\ V = 0$

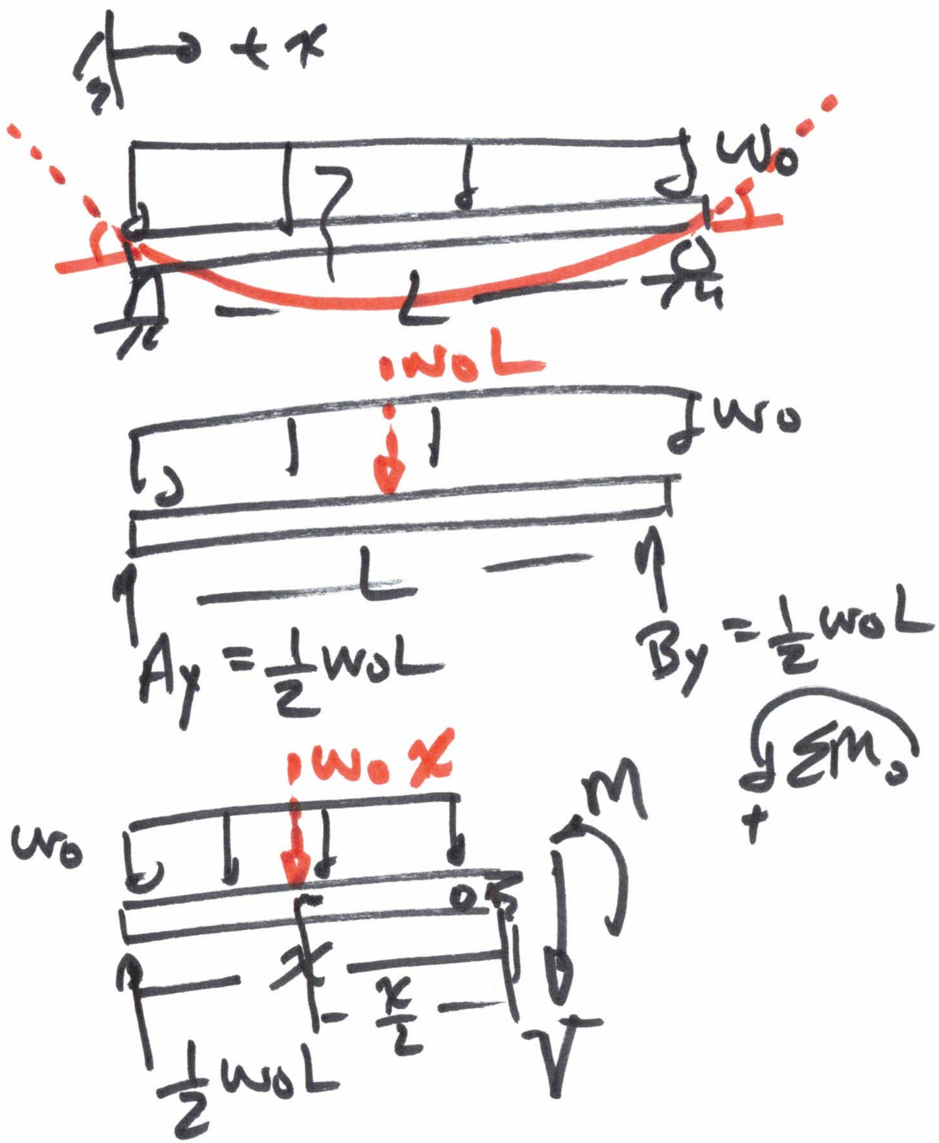
$X = 0,\ V = 0 \quad 0 = 0 - 0 + C_1 0 + C_2$

$C_2 = 0$

$X = L,\ V = 0 \quad 0 = \frac{1}{2} w_0 L \frac{L^3}{6} - \frac{w_0 L^4}{24} + C_1 L$

$$C_1 = -\frac{1}{2} w_0 \frac{L^3}{6} + \frac{w_0 L^3}{24}$$

EXAMPLE:



Find M, then

$EIv'' = M$

$$+M + w_o x \frac{x}{2} - \frac{1}{2} w_o L x = 0$$