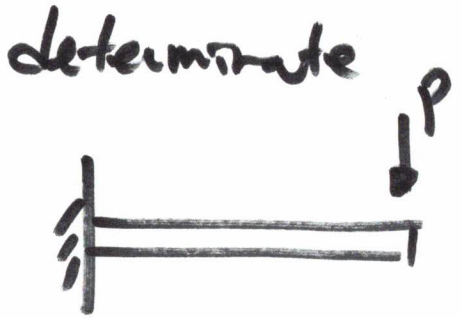
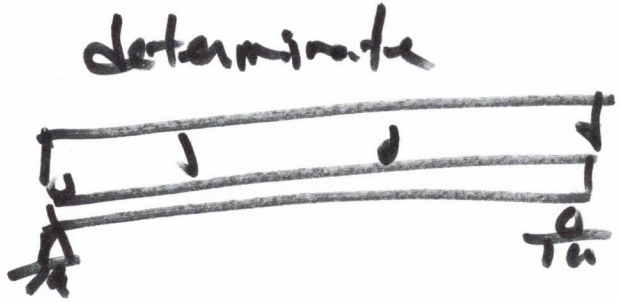
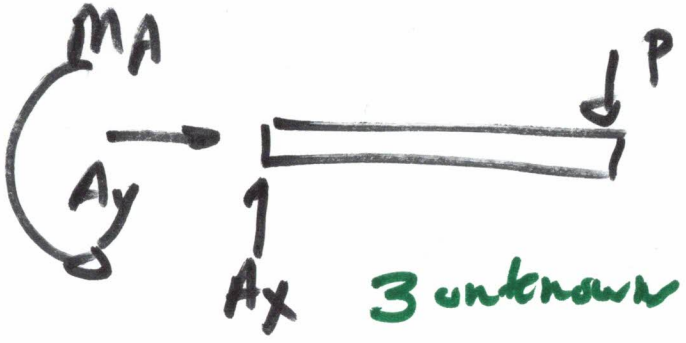
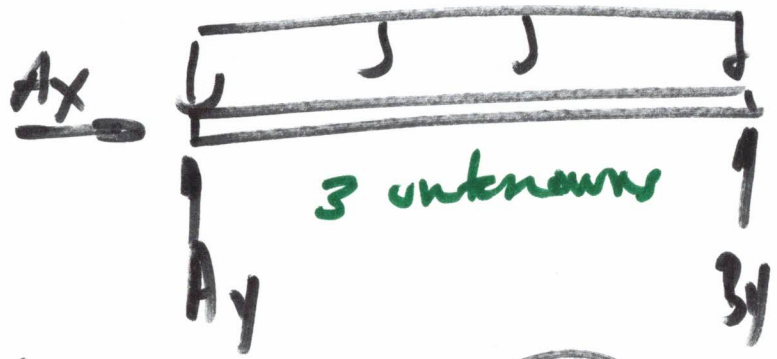
C. Statically Indeterminate Beams

— involve more support reactions than can be determined by statics alone

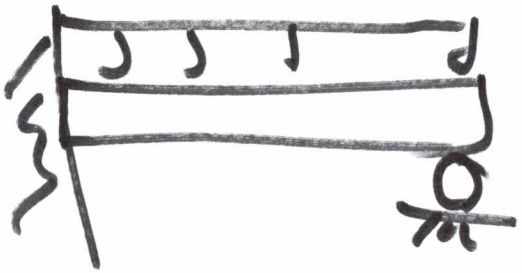


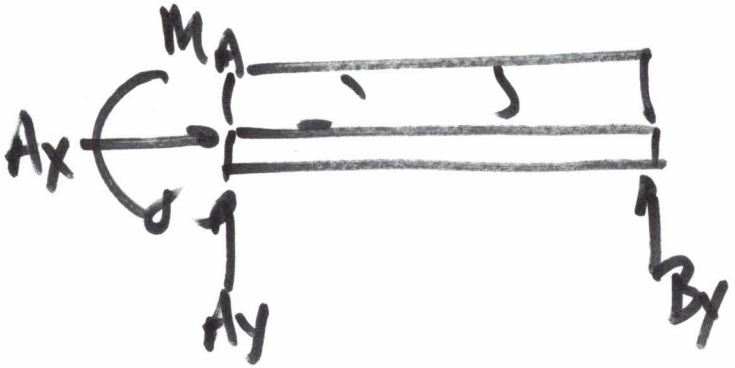




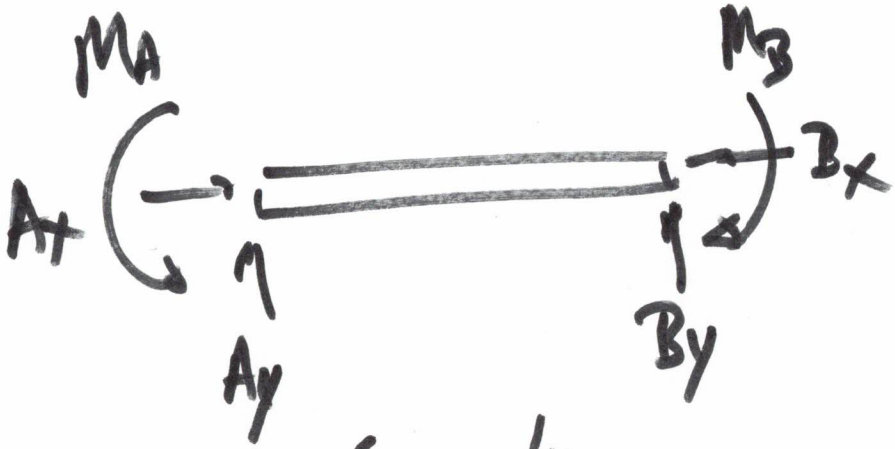


Statics eqns (2-D): $\overrightarrow{\Sigma F_x} \rightarrow^{+}$, $\Sigma F_y \uparrow^{+}$, $\circlearrowleft^{+} \Sigma M_{pt} = 0$



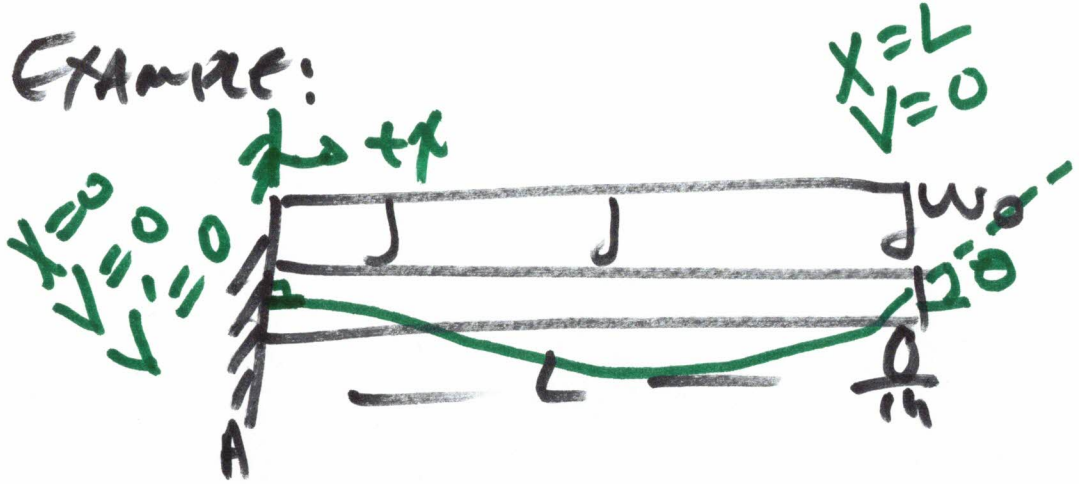


4 unknowns
(3 non-trivial)
2 statics eqns

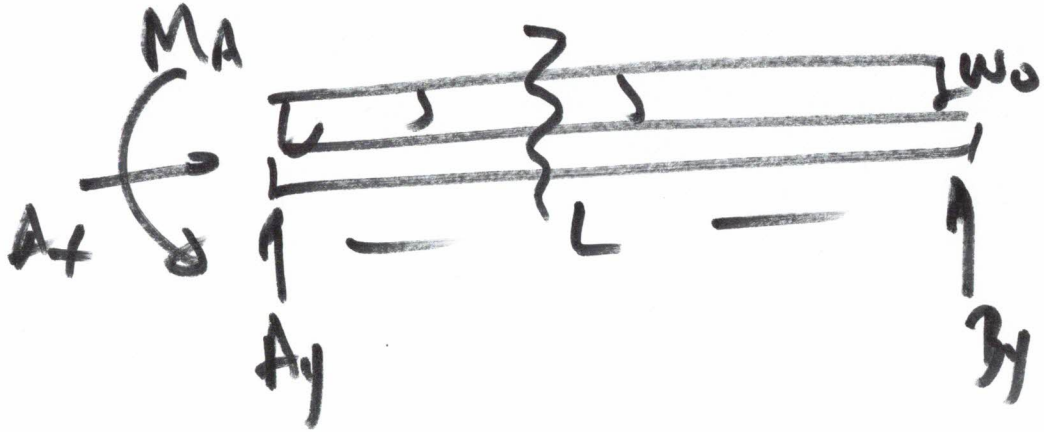


6 unknowns
(4 non-trivial)
2 statics eqns

EXAMPLE:



Find RXN's in terms of applied load



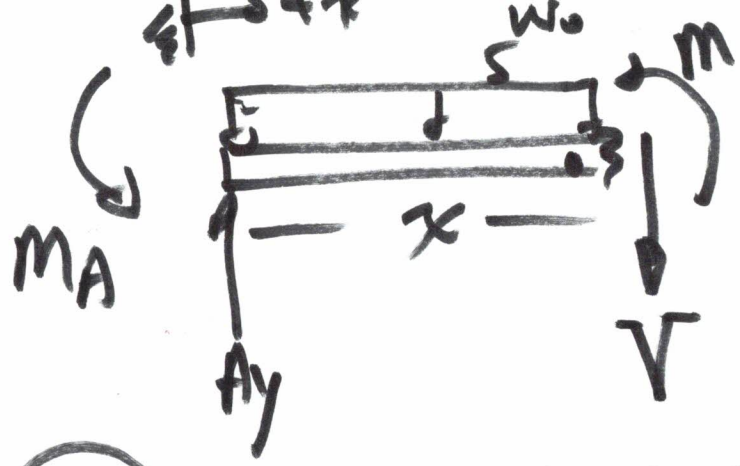
① Apply statics equations

$$\xrightarrow{+}\ \Sigma F_x = 0 \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0 \quad A_y + B_y - w_0 L = 0 \quad (1)$$

$$+\circlearrowleft \Sigma M_A = 0 \quad B_y L - w_0 L \cdot \frac{L}{2} + M_A = 0 \quad (2)$$

3 unknowns



② Find V & V' eqns for beam

$$+\circlearrowleft \Sigma M: \quad M + w_0 x \cdot \frac{x}{2} - A_y x + M_A = 0$$

$$EIV'' = M = A_y x - w_0 \frac{x^2}{2} - M_A$$

$$EI\theta = EIV' = A_y \frac{x^2}{2} - w_0 \frac{x^3}{6} - M_A x + C_1$$

$$EIV = A_y \frac{x^3}{6} - w_0 \frac{x^4}{24} - M_A \frac{x^2}{2} + C_1 x + C_2$$

5 unknowns A_y, B_y, M_A, C_1, C_2

2 eqns from statics, 3 others from Boundary Conditions (B.C.'s)

③ Apply B.C.'s

$x=0, V=0 \Rightarrow C_2 = 0$

$x=0, V' = 0 \Rightarrow C_1 = 0$

$x = L, V = 0$

$$0 = \frac{A_y L^3}{6} - \frac{w_0 L^4}{24} + M_A \frac{L^2}{2} \quad (3)$$

On exams, stop after you apply the B.C.'s.
(plug in B.C.'s)

Some homework may make you solve
M_A, A_y, B_y in terms of w_0

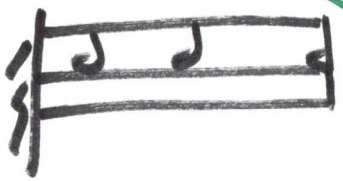
EXAM 3

1) Beam deflection
2) Mohr's Circle
3) Combined loading

On EXAM 3

Determinate

1- Region

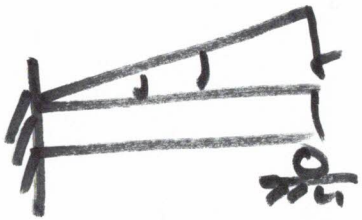


Find C_1 & C_2 from B.C.'s
(Do all algebra)

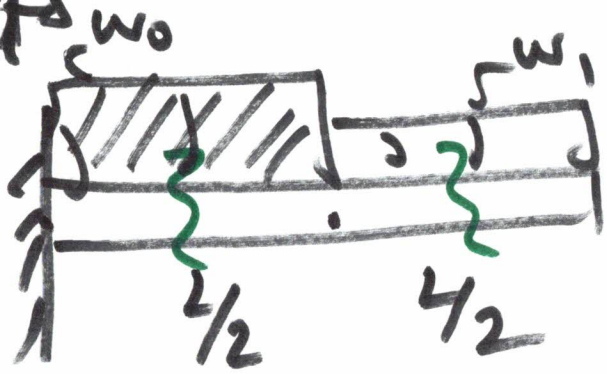
2- Region

Matching B.C.'s
Support B.C.'s
& Matching B.C.'s
C_1, C_2, C_3, C_4
Apply B.C.'s to get 4 eqns (step)

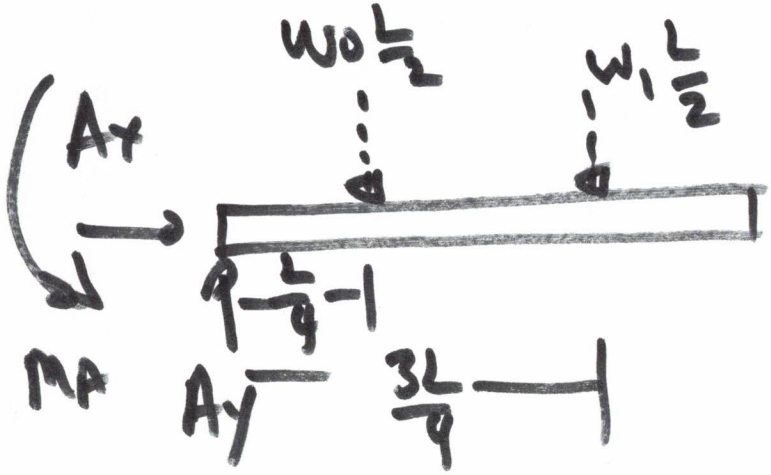
Indeterminate



Support B.C.'s
Statics eqns
generate eqns By applying B.C.'s (step)



Determine



$\rightarrow \sum F_x \quad A_x = 0$

$\uparrow \sum F_y \quad A_y - w_0 \frac{L}{2} - w_1 \frac{L}{2} = 0$

Find A_y

$\circlearrowleft \sum M_A \quad +M_A - w_0 \frac{L}{2} \frac{L}{4} - w_1 \frac{L}{2} \frac{3L}{4} =$

Find M_A

Support B.C.'s

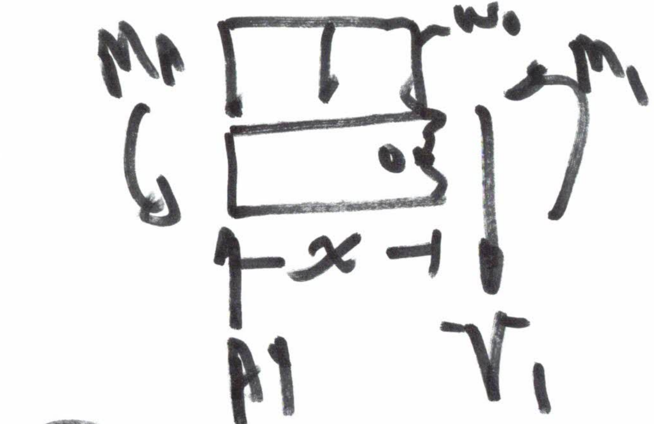
$x = 0 \quad V_1 = 0$

$x = 0 \quad V_1' = 0$

Matching B.C.'s

$x = \frac{L}{2} \quad V_1 = V_2$

$x = \frac{L}{2} \quad V_1' = V_2'$

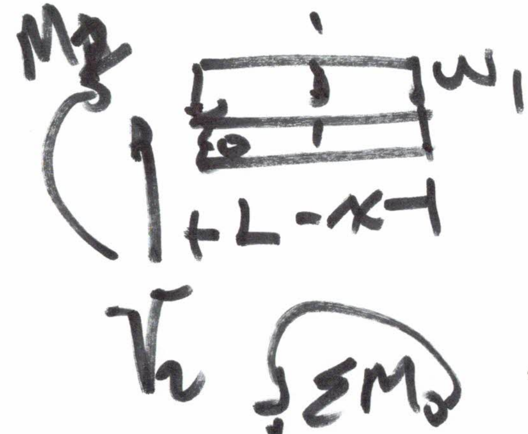


$\circlearrowleft \sum M_0 \quad M_1 + M_A - A_y x + w_0 x \frac{x}{2} = 0$

$EIv_1'' = M_1 = A_y x - M_A - w_0 \frac{x^2}{2}$

$EIv_1' = A_y \frac{x^2}{2} - M_A x - w_0 \frac{x^3}{6} + C_1$

$EIv_1 = A_y \frac{x^3}{6} - M_A \frac{x^2}{2} - w_0 \frac{x^4}{24} + C_1 x + C_2$



$\circlearrowleft \sum M_0 \quad -M_2 - w_1 \frac{(L-x)^2}{2} = 0$

$EIv_2'' = M_2 = -w_1 \frac{(L-x)^2}{2}$

$EIv_2' = +w_1 \frac{(L-x)^3}{6} + C_3$

$EIv_2 = -w_1 \frac{(L-x)^4}{24} + C_3 x + C_4$

$x=0,\ v_1=0 \Rightarrow \underline{C_2=0}$

$x=0,\ v_1'=0 \Rightarrow \underline{C_1=0}$

$x=\frac{L}{2},\ v_1'=v_2'$ (or $EIv_1'=EIv_2'$)

$$A_y\frac{\left(\frac{L}{2}\right)^2}{2}-M_A\frac{L}{2}-w_0\frac{\left(\frac{L}{2}\right)^3}{6}=w_1\frac{\left(\frac{L}{2}\right)^3}{6}+C_3$$

$x=\frac{L}{2}\quad v_1=v_2$

$$A_y\frac{\left(\frac{L}{2}\right)^3}{6}-M_A\frac{\left(\frac{L}{2}\right)^2}{2}-w_0\frac{\left(\frac{L}{2}\right)^4}{24}=-w_1\frac{\left(\frac{L}{2}\right)^4}{24}+C_3\frac{L}{2}+C_4$$

4 eqns for C_1, C_2, C_3, C_4

On EXAM 3,
you will have 1-Region Indeterminate Beam

On final exam, you will have
either 2-region Indeterminate
or
Indeterminate Beam