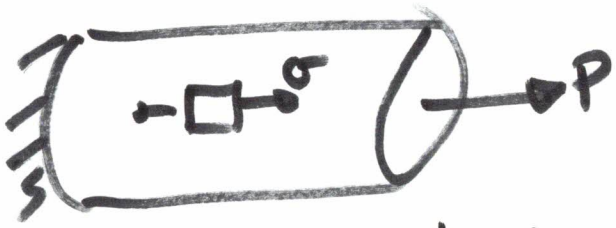
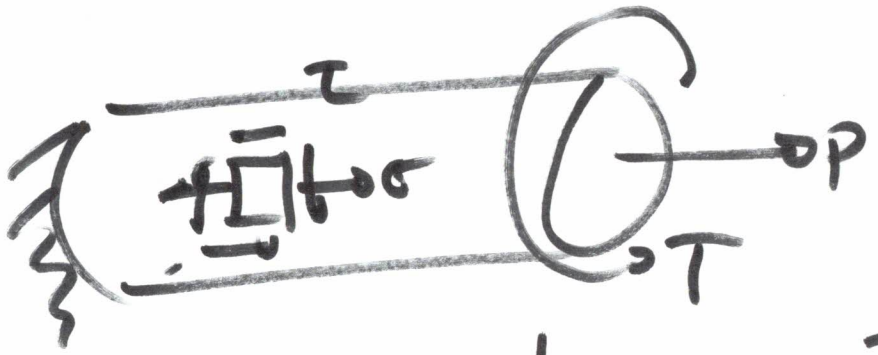
VIII Stress & Strain Transformations



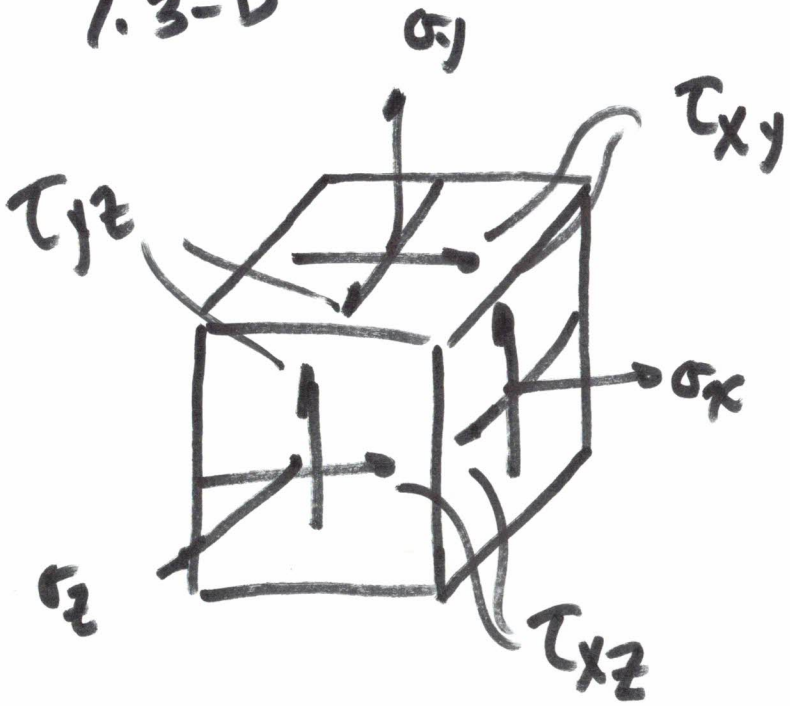
what angle for max shear stress?
what angle for max normal stress?



How about this case?

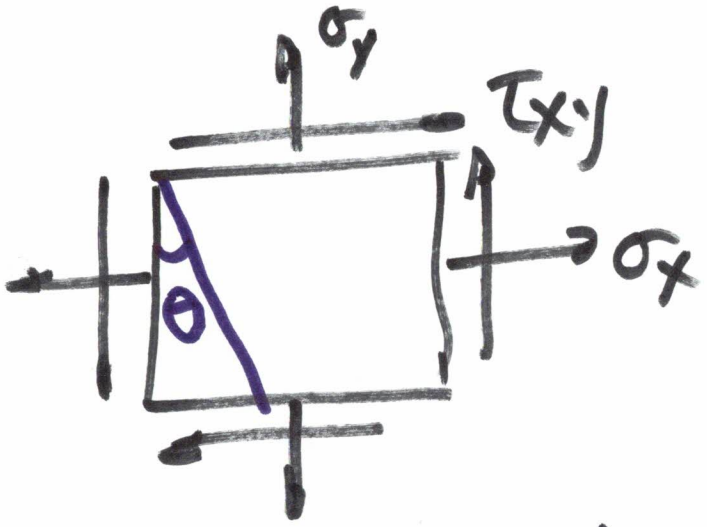
A. States of Stress

1. 3-D



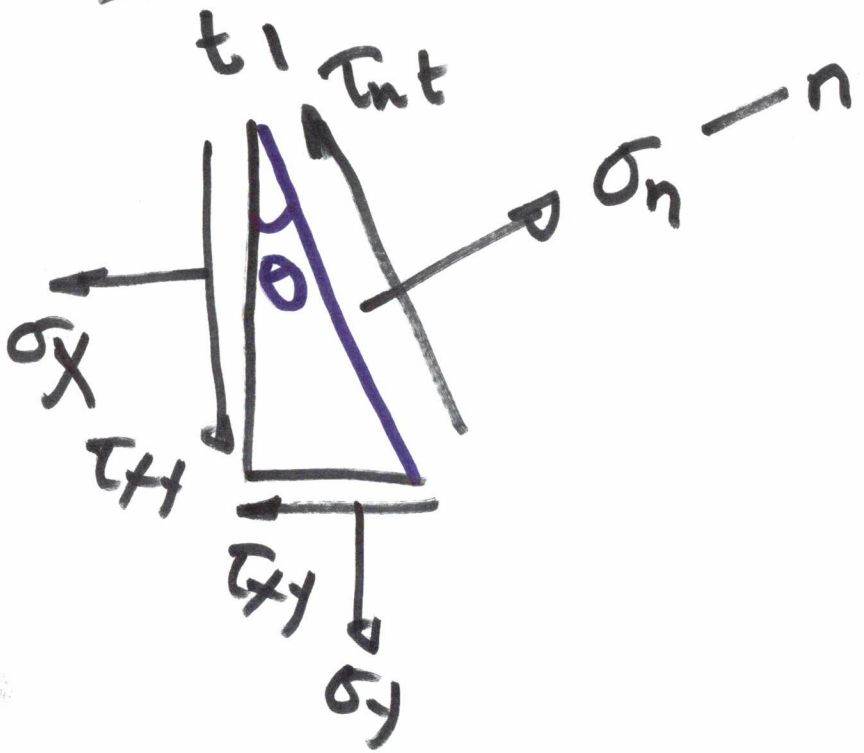
Most general state of stress.

2 plane Stress state

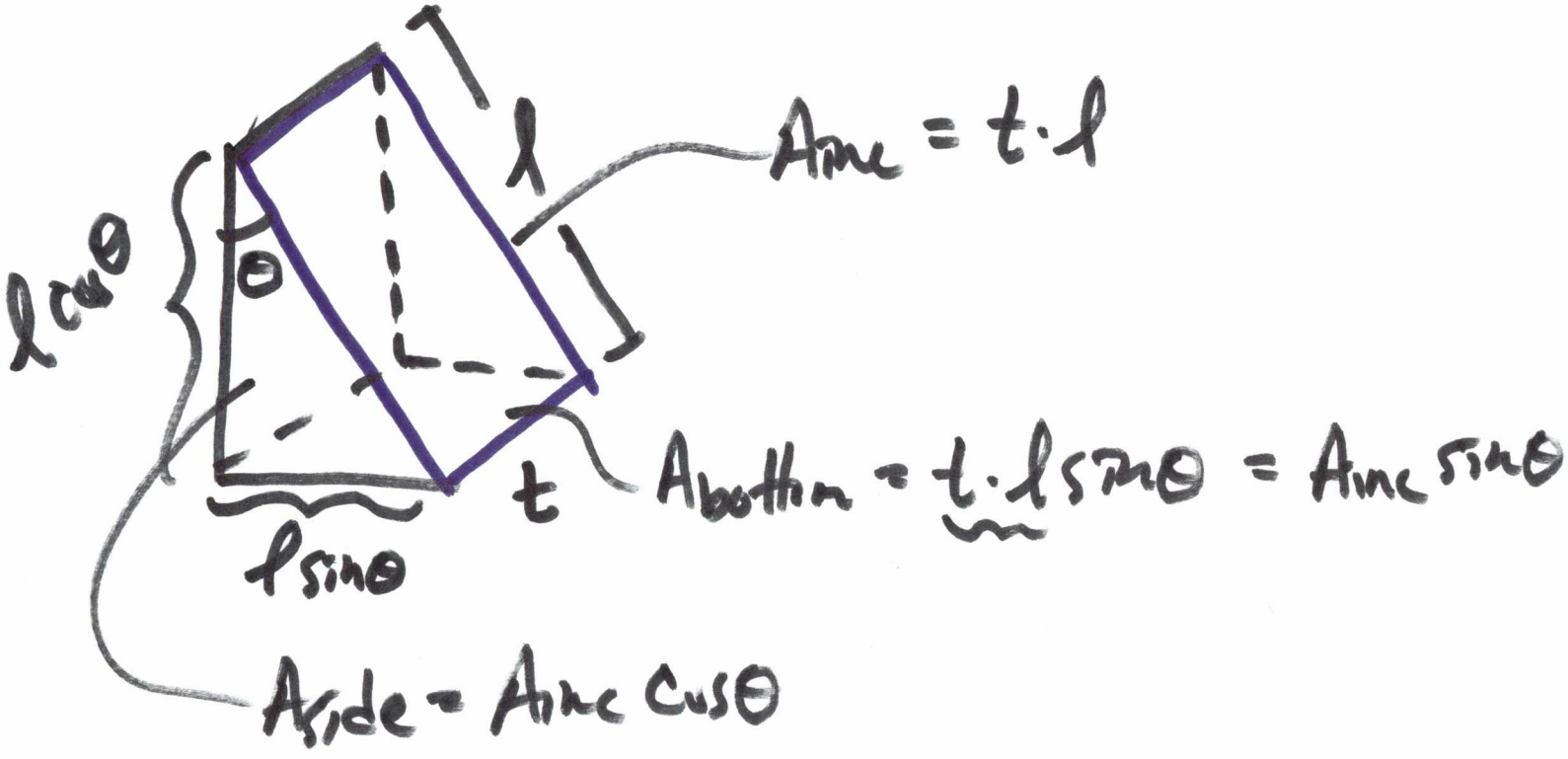


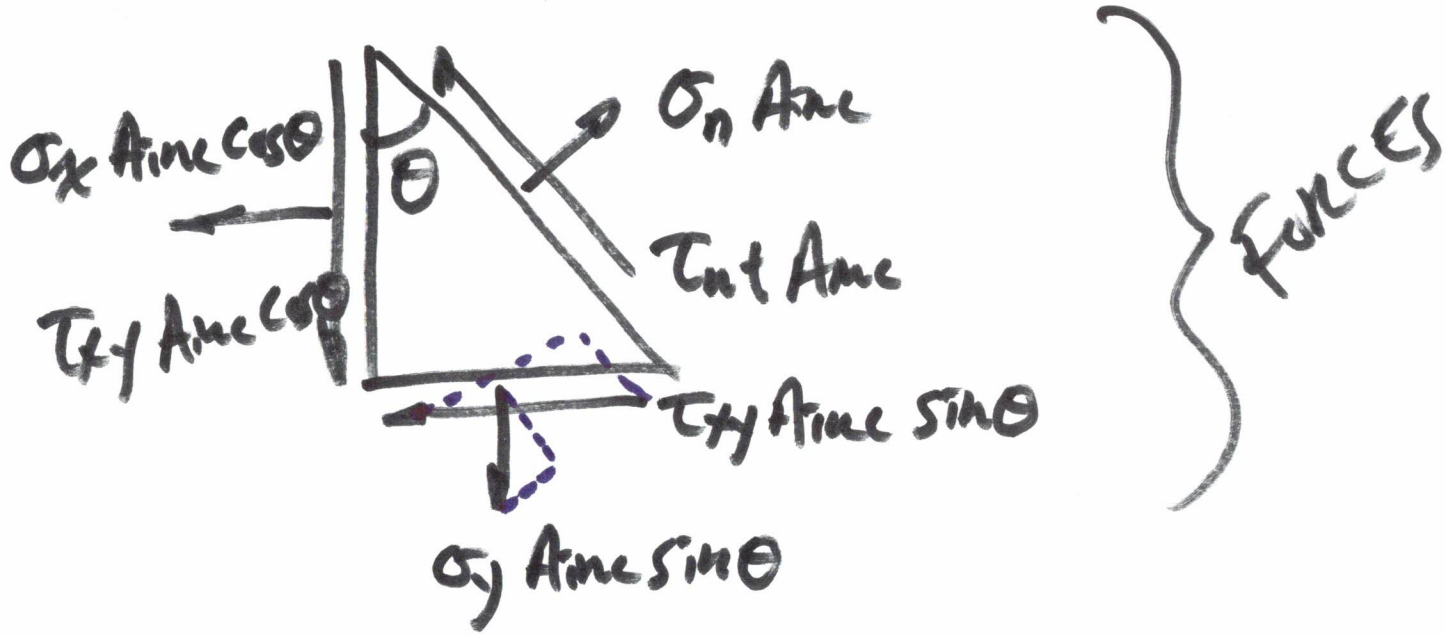
$\sigma_z = 0$, $\tau_{xz}, \tau_{yz} = 0$

B. Transformations of Plane Stress



convert stresses into forces


l
θ
l cosθ
l sinθ
t
$A_{inc} = t \cdot l$
$A_{bottom} = t \cdot l \sin\theta = A_{inc} \sin\theta$
$A_{side} = A_{inc} \cos\theta$


θ
$\sigma_n A_{inc}$
$\tau_{nt} A_{inc}$
$\sigma_x A_{inc} \cos\theta$
$\tau_{xy} A_{inc} \cos\theta$
$\tau_{xy} A_{inc} \sin\theta$
$\sigma_y A_{inc} \sin\theta$
FORCES

$\Sigma F_n \uparrow^+$ $\sigma_n A_{inc} - (\sigma_x A_{inc}\cos\theta)\cos\theta - (\sigma_y A_{inc}\sin\theta)\sin\theta$

$- (\tau_{xy} A_{inc}\cos\theta)\sin\theta - (\tau_{xy} A_{inc}\sin\theta)\cos\theta = 0$

$\Sigma F_t \nearrow^+$ $\tau_{nt} A_{inc} + (\sigma_x A_{inc}\cos\theta)\sin\theta - (\sigma_y A_{inc}\sin\theta)\cos\theta$

$- (\tau_{xy} A_{inc}\cos\theta)\cos\theta + (\tau_{xy} A_{inc}\sin\theta)\sin\theta = 0$

$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

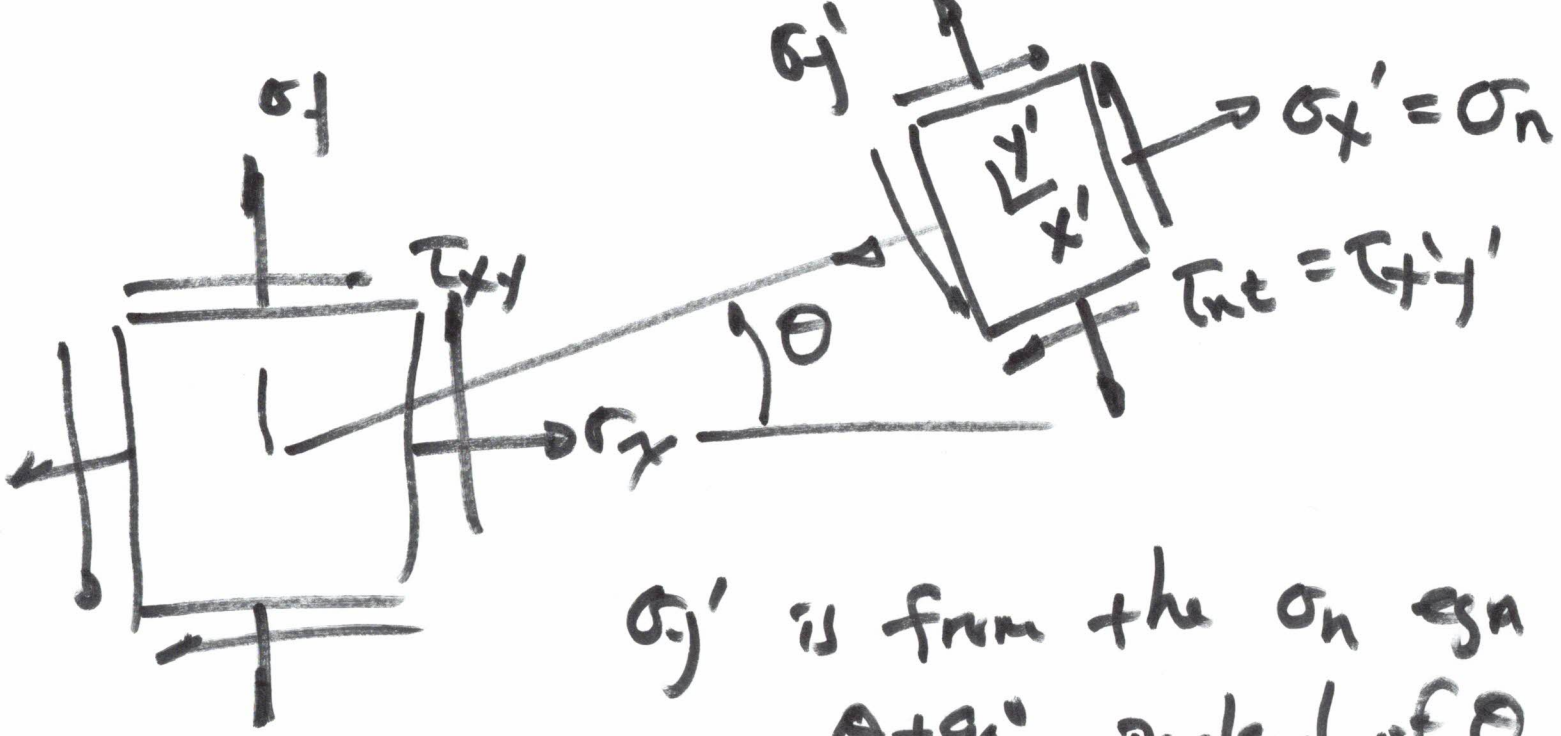
$\tau_{nt} = -\sigma_x \sin\theta\cos\theta + \sigma_y \sin\theta\cos\theta + \tau_{xy}\cos^2\theta - \tau_{xy}\sin^2\theta$

$2\sin\theta\cos\theta = \sin 2\theta$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$



$\sigma_{y'}$ is from the σ_n eqn with $\theta + 90°$ instead of θ

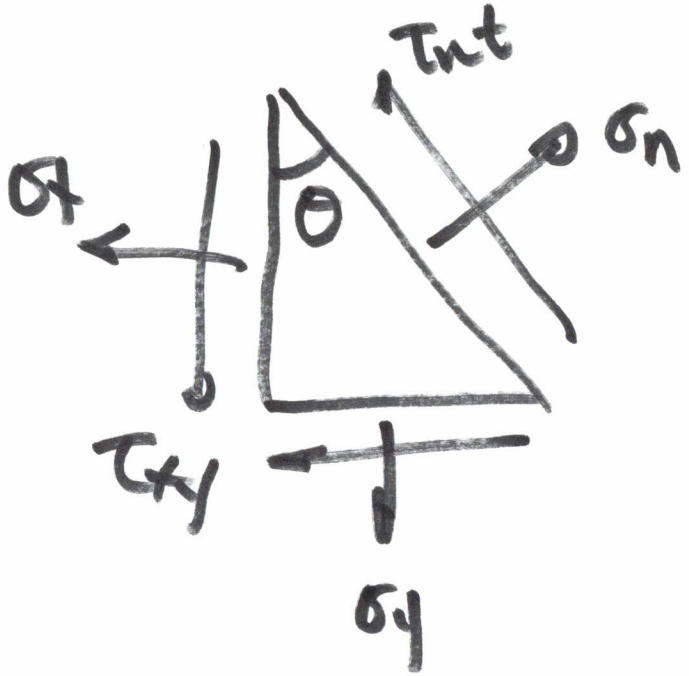
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

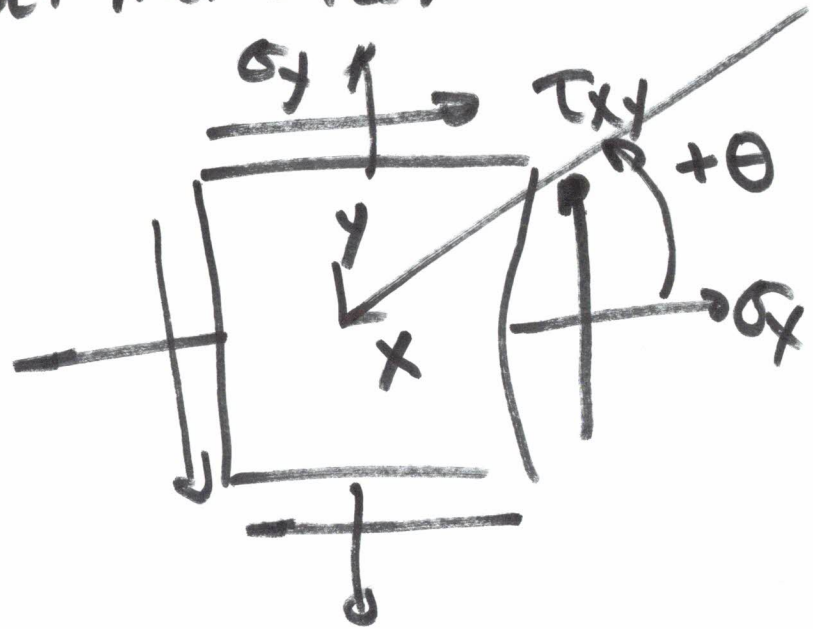
$$\sigma_n = \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

Dont memorize!





$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy} \cos 2\theta$$

note $\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$

EXAMPLE 8.3-5

$\sigma_x = 4.8$ ksi, $\sigma_y = -1.2$
$\tau_{xy} = -4$

$$\sigma_{x'} = \frac{(4.8 - 1.2)}{2} + \left(\frac{4.8 - (-1.2)}{2}\right)\cos 2(20^\circ) + (-4)\sin 2(20^\circ)$$

$$\sigma_{y'} = \frac{4.8 + (-1.2)}{2} - \left(\frac{4.8 - (-1.2)}{2}\right)\cos 2(20^\circ) - (-4)\sin 2(20^\circ)$$

$$\tau_{x'y'} = -\left(\frac{4.8 - (-1.2)}{2}\right)\sin 2(20^\circ) + (-4)\cos 2(20^\circ)$$