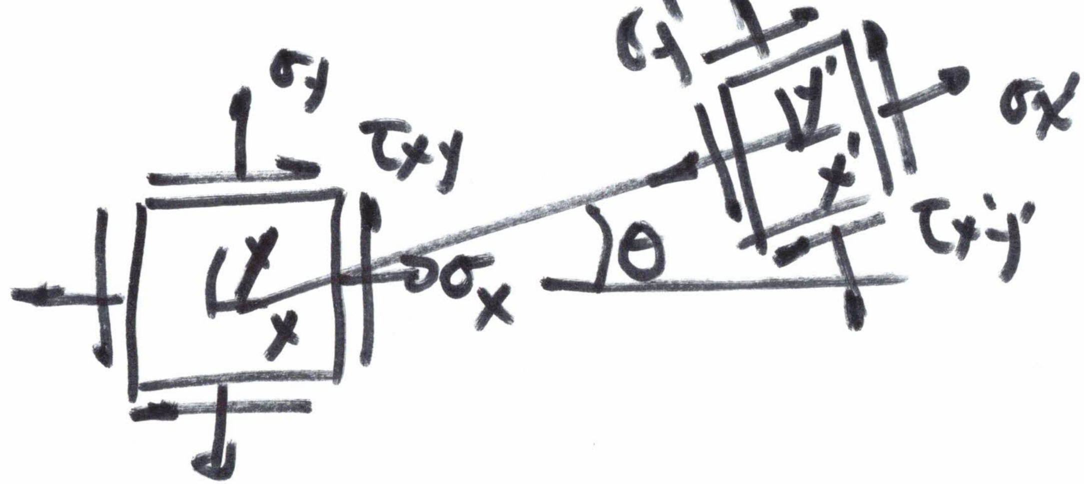


$$\sigma_n = \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

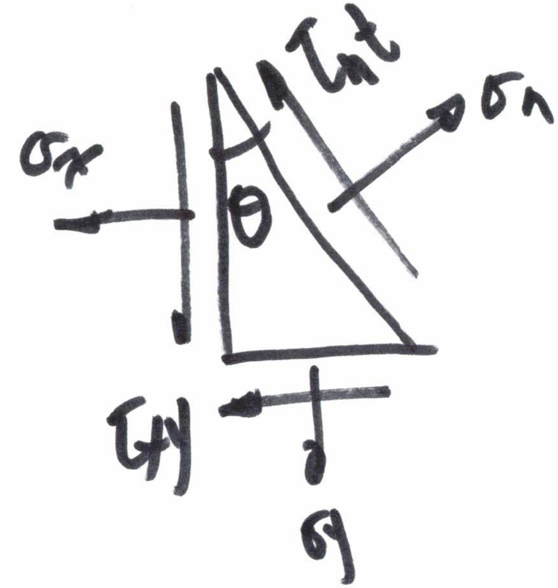
$$\tau_{nt} = \tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Don't memorize

C. Principal & Max shear Stress States

1. principal stress state – orientation of <u>stress block</u> where normal stresses (σ's) are max/minimum. The angle to this stress state is θ_p, the principal angle.

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$



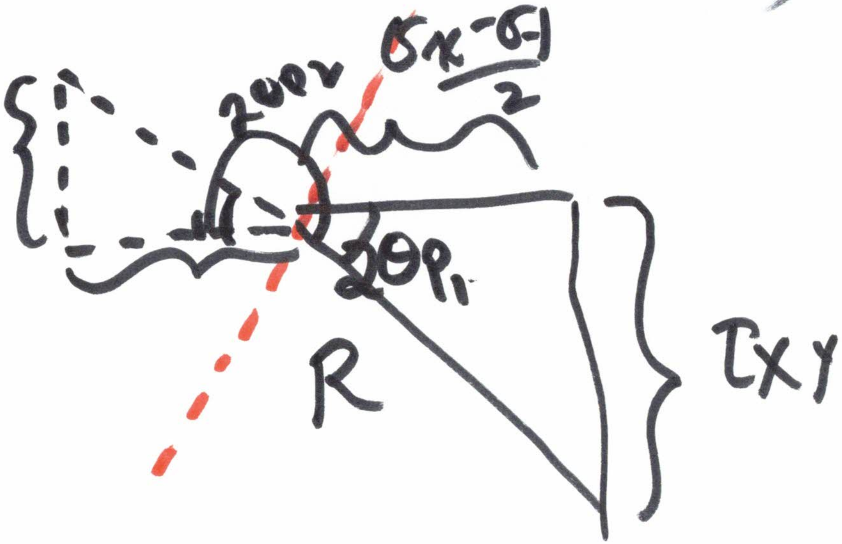
$$\frac{d\sigma_n}{d\theta} = 0 + \left(\frac{\sigma_x - \sigma_y}{2}\right)\left(-\cancel{2}\sin 2\theta_p\right)$$

$$+ \tau_{xy}\,\cancel{2}\cos 2\theta_p = 0.$$

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p = \tau_{xy} \cos 2\theta_p$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$



$$\sin 2\theta_p = \frac{\tau_{xy}}{R}$$

$$\cos 2\theta_p = \frac{\frac{\sigma_x - \sigma_y}{2}}{R}$$

$$\sigma_{n,max} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{R} + \tau_{xy}\frac{\tau_{xy}}{R}$$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{1}{R}\left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]$$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{1}{R}\cdot R^2$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$\sigma_{min} = \sigma_{ave} - R$$

$$\tau_{nt}\Big|_{2\theta_{p1}} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{\tau_{xy}}{R} + \tau_{xy}\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{R} = 0$$

The angle of max σ has zero shear stress.

2. max (in-plane) shear stress state

$$\frac{d\tau_{nt}}{d\theta} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\left(\cancel{2}\cos 2\theta_s\right) + \tau_{xy}\,\cancel{2}\left(-\sin 2\theta_s\right) = 0$$

θ_s - angle to max shear stress state

$$-\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta_s = \tau_{xy}\sin 2\theta_s$$

$$-\frac{\left(\frac{\sigma_x-\sigma_y}{2}\right)}{\tau_{xy}} = \tan 2\theta_s$$

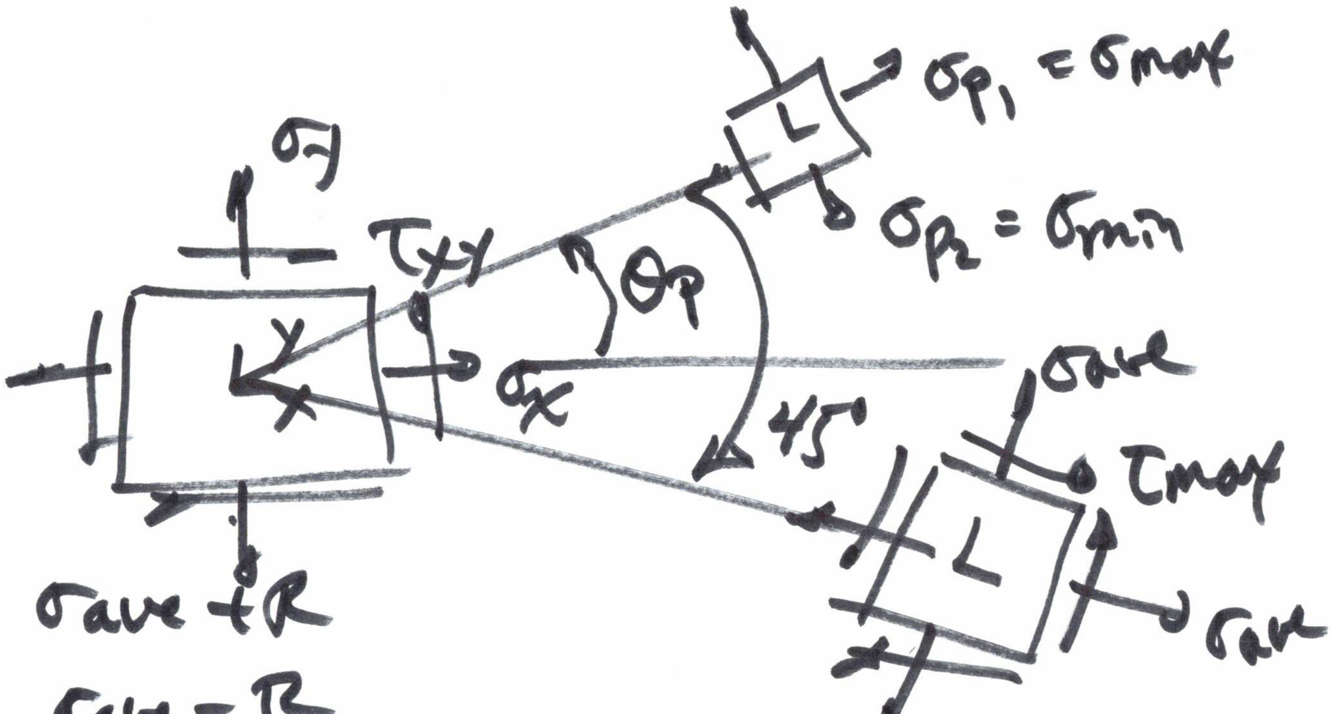
$$2\theta_s = 2\theta_p \pm 90^\circ$$

$$\theta_s = \theta_p \pm 45^\circ$$

$$\tau_{nt,max} = R$$

$$\sigma_n \text{ @ max shear} = \frac{\sigma_x + \sigma_y}{2} = \sigma_{ave}$$

- max normal stress state has zero shear stress
- max shear stress state has <u>average</u> normal stress



$\sigma_{p_1} = \sigma_{ave} + R$

$\sigma_{p_2} = \sigma_{ave} - R$

$\tau_{max} = R$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2}$

D. Mohr's Circle - graphical representation of eqns.

$$\sigma_n - \sigma_{ave} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} - 0 = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

SQUARE & ADD

$$(\sigma_n - \sigma_{ave})^2 + (\tau_{nt} - 0)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \cos^2 2\theta + \tau_{xy}^2 \sin^2 2\theta$$

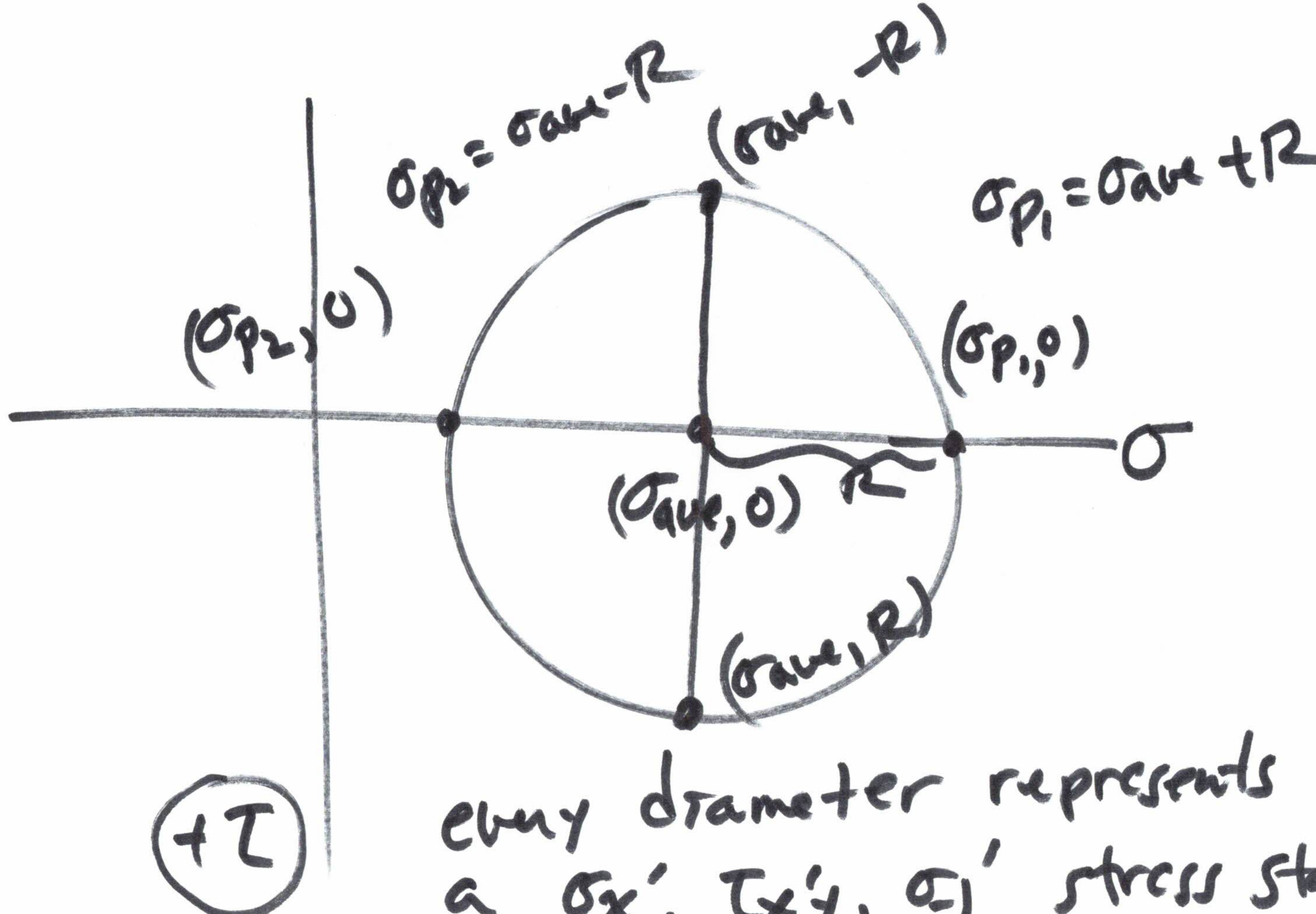
$$\cancel{+2\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta\, \tau_{xy} \sin 2\theta}$$

$$+\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \sin^2 2\theta + \tau_{xy}^2 \cos^2 2\theta$$

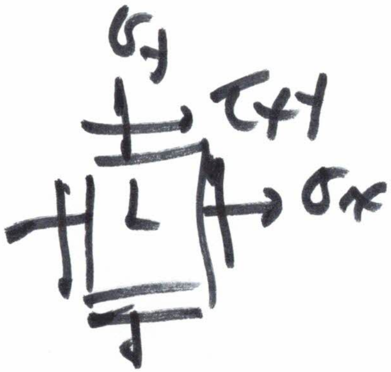
$$\cancel{-2\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta\, \tau_{xy} \cos 2\theta}$$

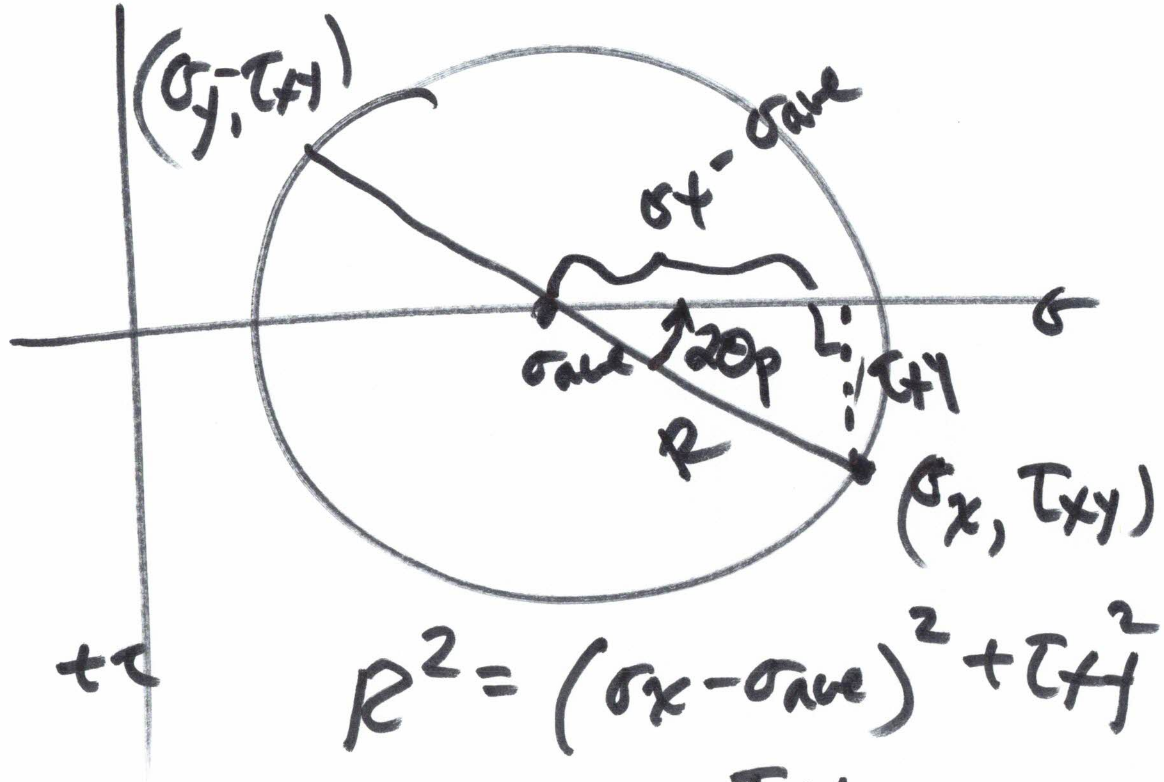
$$(\sigma_n - \sigma_{ave})^2 + (\tau_{nt} - 0)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 = R^2$$

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$



every diameter represents
a σ_x', $\tau_{x'y'}$, σ_y' stress state





$$R^2 = (\sigma_x - \sigma_{ave})^2 + \tau_{xy}^2$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\sigma_x - \sigma_{ave}}$$