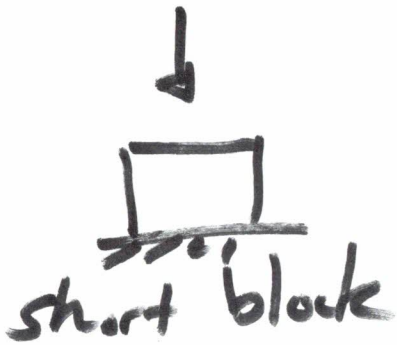
X Columns

Failure of an engineering structure is defined as the inability of the structure to fulfill its intended purpose.



short block

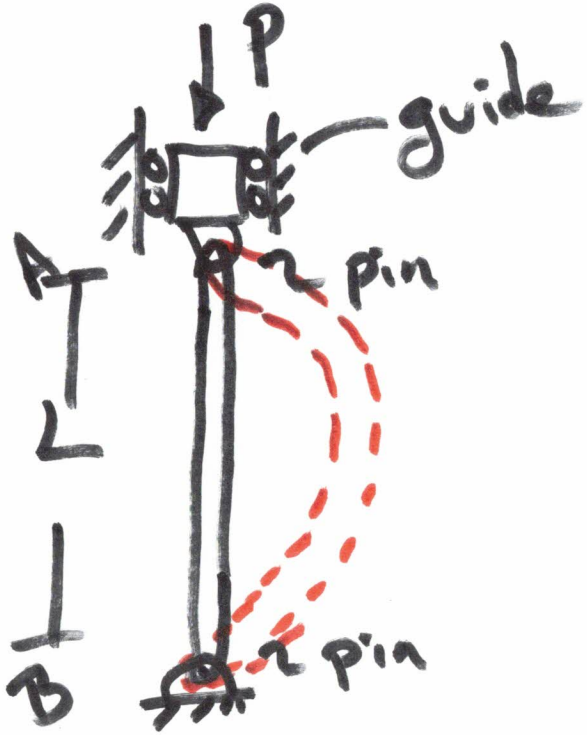
$P_{buckle} < P_{yield}$

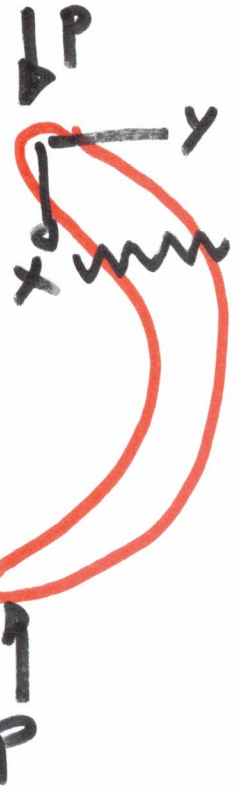
can be much lower

slender column

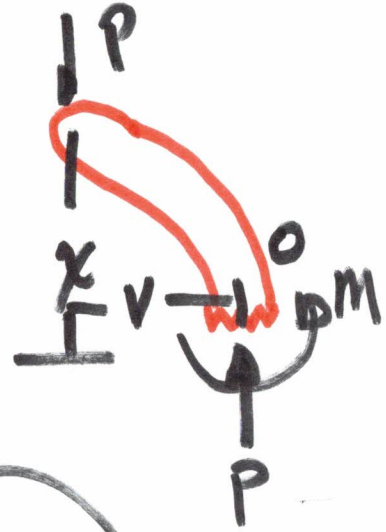
$P_{failure}$ (yield) $= \sigma_{yield} \times Area_{x\text{-section}}$

A. Euler Buckling equation





PBD in deformed state



$\circlearrowleft \Sigma M_o$

$+M + PV = 0$

$M = -PV$

$EIv'' = M = -PV$

$v'' + \frac{P}{EI} v = 0$

$\frac{d^2v}{dx^2} + \frac{P}{EI} v = 0$

let $\alpha^2 = \frac{P}{EI}$

B.C.'s $x=0, V=0$

$x=L, V=0$

$$\frac{d^2V}{dx^2} + \alpha^2 V = 0$$

$$V = A\sin\alpha x + B\cos\alpha x$$

$$V' = \alpha A\cos\alpha x - \alpha B\sin\alpha x$$

$$V'' = -\alpha^2 A\sin\alpha x - \alpha^2 B\cos\alpha x$$

$x=0, V=0 \qquad 0 = 0 + B \quad \Rightarrow B=0$

$x=L, V=0 \qquad 0 = A\sin\alpha L$

either $A=0$ (not interesting)

or

$$\sin\alpha L = 0$$

$$\alpha L = n\pi \qquad n = 1, 2, 3, \ldots$$

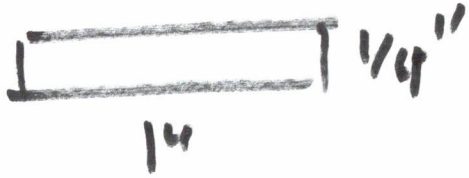
$$\alpha^2 = \frac{P}{EI} \qquad \alpha^2 L^2 = n^2 \pi^2$$

$$\frac{P}{EI} L^2 = n^2 \pi^2$$

$$P_{cR} = \frac{n^2 \pi^2 EI}{L^2}$$

known for final

↑ critical buckling load of column



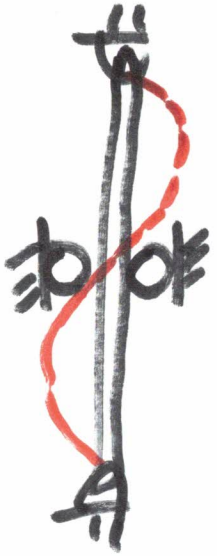
Use I for axis about which the beam buckles.

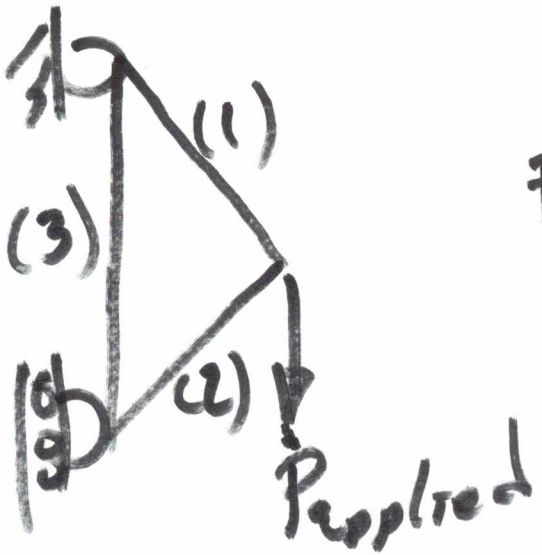
n – mode shape factor

$n = 1$

$n = 2$ – need extra supports

. . .





$$F_2 = \frac{n^2 \pi^2 EI}{L^2} \qquad n = 1$$