Example

1) Find stress in brass and magnesium ($\sigma_m$, $\sigma_b$)

Materials act in parallel, so deformations must be the same.

$$\Delta L_m = \Delta L_b \quad \text{constant section, constant force}$$

$$\left(\frac{PL}{AE}\right)_m = \left(\frac{PL}{AE}\right)_b \quad \Rightarrow \quad P_m = \frac{P_b A_m}{A_b} \frac{E_m}{E_b} = P_b \frac{\pi L^2}{4} \frac{6.5 \times 10^3}{15 \times 10^3}$$

$$P_m = 0.397 P_b \quad (1)$$

Each material carries part of the load.

Two gussets, two unknowns.

$$0.397 P_b + P_b = 10,000 \Rightarrow P_b = 7426 \text{ lbs}$$

$$P_m = 2577 \text{ lbs}$$

$$
\begin{align*}
\sigma_b &= \frac{P_b}{A_b} = \frac{7426}{\pi (1.5^2 - 1^2)} = 7.56 \text{ ksi} \\
\sigma_m &= \frac{P_m}{A_m} = \frac{2577}{\pi \frac{1^2}{4}} = 3.28 \text{ ksi}
\end{align*}
$$

2) Elongation $\Delta L$ or $\Delta L_b$

Use $\Delta L_m = \Delta L_b$

$$\Delta L = \frac{PL_b}{A_b E_b} = \sigma_b \frac{L_b}{E_b} = \frac{(7.52 \times 10^3) (20)}{15 \times 10^3 \times 10^3} = 1.008 \times 10^{-2} \text{ in/day}$$
If the materials were allowed to expand on their own, we would see:

\[
\text{FREE CONCRETE (NO STEEL)} \quad \text{FREE STEEL (NO CONCRETE)} \quad \text{BOTH CONCRETE \& STEEL}
\]

1. **STATIC**: To maintain equilibrium, the force caused by the steel must be equal and opposite to the force caused by the concrete: \(-P_{ST} = +P_{conc} = P\)

2. **Geometry**: materials in parallel \(S_{ST} = S_{conc}\)

3. **Load Deformation**

\[
\sigma_{ST} = \frac{(x \Delta T L)}{A_{ST}} - \frac{(P L)}{A E_{ST}} = (x \Delta T L)_{conc} + \frac{(P L)}{A E_{conc}} = \sigma_{conc}
\]

\[
A_{ST} = 4 \left( \frac{3}{4} \right)^2 \text{ in}^2 = 1.767 \text{ in}^2
\]

\[
A_{con} = 4 \times 4'' = 62.233 \text{ in}^2
\]

\[
\sigma_{ST} = \frac{-P}{A_{ST}} = -1.889 \text{ ksi}
\]

\[
\sigma_{conc} = \frac{P}{A_{con}} = 53.6 \text{ ksi}
\]

\[
\rightarrow P = 3.34 \text{ kips}
\]
Materials in Series Example

\[ \varepsilon_1 = 10 \times 10^3 \text{ ksi} \]
\[ \varepsilon_2 = 30 \times 10^3 \text{ ksi} \]

\[ L_1 = 15 \text{ in} \quad L_2 = 10 \text{ in} \]
\[ d_1 = 0.5 \text{ in} \quad d_2 = 0.25 \text{ in} \]

(Round bars)

Find \( \sigma_1 \) and \( \sigma_2 \)

Need \( F_1 \) and \( F_2 \)

Follow these steps:
1. Overall FBD

\[ F_L = 1000 \text{ lb} \]

\[ F_R \]

\[ \sum F_x = F_R - F_L + 1000 = 0 \]

\[ F_L \text{ and } F_R \text{ are Rxns at the walls.} \]

2. Geometry of Deformation

\[ e_1 + e_2 = 0 \]

3. Load - Deformation eqn.

\[ \frac{F_1 L_1}{A_1 E_1} = e_1 \]

\[ \frac{F_2 L_2}{A_2 E_2} = e_2 \]
\[ \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2} = 0 \]

\[ F_1 = -F_2 \frac{L_2}{L_1} \frac{A_1}{A_2} \frac{E_1}{E_2} \]

Suggestion: do algebra before inserting numbers.

\[ F_1 = -F_2 \frac{10}{15} \frac{\pi h (0.5)^2}{\pi h (0.25)^2} \frac{10}{30} = -F_2 (0.8889) \]

4. Find internal force & external reaction relation. Must use FBD's
From steps 1 & 3:

\[ F_2 - F_1 + 1000 = 0 \]
\[ F_2 = \frac{F_1}{1.8889} F_2 + 1000 = 0 \]

**Suggestion:** Always draw unknown internal forces & external forces so that bar looks like it is in tension.

If it is not, then you will get a negative number.

\[ F_2 = -529 \text{ lbs} \]
\[ F_1 = 470 \text{ lbs} \]

\[ \sigma_1 = \frac{F_1}{A_1}, \quad \sigma_2 = \frac{F_2}{A_2} \text{ etc.} \]
EXAMPLE: SIMILAR TRIANGLES

Find $\sigma_1$, $\sigma_2$, etc.

Need Forces $F_1$ & $F_2$

$L_1 = 450$ mm
$L_2 = 600$ mm
$d_1 = 15$ mm
$d_2 = 20$ mm
$E_1 = E_2 = 200$ GPa
Statics:

\[ \begin{align*}
F_1 \ (510) &= 85(510+310) \\
+F_2 \ (510+310+360) &= 0 \\
510 \ F_1 + 1180 \ F_2 &= 69.7 \times 10^3 \\
F_1 + 2.314 \ F_2 &= 136.7
\end{align*} \]