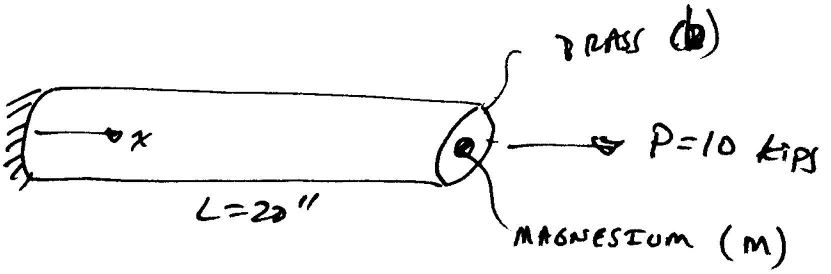
EXAMPLE




$d_m'' = 1.0''$

$E_m = 6.5 \times 10^3$ ksi

$E_b = 15 \times 10^3$ ksi

$d_{outer\,b} = 1.5''$

$d_{inner\,b} = 1.0''$

a) Find stress in BRASS & MAGNESIUM $(\sigma_{x_m}, \sigma_{x_b})$

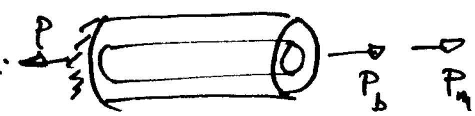
Materials act in parallel, so deformations must be the same.

$\Delta L_m = \Delta L_b$ const x-section, const force

$$\left(\frac{PL}{AE}\right)_m = \left(\frac{PL}{AE}\right)_b \Rightarrow P_m = P_b \frac{A_m}{A_b}\frac{E_m}{E_b} = P_b \frac{\frac{\pi}{4} 1^2}{\pi/4\,(1.5^2 - 1^2)} \cdot \frac{6.5 \times 10^3}{15 \times 10^3}$$

$$P_m = 0.347\, P_b \quad ①$$

EACH MATERIAL CARRIES PART OF THE LOAD:




$P = P_b + P_m$ ②

Two eqns, Two unknowns.

$$0.347\, P_b + P_b = 10{,}000 \Rightarrow P_b = 7426 \text{ lbs}$$

$$P_m = 2577 \text{ lbs}$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{7426}{\frac{\pi}{4}[1.5^2 - 1^2]} = \boxed{7.56 \text{ ksi}}$$

$$\sigma_m = \frac{P_m}{A_m} = \frac{2577}{\frac{\pi}{4} 1^2} = \boxed{3.28 \text{ ksi}}$$

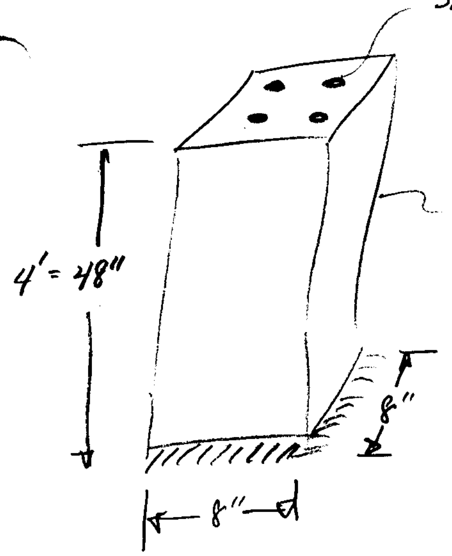
b) Elongation of bar

use $\Delta L_m$ or $\Delta L_b$

$$\Delta L = \Delta L_B = \frac{P_b L_b}{A_b E_b} = \sigma_b \frac{L_b}{E_b} = \frac{(7.56 \times 10^3)(20)}{15 \times 10^3 \times 10^3} = \boxed{1.008 \times 10^{-2} \text{ inches}}$$

2.56

FIND $\sigma$'s



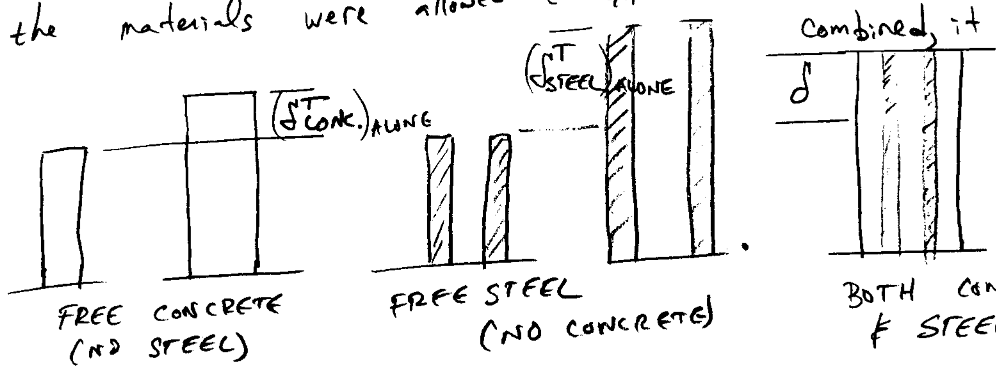

3/4" diameter STEEL $E_s = 29 \times 10^6$ psi $\alpha_s = 6.5 \times 10^{-6} / {}^\circ F$

Concrete $E_c = 3.6 \times 10^6$ psi $\alpha_c = 5.5 \times 10^{-6} / {}^\circ F$

$\Delta T = +80^\circ F$

If the materials were allowed to expand on their own, we would see:

Combined, it is somewhere inbetween:




This makes the steel be in compression & the concrete in tension.

① STATICS: To maintain equilibrium, the force caused by the steel must be equal & opposite to the force caused by the concrete: $-P_{ST} = +P_{CONC} = P$

② geometry: materials in parallel $\delta_{ST} = \delta_{CONC}$

③ load deformation

$$\delta_{ST} = (\alpha \Delta T L)_{ST} - \left(\frac{PL}{AE}\right)_{ST} = (\alpha \Delta T L)_{CONC} + \left(\frac{PL}{AE}\right)_{CONC} = \delta_{CONC.}$$

$A_{ST} = 4(\pi/4)(3/4)^2 \text{ in}^2 = 1.7671 \text{ in}^2$

$A_{CONS} = 4'' \times 4'' - A_{ST} = 62.233 \text{ in}^2$

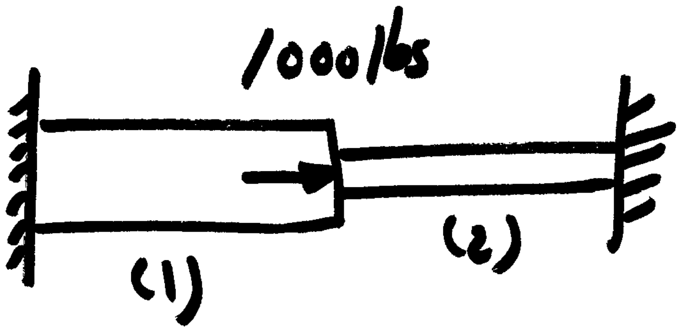
$$6.5(10^{-6})(80)(48) - \frac{P(48)}{(1.767)\,29 \times 10^6} = (5.5 \times 10^{-6})(80)\,48 + \frac{P(48)}{(62.23)(3.6)(10^6)}$$

$\Rightarrow$ $\underline{P = 3.34 \text{ Kips}}$

$$\boxed{\begin{aligned} \sigma_s &= -P/A_s = -1.889 \text{ ksi} \\ \sigma_c &= P/A_c = 53.6 \text{ ksi} \end{aligned}}$$

□

# MATERIALS IN SERIES EXAMPLE




$E_1 = 10 \times 10^3$ ksi

$E_2 = 30 \times 10^3$ ksi

$L_1 = 15$ in $\quad L_2 = 10$ in

$d_1 = 0.5$ in $\quad d_2 = 0.25$ in

(Round bars)

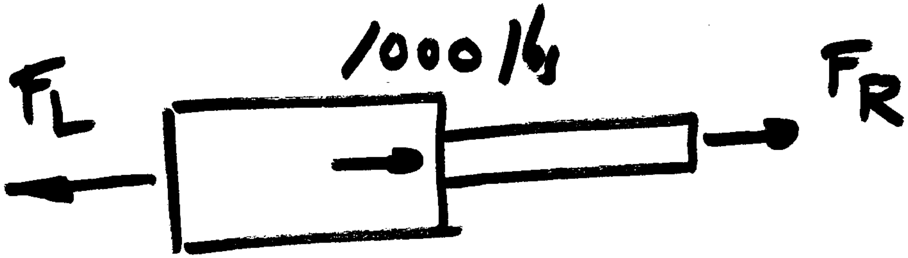
Find $\sigma_1$ & $\sigma_2$

NEED $F_1$ & $F_2$

FOLLOW THESE STEPS:

① Overall FBD

$F_L$ & $F_R$ are RXNS at the walls.




$\overset{+}{\rightarrow} \Sigma F_x \quad F_R - F_L + 1000 = 0$ Statics eqn.

② Geometry of Deformation

$$e_1 + e_2 = 0$$

③ Load - Deformation eqn.

$$\frac{F_1 L_1}{A_1 E_1} = e_1 \qquad \frac{F_2 l_2}{A_2 E_2} = e_2$$

② & ③

$$\frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 E_2} = 0$$

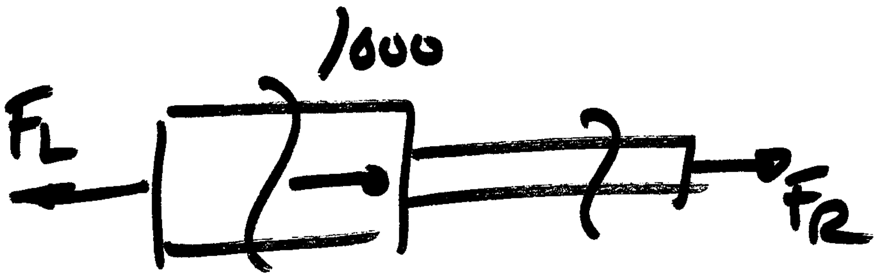
$$F_1 = -F_2 \frac{L_2}{L_1} \frac{A_1}{A_2} \frac{E_1}{E_2}$$

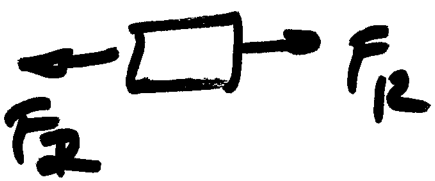
Suggestion: do algebra before inserting numbers.

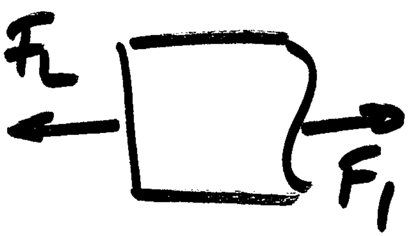
$$F_1 = -F_2 \frac{10}{15} \frac{\pi/4 (0.5)^2}{\pi/4 (0.25)^2} \frac{10}{30} = -F_2 (0.8889)$$

④ Find internal force & external reaction relation.

Must use FBD's




$\underline{F_1 = F_L}$

$\underline{F_2 = F_R}$

Suggestion: Always draw unknown internal forces & external RXNS so that bar looks like it is in tension.

If it is not, then you will get a negative number.

From steps ① & ③

$$F_R - F_L + 1000 = 0$$

$$F_2 - F_1 + 1000 = 0$$

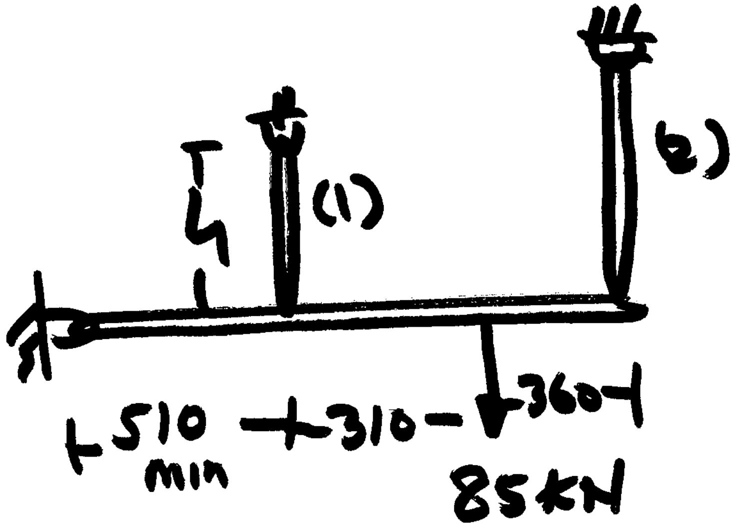
$$F_2 - (-0.8889)F_2 + 1000 = 0$$

$$F_2 = -529 \text{ lbs}$$

$$F_1 = 470 \text{ lbs}$$

$\sigma_1 = \dfrac{F_1}{A_1}$ , $\sigma_2 = \dfrac{F_2}{A_2}$ etc.

EXAMPLE: SIMILAR TRIANGLES

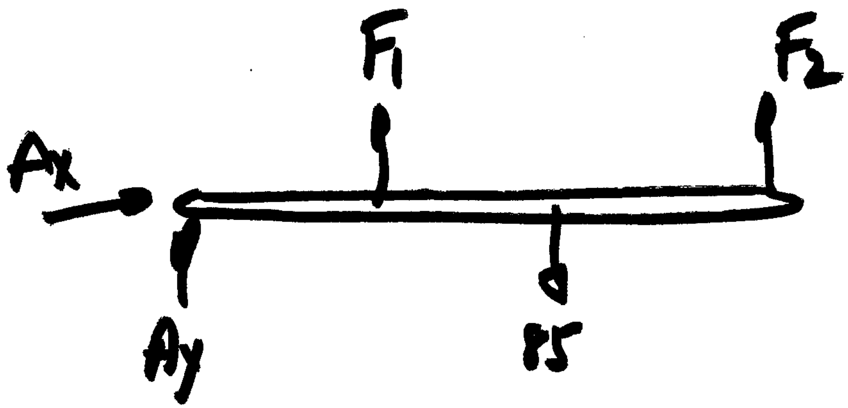



$L_1 = 450$ mm
$L_2 = 600$ mm
$d_1 = 15$ mm
$d_2 = 20$ mm
$E_1 = E_2 = 200$ GPa

Find $\sigma_1$ , $\sigma_2$ etc.

Need Forces $F_1$ & $F_2$

① STATICS :




$\overset{\curvearrowright}{+}\ \Sigma M_A$

$$F_1(510) - 85(510+310) + F_2(510+310+360) = 0$$

$$510F_1 + 1180F_2 = 69.7 \times 10^3$$

$$F_1 + 2.314F_2 = 136.7$$