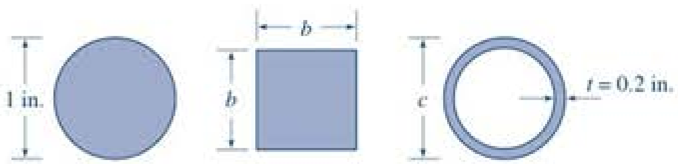
▼ NORMAL STRESS; AXIAL STRESS

MDS 2.1–2.4

For all problems in Section 2.2, ***assume unknown axial forces to be positive in tension. As in Examples 2.1 and 2.2, label tensile-stress answers with (T) and compressive-stress answers with (C).***

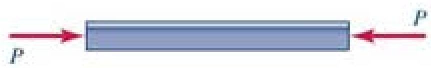
Prob. 2.2-1. A 1-in.-diameter solid bar (1), a square solid bar (2), and a circular tubular member with 0.2-in. wall thickness (3), each supports an axial tensile load of 5 kips. (a) Determine the axial stress in bar (1). (b) If the axial stress in each of the other bars is 6 ksi, what is the dimension, b, of the square bar, and what is the outer diameter, c, of the tubular member?



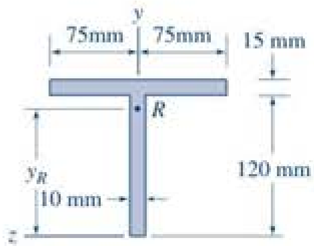
(1) (2) (3)

P2.2-1

Prob. 2.2-2. The structural tee shown in Fig. P2.2-2 supports a compressive load $P = 200$ kN. (a) Determine the coordinate y_R of the point R in the cross section where the load must act in order to produce uniform compressive axial stress in the member, and (b) determine the magnitude of that compressive stress.



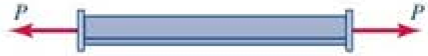
(*a*)



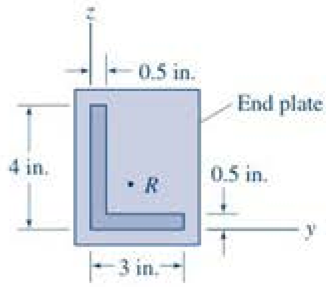
(*b*)

P2.2-2

Prob. 2.2-3. A steel plate is welded onto each end of the structural angle in Fig. P2.2-3 so that a load can be applied at point R, where it will produce uniform axial stress in the member. (a) Determine the coordinates y_R and z_R of the point where the tensile load P must act in order to produce uniform tensile stress in the cross section of the structural angle, and (b) determine the magnitude of that tensile stress if $P = 18$ kips.



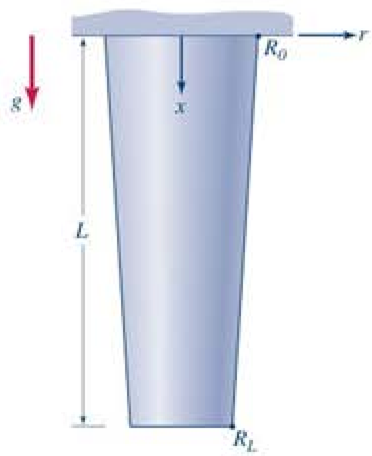
(*a*)



(*b*)

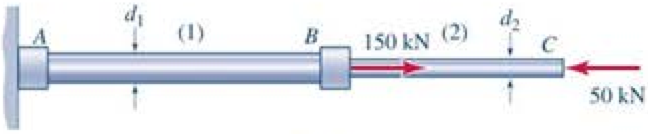
P2.2-3

Prob. 2.2-4. Consider the free-hanging rod shown in Fig. P2.2-4. The rod has the shape of a conical frustum, with radius R_0 at its top and radius R_L at its bottom, and it is made of material with mass density ρ. The length of the rod is L. Determine an expression for the normal stress, $\sigma(x)$, at an arbitrary cross section x $(0 \le x \le L)$, where x is measured downward from the top of the rod.



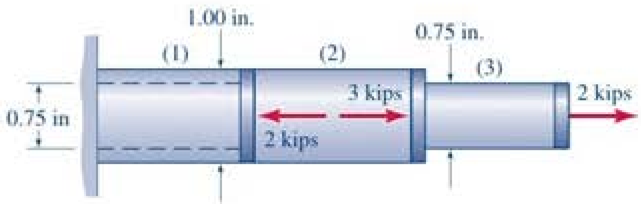
P2.2-4 and P2.3-6

Prob. 2.2-5. A solid brass rod AB and a solid aluminum rod BC are connected together by a coupler at B, as shown in Fig. P2.2-5. The diameters of the two segments are $d_1 = 60$ mm and $d_2 = 50$ mm, respectively. Determine the axial stresses σ_1 (in rod AB) and σ_2 (in rod BC).



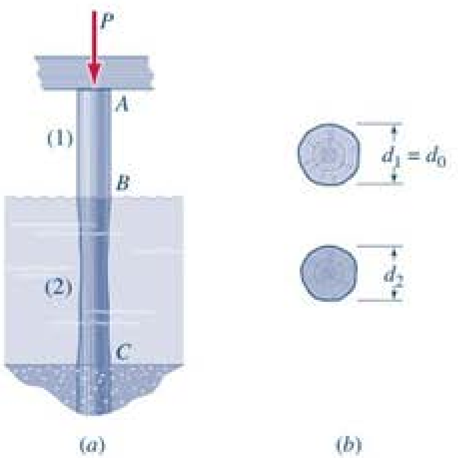
P2.2-5

Prob. 2.2-6. The three-part axially loaded member in Fig. P2.2-6 consists of a tubular segment (1) with outer diameter $(d_o)_1 = 1.00$ in. and inner diameter $(d_i)_1 = 0.75$ in., a solid circular rod segment (2) with diameter $d_2 = 1.00$ in., and another solid circular rod segment (3) with diameter $d_3 = 0.75$ in. The line of action of each of the three applied loads is along the centroidal axis of the member. Determine the axial stresses σ_1, σ_2, and σ_3 in each of the three respective segments.



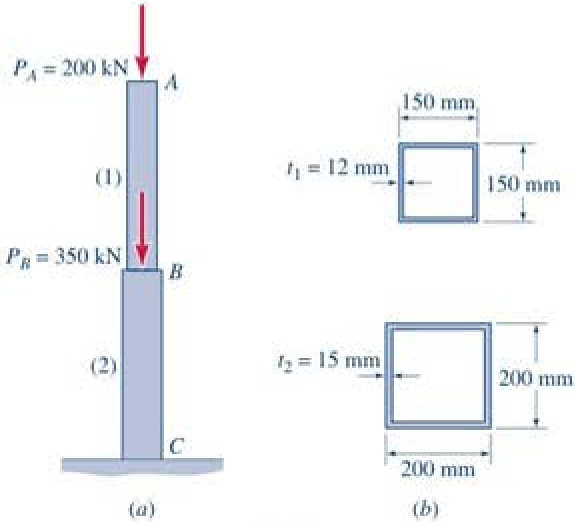
P2.2-6

Prob. 2.2-7. At a local marina the dock is supported on wood piling in the manner shown in Fig. P2.2-7*a*. The top part, *AB*, of one pile is above the normal waterline; the middle part, *BC*, is in direct contact with the water; and the part below *C* is underground. The original diameter of the pile is $d_0 = 12$ in., but action of the water and insects has reduced the diameter of the pile over the part *BC*. (a) If the axial force that the deck exerts on this pile is $P = 200$ kips, what is the axial stress in *AB*? Neglect the weight of the pile itself. (b) An inspector estimates that the diameter of the pile in segment *BC* has been eroded by 5%. What axial stress does the deck load of $P = 200$ kips produce in this damaged part of the pile? (c) If the maximum axial stress allowed in the wood piles is 7.5 ksi (in compression), what is the maximum deck load that this damaged pile can support?



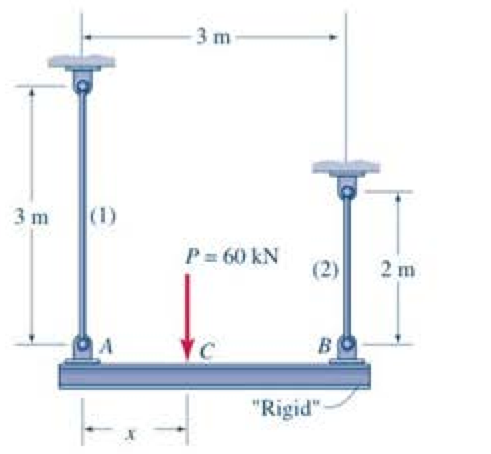
P2.2-7

Prob. 2.2-8. A column in a two-story building is fabricated from square structural tubing having the cross-sectional dimensions shown in Fig. P2.2-8*b*. Axial loads $P_A = 200$ kN and $P_B = 350$ kN are applied to the column at levels *A* and *B*, as shown in Fig. P2.2-8*a*. Determine the axial stress σ_1 in segment *AB* of the column and the axial stress σ_2 in segment *BC* of the column. Neglect the weight of the column itself.



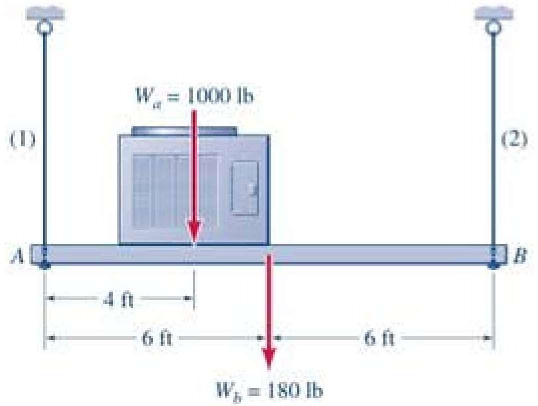
P2.2-8

Prob. 2.2-9. A rigid beam *AB* of total length 3 m is supported by vertical rods at its ends, and it supports a downward load at *C* of $P = 60$ kN, as shown in Fig. P2.2-9. The diameters of the steel hanger rods are $d_1 = 25$ mm and $d_2 = 20$ mm. Neglect the weight of beam *AB* and the rods. (a) If the load is located at $x = 1$ m, what are the stresses σ_{1a} and σ_{2a} in the respective hanger rods. (b) At what distance *x* from *A* must the load be placed such that $\sigma_2 = \sigma_1$, and what is the corresponding axial stress, $\sigma_{1b} = \sigma_{2b}$, in the rods?



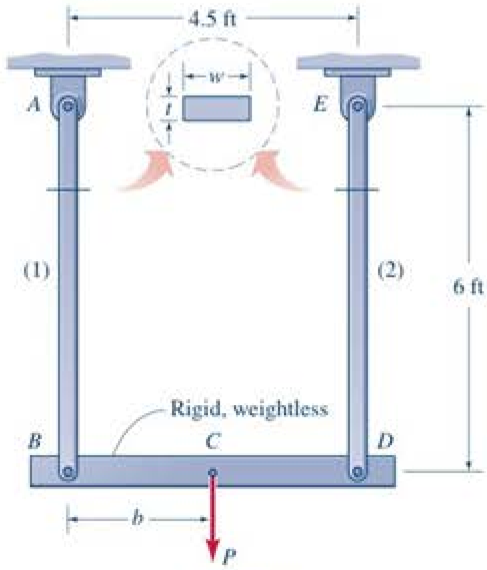
P2.2-9

Prob. 2.2-10. A 12-ft beam *AB* that weighs $W_b = 180$ lb supports an air conditioner that weighs $W_a = 1000$ lb. The beam, in turn, is supported by hanger rods (1) and (2), as shown in Fig. P2.2-10. (a) If the diameter of rod (1) is $\frac{3}{8}$ in., what is the stress, σ_1, in the rod? (b) If the stress in rod (2) is to be the same as the stress in rod (1), what should the diameter of rod (2) be (to the nearest $\frac{1}{32}$ in.)?



P2.2-10

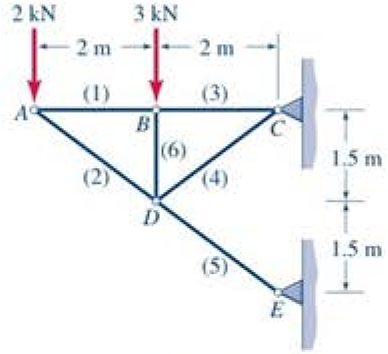
Prob. 2.2-11. A rigid, weightless beam *BD* supports a load *P* and is, in turn, supported by two hanger rods, (1) and (2), as shown in Fig. P2.2-11. The rods are initially the same length $L = 6$ ft and are made of the same material. Their rectangular cross sections have original dimensions ($w_1 = 1.5$ in., $t_1 = 0.75$ in.) and ($w_2 = 2.0$ in., $t_2 = 1.0$ in.), respectively. (a) At what location, *b*, must the load *P* act if the axial stress in the two bars is to be the same, i.e., $\sigma_1 = \sigma_2$? (b) What is the magnitude of this tensile stress if a load of $P = 40$ kips is applied at the location determined in Part (a)?



P2.2-11

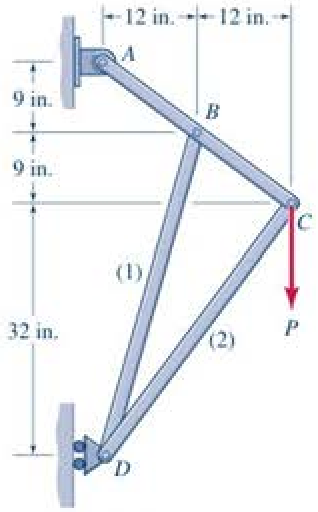
Prob. 2.2-12. Each member of the truss in Fig. P1.4-1 is a solid circular rod with diameter $d = 0.50$ in. Determine the axial stresses σ_1, σ_2, and σ_3 in members (1), (2), and (3), respectively. (See Prob. 1.4-1.)

Prob. 2.2-13. Each member of the truss in Fig. P2.2-13 is a solid circular rod with diameter $d = 10$ mm. Determine the axial stress σ_1 in the truss member (1) and the axial stress σ_6 in the truss member (6).



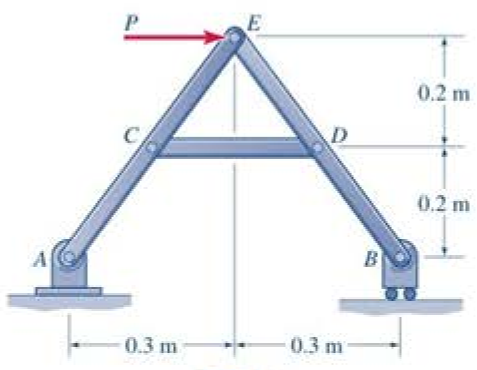
P2.2-13

Prob. 2.2-14. The three-member frame structure in Fig. P2.2-14 is subjected to a downward vertical load *P* at pin *C*. The pins at *B*, *C*, and *D* apply axial loads to members *BD* and *CD*, whose cross-sectional areas are $A_1 = 0.5$ in^2 and $A_2 = 1.0$ in^2, respectively. (a) If the axial stress in member *BD* is $\sigma_1 = 1200$ psi, what is the value of force *P*? (b) What is the corresponding axial stress, σ_2, in member *CD*?



P2.2-14

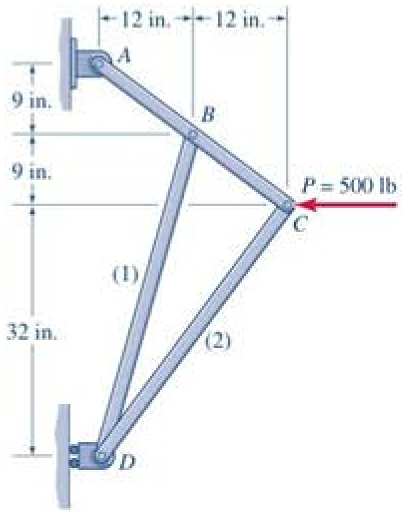
Prob. 2.2-15. The three-member frame in Fig. P2.2-15 is subjected to a horizontal load *P* at pin *E*. The pins at *C* and *D* apply an axial load to cross-brace member *CD*, which has a rectangular cross section measuring 30 mm × 50 mm. If $P = 210$ kN, what is the axial stress in member *CD*?



P2.2-15

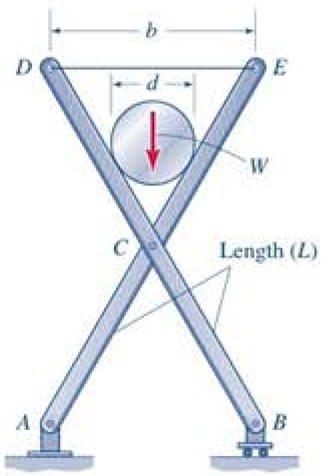
Prob. 2.2-16. The pins at B and D in Fig. P1.4-17 apply an axial load to diagonal bracing member BD. If BD has a rectangular cross section measuring 0.50 in. × 2.00 in., what is the axial stress in member BD when the load is w_0 = 220 lb/ft?

Prob. 2.2-17. The three-member frame structure in Fig. P2.2-17 is subjected to a horizontal load P = 500 lb at pin C. The pins at B, C, and D apply axial loads to members BD and CD. If the stresses in members BD and CD are σ_1 = 1200 psi and σ_2 = −820 psi, what are the respective cross-sectional areas of the two members?



P2.2-17

*__Prob. 2.2-18.__ A cylinder of weight W and diameter d rests between thin, rigid members AE and BD, each of length L. Friction between the cylinder and its supports is negligible. The members are joined at their midpoint C by a frictionless pin, and they are prevented from collapsing by a restraining wire DE of length b and cross-sectional area A. Consider W, L, d, and A to be given, and determine an expression that relates the axial stress in wire DE to its length b.

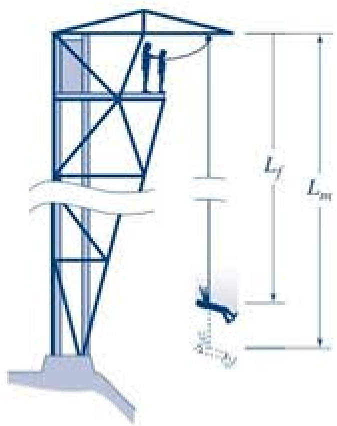


P2.2-18

▼ EXTENSIONAL STRAIN

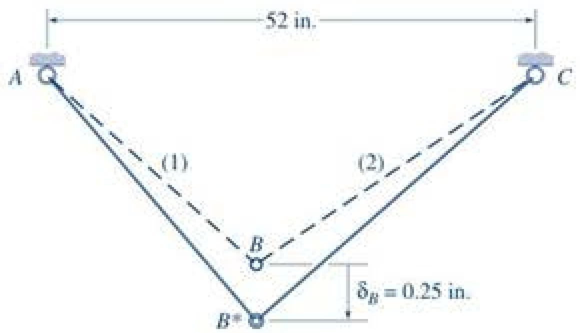
Where both undeformed and deformed configurations are shown, the undeformed configuration is shown with dashed lines and the deformed configuration is shown with solid lines. Points on the deformed body are indicated by an asterisk(*).

Prob. 2.3-1. When the bungee jumper in Fig. P2.3-1 stands on the platform, the unstretched length of the bungee cord is L = 15.0 ft. (a) When the jumper "hits bottom," the maximum extended length of the bungee cord is L_m = 41.4 ft. Assuming that the bungee cord stretches uniformly along its length, determine the extensional strain ϵ_m in the bungee cord at this point. (b) After bouncing a few times, the bungee jumper comes to rest with the final length of the bungee cord being L_f = 32.4 ft. What is the final strain, ϵ_f?



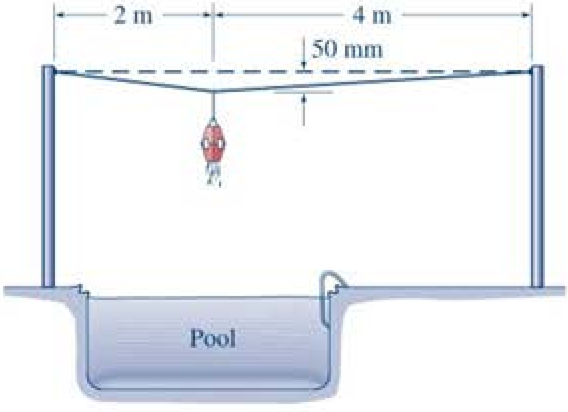
P2.3-1

Prob. 2.3-2. Wire AB of length L_1 = 30 in. and wire BC of length L_2 = 36 in. are attached to a ring at B. Upon loading, point B moves vertically downward by an amount δ_B = 0.25 in. Determine the extensional strains ϵ_1 and ϵ_2 in wires (1) and (2), respectively.



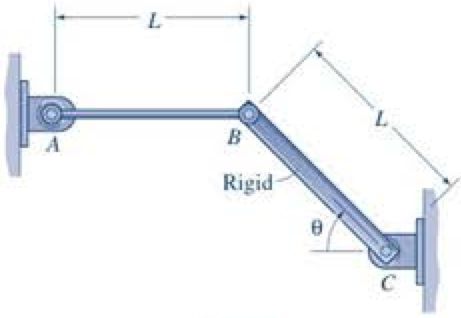
P2.3-2

Prob. 2.3-3. A wire is used to hang a lantern over a pool. Neglect the weight of the wire, and assume that it is taut, but strain free, before the lantern is hung. When the lantern is hung, it causes a 50-mm sag in the wire. Determine the extensional strain in the wire with the lantern hanging as shown.



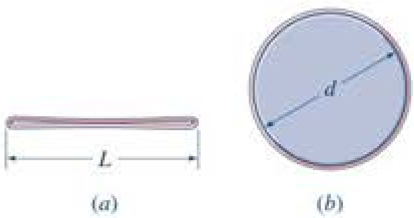
P2.3-3

^{C}Prob. 2.3-4. A "rigid" beam BC of length L is supported by a fixed pin at C and by an extensible rod AB, whose original length is also L. When $\theta = 45°$, rod AB is horizontal and strain free, that is, $\epsilon(\theta = 45°) = 0$. (a) Determine an expression for $\epsilon(\theta)$, the strain in rod AB, as a function of the angle θ shown in Fig. P2.3-4, valid for $45° \le \theta \le 90°$. (b) Write a computer program and use it to plot the expression for $\epsilon(\theta)$ for the range $45° \le \theta \le 90°$.



P2.3-4

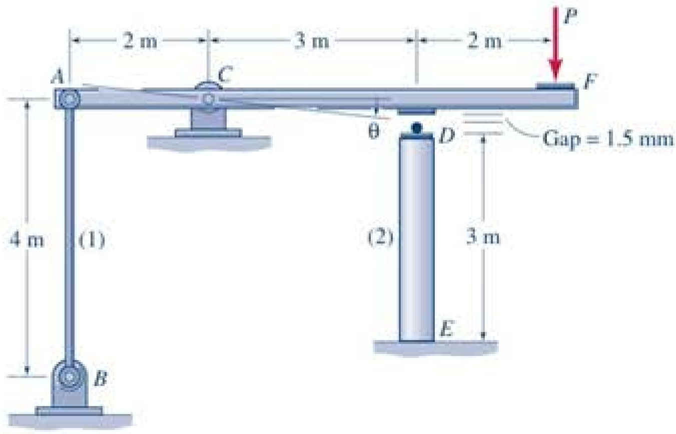
Prob. 2.3-5. When a rubber band is uniformly stretched around the solid circular cylinder in Fig. P2.3-5*b*, its extensional strain is $\epsilon = 0.025\ \frac{\text{mm}}{\text{mm}}$. If the diameter of the cylinder is $d = 100$ mm, what is the unstretched length of the rubber band (i.e., length L in Fig. P2.3-5*a*)? (Neglect the thickness of the rubber band.)



P2.3-5

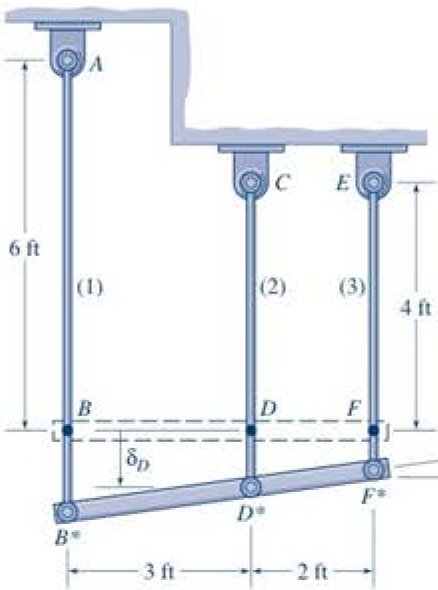
Prob. 2.3-6. Determine an expression for the extensional strain $\epsilon(x)$ at cross section x $(0 \le x \le L)$ of the hanging conical frustum in Prob. 2.2-4, and (b) determine an expression for the total elongation, e, of this hanging conical frustum in Prob. 2.2-4. The conical-frustum rod is made of material for which $\sigma = E\epsilon$, with E = const.

Prob. 2.3-7. For small loads, P, the rotation of "rigid" beam AF in Fig. P2.3-7 is controlled by the stretching of rod AB. For larger loads, the beam comes into contact with the top of column DE, and further resistance to rotation is shared by the rod and the column. Assume (and later show that this is a valid assumption) that the angle θ through which beam AF rotates is small enough that points on the beam essentially move vertically, even though they actually move on circular paths about the fixed pin at C. (a) A load P is applied at end F that is just sufficient to close the 1.5-mm gap between the beam and the top of the column at D. What is the strain, ϵ_1, in rod AB for this value of load P? (b) If load P is increased further until $\epsilon_1 = 0.001\ \frac{\text{mm}}{\text{mm}}$, what is the corresponding strain, ϵ_2, in column DE?



P2.3-7

Prob. 2.3-8. Vertical rods (1), (2), and (3) are all strain free when they are initially pinned to a straight, rigid,

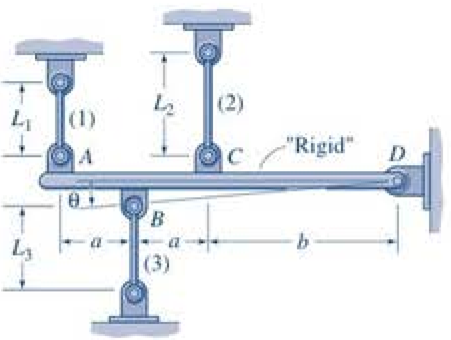


P2.3-8

horizontal beam BF. Subsequently, heating of the rods causes them to elongate and leaves the beam in the position denoted by $B^*D^*F^*$. Point D moves vertically downward by a distance $\delta_D = 0.20$ in., and the inclination angle of the beam is $\theta = 0.4°$ in the counterclockwise sense, as indicated on Fig. P2.3-8. Determine the strains ϵ_1, ϵ_2, and ϵ_3 in the three rods.

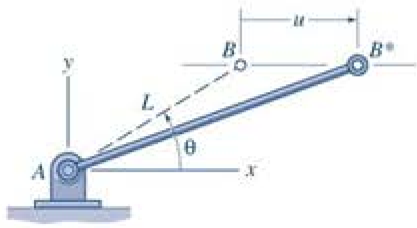
***Prob. 2.3-9.** A "rigid" beam AD is supported by a smooth pin at D and by vertical rods attached to the beam at points A, B, and C. The rods are all strain free when the beam is horizontal ($\theta = 0$). Subsequently, rod (2) is heated until its extensional strain reaches the value $\epsilon_2 = 80.0(10^{-6})\ \frac{\text{in.}}{\text{in.}}$. (a) Determine the value of the (counterclockwise) beam angle θ that corresponds to the strain $\epsilon_2 = 80.0(10^{-6})\ \frac{\text{in.}}{\text{in.}}$. (b) Determine the corresponding extensional strains ϵ_1 and ϵ_3.

$L_1 = L_3 = 30$ in., $L_2 = 40$ in., $a = 20$ in., $b = 60$ in.



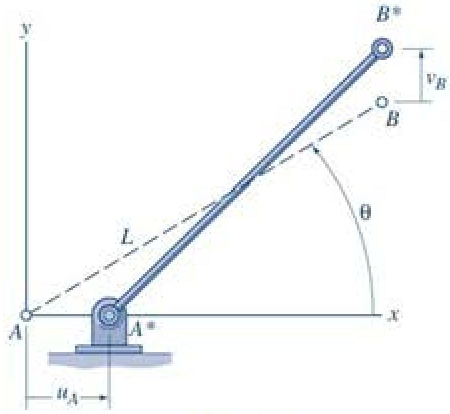
P2.3-9

Prob. 2.3-10. A rod AB, whose unstretched length is L, is originally oriented at angle θ counterclockwise from the $+x$ axis. (a) If the rod is free to rotate about a fixed pin at A, what is the extensional strain in the rod when end B moves a distance u in the $+x$ direction to point B^*? Express your answer in terms of the displacement u, the original length L, and the original angle θ. (b) Simplify the answer you obtained for Part (a), obtaining a small-displacement approximation that is valid if $u \ll L$. (Note: This result will be used in Section 3.10.)



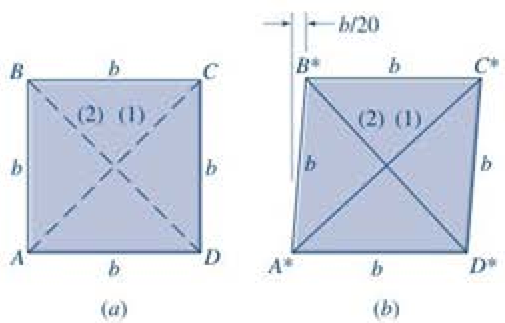
P2.3-10

Prob. 2.3-11. Rod AB, whose undeformed length is L, is originally oriented at angle θ counterclockwise from the $+x$ axis (the dashed line in Fig. P2.3-11). Both ends of the rod are free to move in the x–y plane, while the rod remains straight. (a) Derive an expression for the extensional strain, ϵ, in the rod when end A moves a distance u_A in the $+x$ direction to point A^* and end B moves a distance v_B in the $+y$ direction to point B^*. Express your answer in terms of u_A, v_B, L, and θ. (b) Simplify your answer for Part (a), obtaining a small-displacement approximation that is valid if $u_A \ll L$ and $v_B \ll L$. (Note: The displacements u_A and v_B are exaggerated in Fig. P2.3-11.)



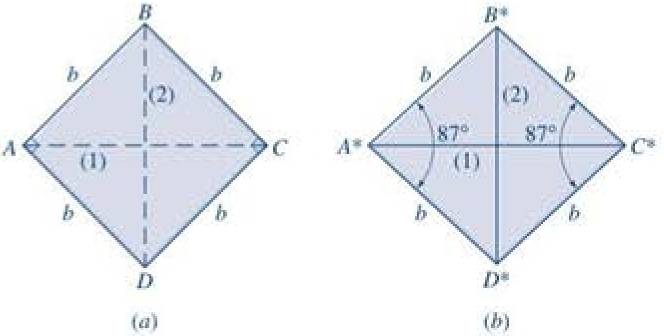
P2.3-11

Prob. 2.3-12. A thin sheet of rubber in the form of a square (Fig. P2.3-12*a*) is uniformly deformed into the parallelogram shape shown in Fig. P2.3-12*b*. All edges remain the same length, b, as the sheet deforms. (a) Compute the extensional strain ϵ_1 of diagonal AC. (b) Compute the extensional strain ϵ_2 of diagonal BD.



P2.3-12

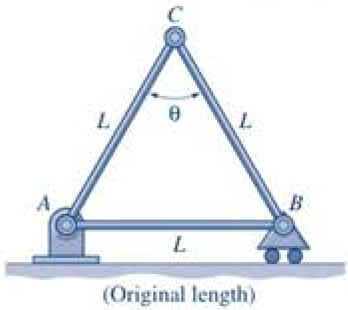
Prob. 2.3-13. A thin sheet of rubber in the form of a square (Fig. P2.3-13*a*) is uniformly deformed into the parallelogram shape shown in Fig. P2.3-13*b*. All edges remains the same length, b, as the sheet deforms. (a) Compute the extensional strain ϵ_1 of diagonal AC. (b) Compute the extensional strain ϵ_2 of diagonal BD.



P2.3-13

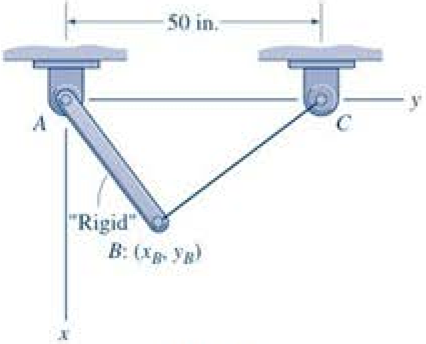
▼ THERMAL STRAIN

Prob. 2.3-14. At the reference temperature, three identical rods of length L form a pin-jointed truss in the shape of an equilateral triangle, ABC, as shown in Fig. P2.3-14. Determine the angle θ as a function of ΔT, the temperature increase of tie rod AB. Rods AC and BC remain at the reference temperature. The properties of the rods are: area $= A$, modulus of elasticity $= E$, and coefficient of thermal expansion $= \alpha$.



P2.3-14

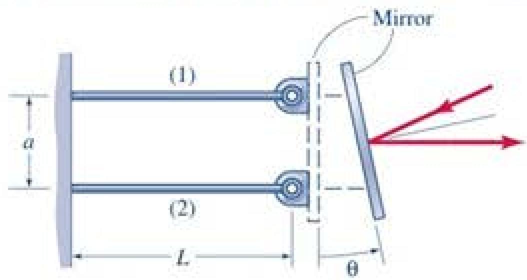
Prob. 2.3-15. As shown in Fig. P2.3-15, a "rigid" beam AB of length $L_{AB} = 30$ in. is supported by a wire BC that is 40 in. long at the reference temperature of $T_0 = 70$°F. Determine an expression that relates the horizontal coordinate y_B of point B to the temperature T of the wire BC in °F if the coefficient of thermal expansion of the wire is $\alpha = 8 \times 10^{-6}$/°F. (Hint: Use the trigonometric *law* of *cosines*.)



P2.3-15

Prob. 2.3-16. A steel pipe ($\alpha = 8 \times 10^{-6}$/°F) has a nominal inside diameter $d = 4.06$ in. at 70°F. (a) What is the inside diameter if the pipe carries steam that raises its temperature to 212°F? (Assume that the outside of the pipe is insulated so that the pipe reaches a uniform temperature of 212°F.) (b) How much would the steam-carrying pipe increase in length if it is originally 40 ft long, if its ends are not restrained against axial motion, and if the temperature increases from 70°F to 212°F?

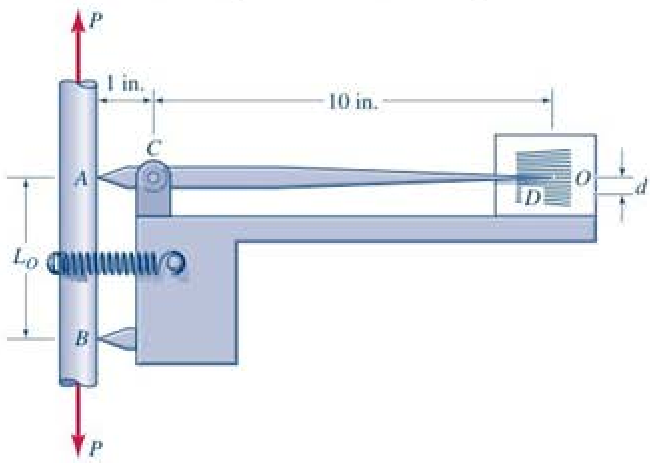
Prob. 2.3-17. The angular orientation, θ, of a "rigid" mirror is controlled by the lengths of rods (1) and (2), as shown in Fig. P2.3-17. At the reference temperature, the rods are the same length: $L_1 = L_2 = 2.00$ m. The distance between the rods is $a = 1.2$ m. (a) If the thermal coefficient of the rods is $\alpha_1 = \alpha_2 = 14 \times 10^{-6}$/°C, determine an expression for the angle θ (in radians) as a function of ΔT_1 and ΔT_2. (b) If the maximum temperature difference that can be achieved between the temperature changes ΔT_1 and ΔT_2 of the rods is 30°C, what is the maximum mirror rotation angle that can be achieved? (Assume that the rods are uniformly heated or cooled along their lengths.)



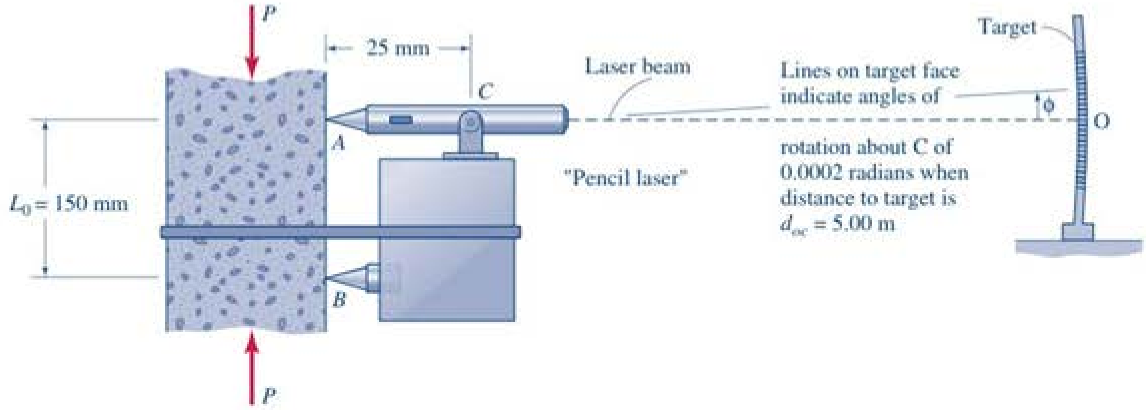
P2.3-17

▼ MEASUREMENT OF STRAIN

Prob. 2.4-1. A *mechanical extensometer* uses the lever principle to magnify the elongation of a test specimen enough to make the elongation (or contraction) readable. The extensometer shown in Fig. P2.4-1 is held against the test specimen by a spring that forces two sharp points against the specimen at A and B. The pointer AD pivots about a pin at C, so that the distance between the contact points at A and B is exactly $L_0 = 6$ in. (the *gage length*, or gauge length, of this extensometer) when the pointer points to the origin, O, on the scale. In a particular test, the extensometer arm points "precisely" at point O when the load P is zero. Later in the test, the 10-in.-long pointer points a distance $d = 0.12$ in. below point O. What is the current extensional strain in the test specimen at this reading?



P2.4-1



P2.4-2

Prob. 2.4-2. A "pencil" *laser extensometer*, like the mechanical lever extensometer in Prob. 2.4-1, measures elongation (from which extensional strain can be computed) by multiplying the elongation. In Fig. P2.4-2 the laser extensometer is being used to measure strain in a reinforced concrete column. The target is set up across the room from the test specimen so that the distance from the fulcrum, C, of the laser to the reference point O on the target is $d_{oc} = 5$ m. Also, the target is set so that the laser beam points directly at point O on the target when the extensometer points are exactly $L_0 =$ 150 mm apart on the specimen, and the cross section at B does not move vertically. At a particular value of (compressive) load P, the laser points upward by an angle that is indicated on the target to be $\phi = 0.0030$ rad. Determine the extensional strain in the concrete column at this load value.

▼ STRESS-STRAIN CURVES MDS 2.5 & 2.6

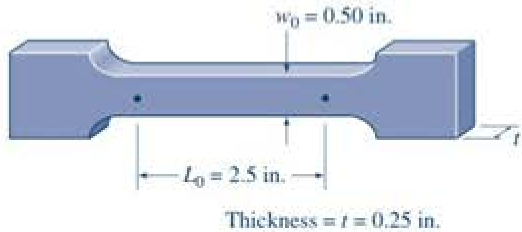
Problems 2.4-3 through 2.4-6. ***You are strongly urged to use a computer program (e.g., MDSolids or a spreadsheet program) to plot the stress-strain diagrams for these problems. In some cases it will be advantageous to make two plots, one covering the initial few points and one covering the entire dataset.***

^{C}Prob. 2.4-3. The data in Table P2.4-3 was obtained in a tensile test of a flat-bar steel specimen having the dimensions shown in Fig. P2.4-3. (a) Plot a curve of engineering stress, σ, versus engineering strain, ϵ, using the given data.

TABLE P2.4-3. Tension-test Data; Flat Steel Bar

P (kips)	ΔL (in.)	P (kips)	ΔL (in.)
1.2	0.0008	6.25	0.0060
2.4	0.0016	6.50	0.0075
3.6	0.0024	6.65	0.0100
4.8	0.0032	6.85	0.0125
5.7	0.0040	6.90	0.0150
5.95	0.0050	—	—

(b) Determine the modulus of elasticity of this material. (c) Use the 0.2%-offset method to determine the yield strength of this material, σ_{YS}.

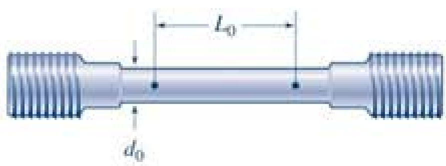


P2.4-3

^{C}Prob. 2.4-4. A standard ASTM tension specimen (diameter $= d_0 = 0.505$ in., gage length $= L_0 = 2.0$ in.) was used to obtain the load-elongation data given in Table P2.4-4. (a) Plot a curve of engineering stress, σ, versus engineering strain, ϵ, using the given data. (b) Determine the modulus of elasticity of this material. (c) Use the 0.2%-offset method to determine the yield strength of this material, σ_{YS}.

TABLE P2.4-4. Tension-test Data; ASTM Tension Specimen

P (kips)	ΔL (in.)	P (kips)	ΔL (in.)
1.9	0.0020	10.0	0.0145
3.8	0.0040	10.4	0.0180
5.7	0.0060	10.65	0.0240
7.6	0.0080	11.00	0.0300
9.0	0.0100	11.05	0.0360
9.5	0.0120	—	—



P2.4-4, P2.4-5, P2.4-6, and P2.4-8

[C]**Prob. 2.4-5.** A tension specimen (diameter = d_0 = 13 mm, gage length = L_0 = 50 mm) was used to obtain the load-elongation data given in Table P2.4-5. (a) Plot a curve of engineering stress, σ, versus engineering strain, ϵ, using the given data. (b) Determine the modulus of elasticity of this material. (c) Use the 0.2%-offset method to determine the yield strength of this material, σ_{YS}. (d) Determine the tensile ultimate stress, σ_{TU}.

TABLE P2.4-5. Tension-test Data

P (kN)	ΔL (mm)	P (kN)	ΔL (mm)
0.0	0.000	27.5	1.68
9.3	0.050	28.4	2.00
14.9	0.200	28.6	2.33
17.7	0.325	28.9	2.68
22.4	0.675	28.4	3.00
25.2	1.00	27.5	3.33
26.6	1.33	26.1	3.68

[C]**Prob. 2.4-6.** A tension specimen (diameter = d_0 = 0.500 in., gage length = L_0 = 2.0 in.) was used to obtain the load-elongation data given in Table P2.4-6. (a) Plot a curve of engineering stress, σ, versus engineering strain, ϵ, using the given data. (b) Determine the modulus of elasticity of this material. (c) Use the 0.2%-offset method to determine the yield strength of this material, σ_{YS}. (d) Determine the tensile ultimate stress, σ_{TU}.

TABLE P2.4-6. Tension-test Data

P (kips)	ΔL (in.)	P (kips)	ΔL (in.)
0.0	0.000	12.5	0.060
5.2	0.005	12.7	0.070
9.4	0.009	12.9	0.080
9.7	0.010	13.0	0.090
10.0	0.013	13.1	0.100
10.6	0.020	13.2	0.110
11.3	0.030	13.2	0.120
11.8	0.040	13.0	0.130
12.2	0.050	12.6	0.138

Prob. 2.4-7. Tension specimens (diameter = d_0 = 0.500 in., gage length = L_0 = 2.00 in.) made of structural materials A and B are tested to failure in tension. (a) At failure the distances between the gage marks are L_{Af} = 2.90 in. and L_{Bf} = 2.22 in.; the corresponding diameters at the failure cross sections are d_{Af} = 0.263 in. and d_{Bf} = 0.471 in., respectively. Determine the *percent elongation in* 2 *in.* and the *percent reduction in area* for these two materials, and classify each material as either *brittle* or *ductile*. (b) From these tensile tests the following data are also obtained: $E_A = 10.0 \times 10^3$ ksi, $(\sigma_Y)_A$ = 5 ksi, $(\sigma_U)_A$ = 13 ksi; $E_B = 10.4 \times 10^3$ ksi, $(\sigma_Y)_B$ = 73 ksi, $(\sigma_U)_B$ = 83 ksi. From the data given here, make rough sketches (to scale) of the stress-strain diagrams of materials A and B.

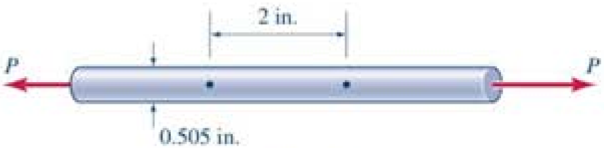
Prob. 2.4-8. Numerous reference sources (e.g., see footnote 13 on p. 41) provide information on the mechanical properties of structural materials—aluminum alloys, copper, nylon, steel, titanium, etc.—including σ-ϵ curves, like those in Figs. 2.12 and 2.13. You will find such resources in your technical library or, perhaps, on the Internet. Obtain, for one particular material, a "room-temperature" stress-strain diagram and any other information on the material that you are able to find. (a) Make a copy of this information to hand in. Be sure to write down complete bibliographic information about your source (e.g., see References on p. R-1). (b) From the room-temperature stress-strain diagram, determine as many mechanical properties as you can (e.g., E, σ_Y). (c) Write a brief paragraph discussing appropriate uses for this material. For example, a particular aluminum alloy may be most useful in sheet form; another alloy is more widely used for extrusions.

▼ MECHANICAL PROPERTIES OF MATERIALS

MDS 2.7

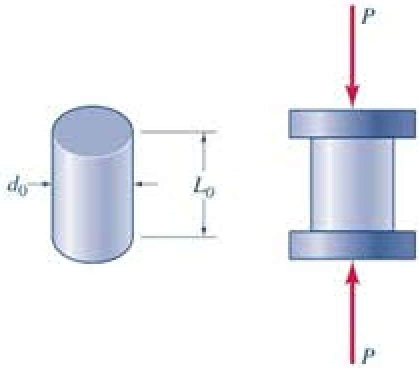
In Problems 2.6-1 through 2.6-10, *dimensions that are shown on the figures, or dimensions that are labeled with subscript 0 (e.g., d_0, L_0), are dimensions of the specimen without any load applied.*

Prob. 2.6-1. A tensile test is performed on an aluminum specimen that is 0.505 in. in diameter using a gage length of 2 in., as shown in Fig. P2.6-1. (a) When the load is increased by an amount P = 2 kips, the distance between gage marks increases by an amount ΔL = 0.00196 in. Calculate the value of the modulus of elasticity, E, for this specimen. (b) If the proportional limit stress for this specimen is σ_{PL} = 45 ksi, what is the distance between gage marks at this value of stress?



P2.6-1

Prob. 2.6-2. A short brass cylinder (d_0 = 15 mm, L_0 = 25 mm) is compressed between two perfectly smooth, rigid plates by an axial force P = 20 kN, as shown in Fig. P2.6-2. (a) If the measured shortening of the cylinder due to this force is 0.0283 mm, what is the modulus of elasticity, E, for this brass specimen? (b) If the increase in diameter due to the load P is 0.0058 mm, what is the value of Poisson's ratio, ν?



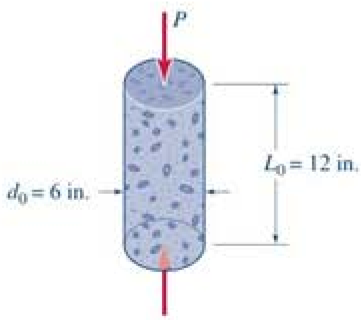
P2.6-2

Prob. 2.6-3. A tensile specimen of a certain alloy has an initial diameter of 0.500 in. and a gage length of 8.00 in. Under a load P = 4500 lb, the specimen reaches its proportional limit and is elongated by 0.0118 in. At this load the diameter is reduced by $2.52(10^{-4})$ in. Determine the following material properties: (a) the modulus of elasticity, E, (b) Poisson's ratio, ν, and (c) the proportional limit, σ_{PL}.

Prob. 2.6-4. A tensile force of 400 kN is applied to a uniform segment of an ASTM-A36 structural steel bar. The cross section is a 50 mm × 50 mm square, and the length of the segment being tested is 200 mm. Using A36 steel data from Table F.2, (a) determine the change in the cross-sectional dimension of the bar, and (b) determne the change in volume of the 200 mm segment being tested.

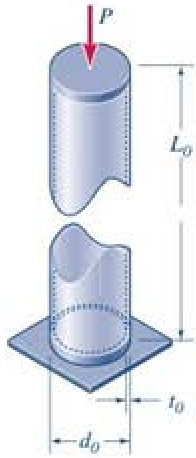
Prob. 2.6-5. A cylindrical rod with an initial diameter of 8 mm is made of 6061-T6 aluminum alloy. When a tensile force P is applied to the rod, its diameter decreases by 0.0101 mm. Using the appropriate aluminum-alloy data from Table F.2, determine (a) the magnitude of the load P, and (b) the elongation over a 200 mm length of the rod.

Prob. 2.6-6. Under a compressive load of P = 24 kips, the length of the concrete cylinder in Fig. P2.6-6 is reduced from 12 in. to 11.9970 in., and the diameter is increased from 6 in. to 6.0003 in. Determine the value of the modulus of elasticity, E, and the value of Poisson's ratio, ν. Assume linearly elastic deformation.



P2.6-6

Prob. 2.6-7. A steel pipe column of initial length L_0 = 4 m, initial outer diameter d_0 = 100 mm, and initial wall thickness t_0 = 10 mm is subjected to an axial compressive load P = 200 kN, as shown in Fig. P2.6-7. If the steel has a modulus of elasticity E = 200 GPa and Poisson's ratio ν = 0.29, determine: (a) the change, ΔL, in the length of the column, and (b) the change, Δt, in the wall thickness.

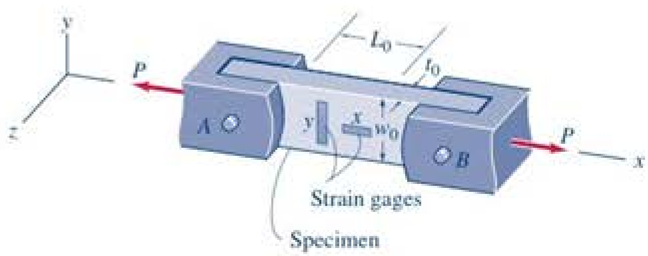


P2.6-7

Prob. 2.6-8. The cylindrical rod in Fig. P2.6-8 is made of annealed (soft) copper with modulus of elasticity $E = 17 \times 10^3$ ksi and Poisson's ratio ν = 0.33, and it has an initial diameter d_0 = 1.9998 in. For compressive loads less than a "critical load" P_{cr}, a ring with inside diameter d_r = 2.0000 in. is free to slide along the cylindrical rod. What is the value of the critical load P_{cr}?

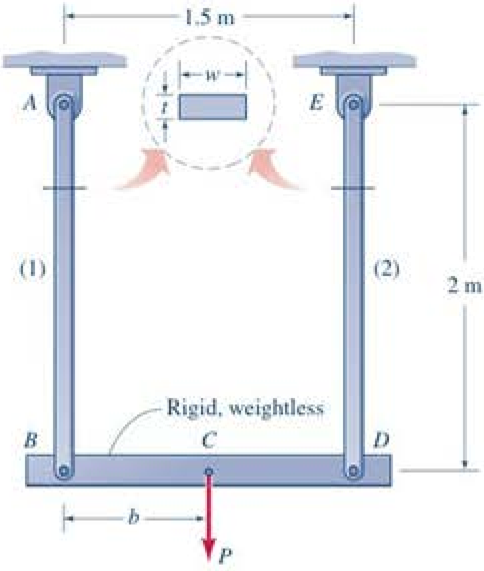
P2.6-8

Prob. 2.6-9. A rectangular aluminum bar (w_0 = 2.0 in., t_0 = 0.5 in.) is subjected to a tensile load P by pins at A and B (Fig. P2.6-9). Strain gages (which are described in Section 8.10) measure the following strains in the longitudinal (x) and transverse (y) directions: $\epsilon_x = 566\mu$, and $\epsilon_y = -187\mu$. (a) What is the value of Poisson's ratio for this specimen? (b) If the load P that produces these values of ϵ_x and ϵ_y is P = 6 kips, what is the modulus of elasticity, E, for this specimen? (c) What is the change in volume, ΔV, of a segment of bar that is initially 2 in. long? (Hint: $\epsilon_z = \epsilon_y$.)



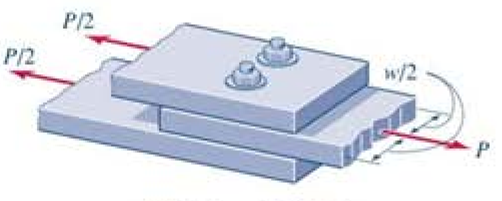
P2.6-9

Prob. 2.6-10. A rigid, weightless beam BD supports a load P and is, in turn, supported by two hanger rods, (1) and (2), as shown in Fig. P2.6-10. The rods are initially the same length $L = 2$ m and are made of the same material. Their rectangular cross sections have original dimensions ($w_1 = 40$ mm, $t_1 = 20$ mm) and ($w_2 = 50$ mm, $t_2 = 25$ mm), respectively. $L_{BD} = 1.5$ m. (a) At what location, b, must the load P act if the rigid bar BD is to remain horizontal when the load is applied? (b) If the longitudinal strain in the hanger rods is $\epsilon_1 = \epsilon_2 = 500\mu$ when the load is $P = 205$ kN, what is the value of the modulus of elasticity, E? (c) If application of load P in the manner described in Parts (a) and (b) above causes the dimension w_2 of hanger rod (2) to be reduced from 50 mm to 49.9918 mm, what is the value of Poisson's ratio, ν, for this material?



P2.6-10

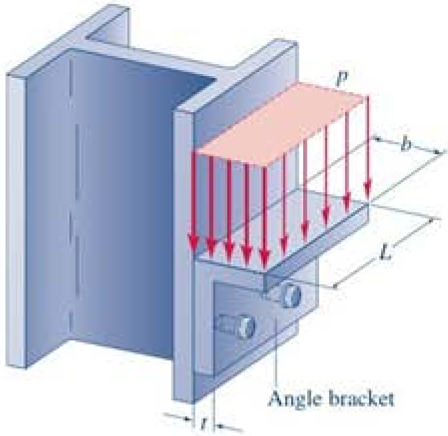
Prob. 2.7-1. Two bolts are used to form a joint connecting rectangular bars in tension, as shown in Fig. P2.7-1. If the bolts have a diameter of 3/8 in., and the load is $P = 20$ kips, determine the average shear stress on the bolt surfaces that are subjected to direct shear.



P2.7-1 and P2.7-2

^{D}Prob. 2.7-2. Two bolts are used to form a joint connecting rectangular bars in tension, as shown in Fig. P2.7-2. Determine the required diameter of the bolts if the average shear stress for the bolts is not to exceed 140 MPa for the given loading of $P = 80$ kN.

Prob. 2.7-3. An angle bracket, whose thickness is $t = 12.7$ mm, is attached to the flange of a column by two 15-mm-diameter bolts, as shown in Fig. P2.7-3. A floor joist that frames into the column exerts a uniform downward pressure of $p = 2$ MPa on the top face of the angle bracket. The dimensions of the loaded face are $L = 152$ mm and $b = 76$ mm. Determine the average shear stress, τ_{avg}, in the bolts. (Neglect the friction between the angle bracket and the column.)

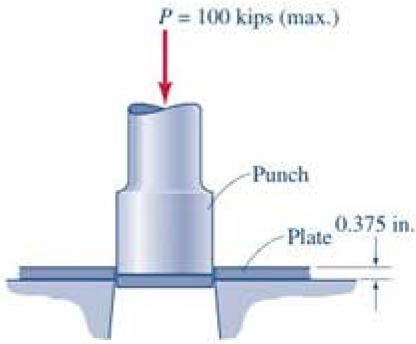


P2.7-3

▼ SHEAR STRESS

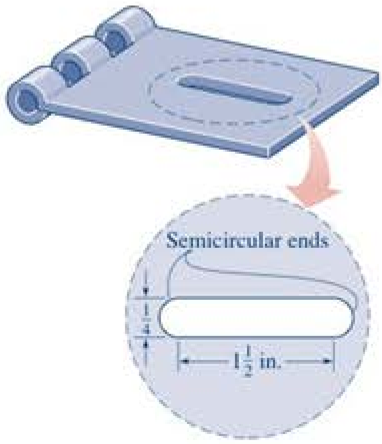
MDS 2.8–2.11

^{D}Prob. 2.7-4. A 100-kip capacity hydraulic punch press is used to punch circular holes in a 3/8-in.-thick aluminum plate, as illustrated in Fig. P2.7-4. If the average punching shear resistance of this plate is 30 ksi, what is the maximum diameter of hole that can be punched?



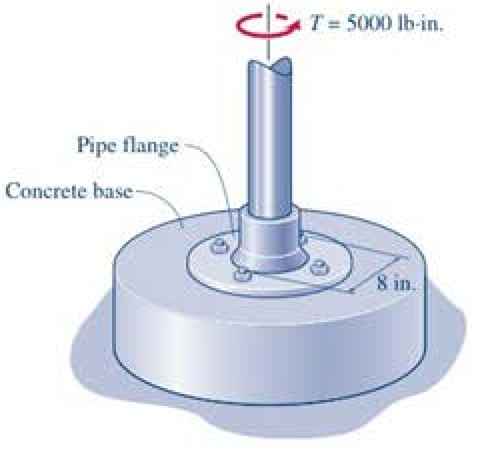
P2.7-4

Prob. 2.7-5. The hole in a hasp plate is punched out by a hydraulic punch press similar to the one in Prob. 2.7-4, with a punch in the shape of the "rectangular" hole as illustrated in Fig. P2.7-5. If the hasp plate is $\frac{1}{16}$-in.-thick steel with an average punching shear resistance of 38 ksi, what is the required punch force P?



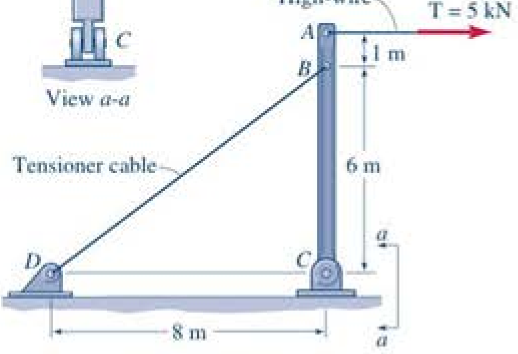
P2.7-5

Prob. 2.7-6. A pipe flange is attached by four bolts, whose effective diameter is 0.425 in., to a concrete base. The bolts are uniformly spaced around an 8-in.-diameter bolt circle, as shown in Fig. P2.7-6. If a twisting couple $T = 5000$ lb · in. is applied to the pipe flange, as shown in Fig. P2.7-6, what is the average shear stress in each of the four bolts? Neglect friction between the pipe flange and the concrete base.



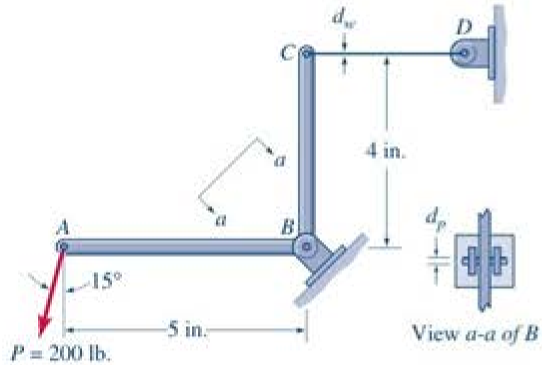
P2.7-6

Prob. 2.7-7. The high-wire for a circus act is attached to a vertical beam AC and is kept taut by a tensioner cable BD, as illustrated in Fig. P2.7-7. At C, the beam AC is attached by a 10-mm-diameter bolt to the bracket shown in View a-a. Determine the average shear stress in the bolt at C if the tension in the high-wire is 5 kN. (Assume that the high-wire is horizontal, and neglect the weight of AC.)



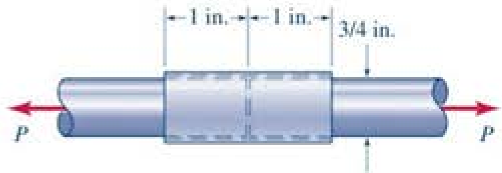
P2.7-7

Prob. 2.7-8. An angle bracket ABC is restrained by a high-strength steel wire CD, and it supports a load P at A, as shown in Fig. P2.7-8. The diameter of the wire CD is $d_w = \frac{1}{8}$ in., and the diameter of the pin at B is $d_p = \frac{3}{8}$ in. Determine the tensile stress in wire CD and the average direct shear stress in the pin at B.



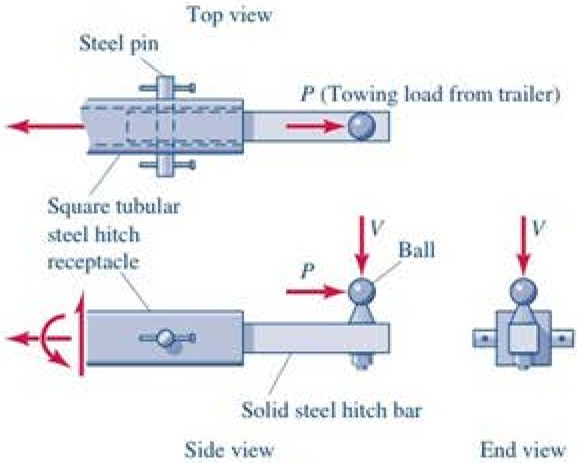
P2.7-8

Prob. 2.7-9. Two $\frac{3}{4}$-in. nylon rods are spliced together by gluing a 2-in. section of plastic pipe over the rod ends, as shown in Fig. P2.7-9. If a tensile force of $P = 500$ lb is applied to the spliced nylon rod, what is the average shear stress in the glue joint between the pipe and the rods?



P2.7-9

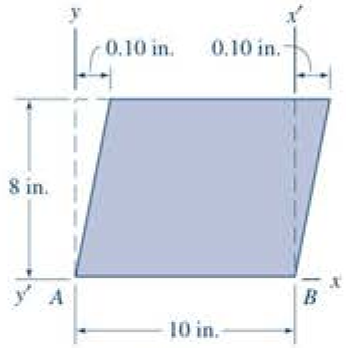
Prob. 2.7-10. Loads P (pull) and V (vertical) in Fig. P2.7-10 are exerted on the ball of a trailer hitch by the trailer it is towing. The ball is bolted to a solid square steel bar which, in turn, fits into a square tubular steel receptacle that is attached to the tow vehicle. A steel pin transmits load from the solid bar to the tubular receptacle. (a) Visit one or more parking lots and see if you can spot such a trailer hitch. (You might need to visit a trailer rental agency or a boat dealership.) Briefly report on what you saw, including sketches that indicate (approximate) dimensions of the hitch parts. (b) Draw a free-body diagram of the solid steel hitch bar together with the ball, indicating how you think loads P and V are transmitted to the tubular receptacle. Indicate specifically what loading the steel pin transmits.



P2.7-10

▼ SHEAR STRAIN

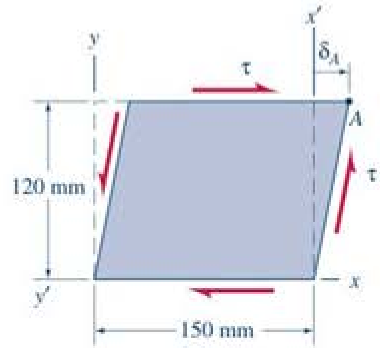
Prob. 2.7-11. A rectangular plate (dashed lines show original configuration) is uniformly deformed into the shape of a parallelogram (shaded figure) as shown in Fig. P2.7-11. (a) Determine the average shear strain, call it $\gamma_{xy}(A)$, between lines in the directions x and y shown in the figure. (b) Determine the average shear strain, call it $\gamma_{x'y'}(B)$, between lines in the directions of x' and y' shown in the figure. (Hint: Don't forget that shear strain is a signed quantity, that is, it can be either positive or negative.)



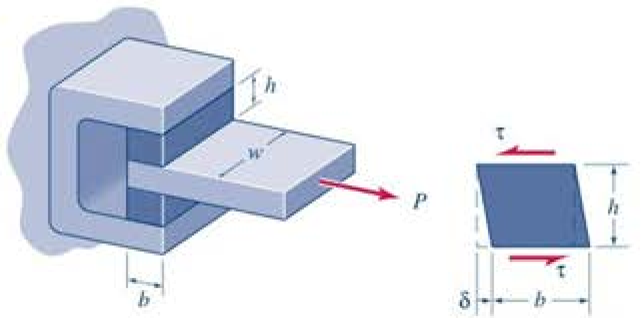
P2.7-11

Prob. 2.7-12. Shear stress τ produces a shear strain γ_{xy} (between lines in the x direction and lines in the y direction) of $\gamma_{xy} = 1200\,\mu$ (i.e., $\gamma = 0.0012\ \frac{\text{m}}{\text{m}}$). (a) Determine the horizontal displacement δ_A of point A. (b) Determine the shear strain $\gamma_{x'y'}$ between the lines in the x' direction and the y' direction, as shown on Fig. P2.7-12.

Prob. 2.7-13. Two identical symmetrically placed rubber pads transmit load from a rectangular bar to a C-shaped bracket, as shown in Fig. P2.7-13. (a) Determine the average shear stress, τ, in the rubber pads on planes parallel to the top and bottom surfaces of the pads if $P = 250$ N and the dimensions of the rubber pads are: $b = 50$ mm, $w = 80$ mm, and $h = 25$ mm. (Although the load is transmitted predominately by shearing deformation, the pads are not undergoing pure shear. However, you can still calculate the *average* shear stress and *average* shear strain.) (b) If the shear modulus of elasticity of the rubber is $G_r = 0.6$ MPa, what is the average shear strain, γ, related to the average shear stress τ computed in Part (a)? (c) Based on the average shear strain determined in Part (b), what is the relative displacement, δ, between the rectangular bar and the C-shaped bracket when the load $P = 250$ N is applied?



P2.7-12



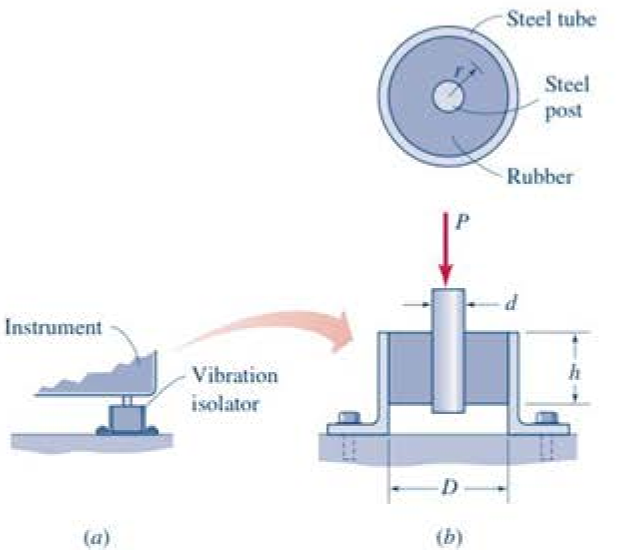
(*a*) Configuration of rubber load-transfer pads.

(*b*) Deformed rubber pad.

P2.7-13 and P2.7-14

[D]Prob. 2.7-14. Two identical, symmetrically placed rubber pads transmit load from a rectangular bar to a C-shaped bracket, as shown in Fig. 2.7-14. The dimensions of the rubber pads are: $b = 3$ in., $w = 4$ in., and $h = 2$ in. The shear modulus of elasticity of the rubber is $G_r = 100$ psi. If the maximum relative displacement between the bar and the bracket is $\delta_{max} = 0.25$ in., what is the maximum value of load P that may be applied? (Use average shear strain and average shear stress in solving this problem.)

***Prob. 2.7-15.** Vibration isolators like the one shown in Fig. P2.7-15 are used to support sensitive instruments. Each isolator consists of a hollow rubber cylinder of outer diameter D, inner diameter d, and height h. A steel center post of diameter d is bonded to the inner surface of the rubber cylinder, and the outer surface of the rubber cylinder is bonded to the inner surface of a steel-tube base. (a) Derive an expression for the average shear stress in the rubber as a function of the distance r from the center of the isolator. (b) Derive an expression relating the load P to the downward displacement of the center post, using G as the shear modulus of the rubber, and assuming that the steel post and steel tube are rigid (compared with the rubber). (Hint: Since the shear strain varies with the distance r from the center, an integral is required.)

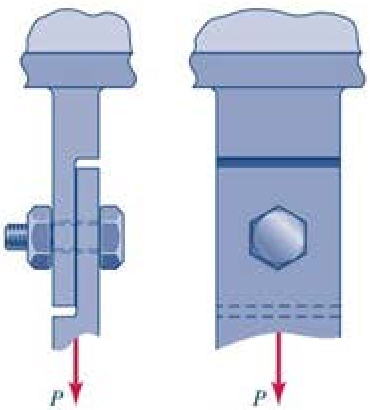


P2.7-15

▼ DESIGN FOR AXIAL LOADS AND DIRECT SHEAR

MDS 2.12–2.15

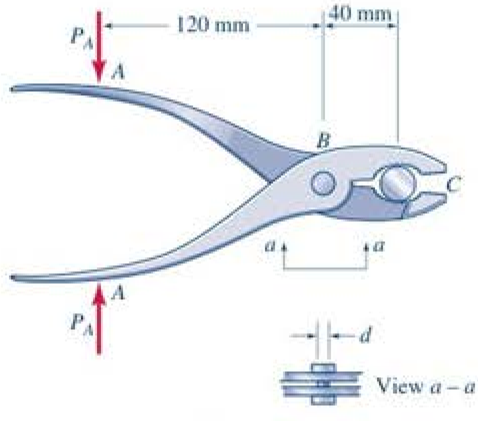
[D]Prob. 2.8-1. A bolted lap joint is used to connect a rectangular bar to a hanger bracket, as shown in Fig. P2.8-1. If the allowable shear stress in the bolt is 15 ksi, and the allowable tensile load on the rectangular bar is to be $P_{allow.} = 2$ kips, what is the required minimum diameter of the bolt shank in inches?



P2.8-1 and P2.8-2

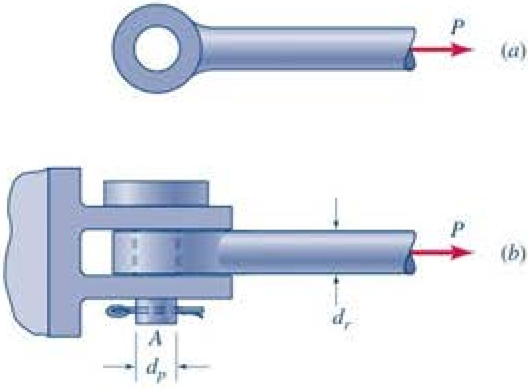
[D]Prob. 2.8-2. A bolted lap joint is used to connect a rectangular bar to a hanger bracket, as shown in Fig. P2.8-2. If the allowable shear stress in the bolt is 80 MPa, and the diameter of the bolt shank 15 mm, what is the allowable tensile load on the rectangular bar, $P_{allow.}$, in kN?

[D]Prob. 2.8-3. The pin that holds the two halves of a pair of pliers together at *B* has a diameter $d = 6.35$ mm and is made of steel for which $\tau_{allow.} = 75$ MPa. What is the allowable force $(P_C)_{allow.}$ (not shown) that can be exerted on the round rod at *C* by each jaw, assuming that the corresponding force P_A is applied to the handles at each of the two places marked *A* in Fig. P2.8-3?



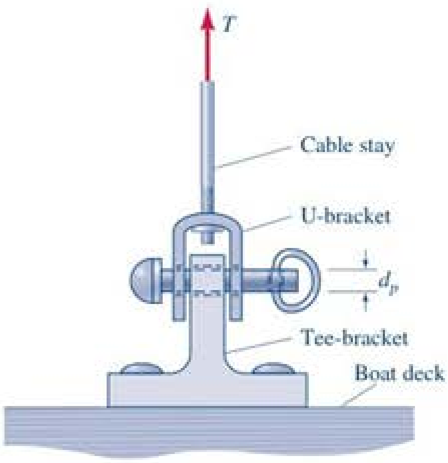
P2.8-3

[D]Prob. 2.8-4. The brass eye-bar in Fig. P2.8-4*a* has a diameter $d_r = 0.500$ in. and is attached to a support bracket by a brass pin of diameter $d_p = 0.375$ in. If the allowable shear stress in the pin is 12 ksi and the allowable tensile stress in the bar is 18 ksi, what is the allowable tensile load $P_{allow.}$?



P2.8-4

[D]Prob. 2.8-5. The forestay (cable) on a sailboat is attached to a tee-bracket on the deck of the boat by a (removable) stainless steel pin. If the allowable shear stress in the pin is $\tau_{allow.} = 11$ ksi, and the diameter of the pin is $d_p = 0.25$ in., what is the allowable tensile force, $T_{allow.}$, in the stay?



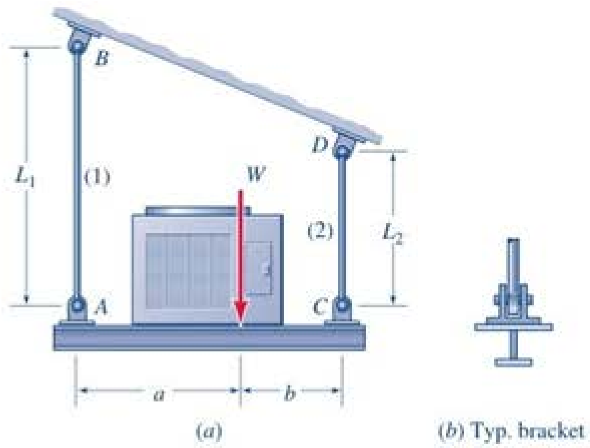
P2.8-5

DProb. 2.8-6. A compressor of weight W is suspended from a sloping ceiling beam by long rods AB and CD of diameters d_1 and d_2, respectively, as shown in Fig. P2.8-6*a*. A typical bracket is shown in Fig. P2.8-6*b*. Using the data given below, determine the allowable compressor weight, $W_{allow.}$ (Neglect the weight of the platform between A and C, and neglect the weight of the two rods. Also, assume that rod AB and the pins at A and B are large enough that they do not need to be considered.)

$$\text{Rod } CD\text{: } d_2 = 10 \text{ mm}, \sigma_{allow.} = 85 \text{ MPa}$$

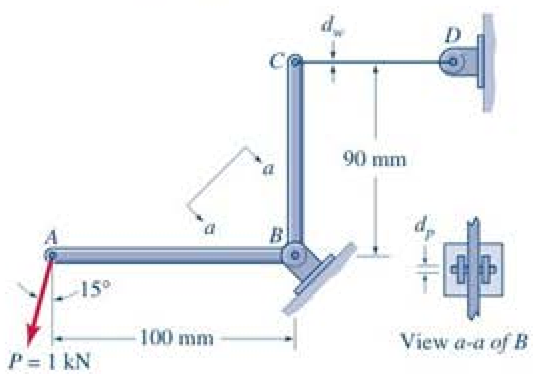
$$\text{Pins at } C \text{ and } D\text{: } d_p = 7 \text{ mm}, \tau_{allow.} = 100 \text{ MPa}$$

$$a = 0.75 \text{ m}, b = 0.50 \text{ m}$$



P2.8-6

DProb. 2.8-7. An angle bracket ABC is restrained by a high-strength steel wire CD, and it supports a load P at A, as shown in Fig. P2.8-7. The strength properties of the wire and the shear pin at B are $\sigma_y = 350$ MPa (wire), and $\tau_U = 300$ MPa (pin at B). If the wire and pin are to be sized to provide a factor of safety against yielding of the wire of $FS_\sigma = 3.3$ and a factor of safety against ultimate shear failure of the pin of $FS_\tau = 3.5$, what are the required diameters of the wire (to the nearest mm) and the pin (to the nearest mm)?

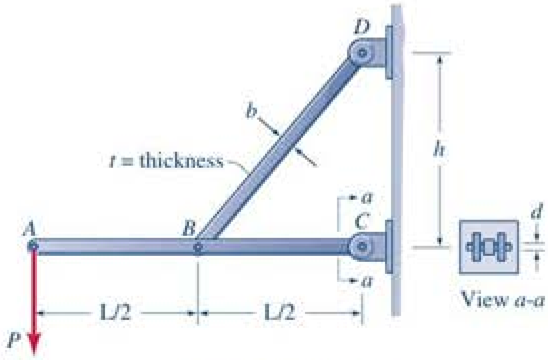


P2.8-7

DProb. 2.8-8. Boom AC in Fig. P2.8-8 is supported by a rectangular steel bar BD, and it is attached to a bracket at C by a high-strength steel pin. Assume that the pin at B is adequate to sustain the loading applied to it, and that the design-critical components are the bar BD and the pin at C. The factor of safety against failure of BD by yielding is $FS_\sigma = 3.0$, and the factor of safety against ultimate shear failure of the pin at C is $FS_\tau = 3.3$. (a) Determine the required thickness, t, of the rectangular bar BD, whose width is b. (b) Determine the required diameter, d, of the pin at C.

$$P = 2400 \text{ lb}, \quad L = 6 \text{ ft}, \quad h = 4 \text{ ft}$$

$$\text{Bar } BD\text{: } b = 1 \text{ in.}, \quad \sigma_Y = 36 \text{ ksi}, \quad \text{Pin } C\text{: } \tau_U = 60 \text{ ksi}$$



P2.8-8 and P2.8-9

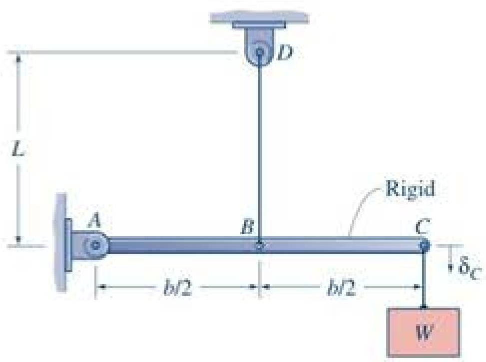
DProb. 2.8-9. Solve Prob. 2.8-8 using the following data:

$$P = 10 \text{ kN}, \quad L = 3 \text{ m}, \quad h = 2 \text{ m}$$

$$\text{Bar } BD\text{: } b = 25 \text{ mm}, \quad \sigma_Y = 250 \text{ MPa},$$

$$\text{Pin } C\text{: } \tau_U = 400 \text{ MPa}$$

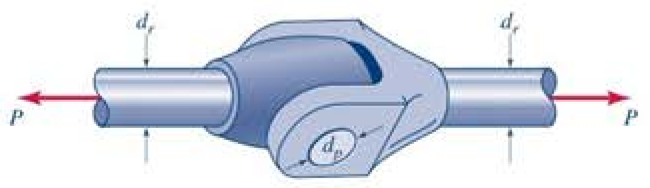
DProb. 2.8-10. A load W is to be suspended from a cable at end C of a rigid beam AC, whose length is $b = 3$ m. Beam AC, in turn, is supported by a steel rod of diameter $d = 25$ mm and length $L = 2.5$ m. Rod BD is made of steel with a yield point $\sigma_Y = 250$ MPa, and modulus of elasticity $E = 200$ GPa. If the maximum displacement at C is $(\delta_C)_{max} = 10$ mm, and there is to be a factor of safety with respect to yielding of BD of $FS_\sigma = 3.3$ and with respect to displacement of $FS_\delta = 3.0$, what is the allowable weight that can be suspended from the beam at C?



P2.8-10 and P2.8-11

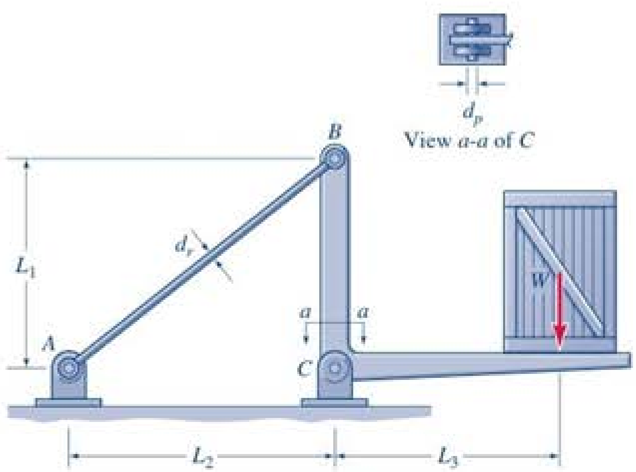
^{D}**Prob. 2.8-11.** A load of $W = 2$ kips is suspended from a cable at end C of a rigid beam AC, whose length is $b = 8$ ft. Beam AC, in turn, is supported by a steel rod BD ($E = 30 \times 10^3$ ksi) of diameter d and length $L = 12$ ft. The rod BD is to the sized so that there will be a factor of safety with respect to yielding of $FS_\sigma = 4.0$ and a factor of safety with respect to deflection of $FS_\delta = 3.0$. The yield strength of rod BD is $\sigma_Y =$ 50 ksi, and the maximum displacement at C is limited to $(\delta_C)_{max} = 0.25$ in. $\left(\text{i.e., } (\delta_C)_{allow} = \frac{(\delta_C)_{max}}{FS_\delta}\right)$. Determine the required diameter, d, of rod BD to the nearest $\frac{1}{8}$ in.

^{D}**Prob. 2.8-12.** A tension rod is spliced together by a pin-and-yoke type connector, as shown in Fig. P2.8-12. The tension rod is to be designed for an allowable load of $P_{allow.} = 3$ kips. If the allowable tensile stress in the rods is $\sigma_{allow.} = 15$ ksi, and the allowable shear stress in the pin is $\tau_{allow.} = 12$ ksi, determine (to the nearest $\frac{1}{16}$ in.) (a) the smallest diameter, d_r, of rod that can be used, and (b) the smallest diameter, d_p, of pin that can be used.



P2.8-12

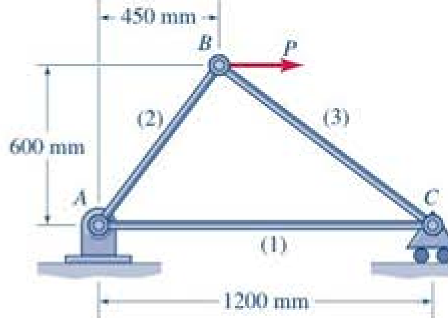
^{D}**Prob. 2.8-13.** The L-shaped loading frame in Fig. P2.8-13 is supported by a high-strength shear pin ($d_p = 0.5$ in, $\tau_U = 50$ ksi) and by a tie-rod AB ($d_r = 0.625$ in., $\sigma_Y = 50$ ksi). Both the tie-rod and the pin are to be sized with a factor of safety of $FS = 3.0$, the tie-rod with respect to tensile yielding, and the shear pin with respect to ultimate shear failure. Determine the allowable platform load, $W_{allow.}$. Let $L_1 = 3$ ft, $L_2 = L_3 = 4$ ft.



P2.8-13 and P2.8-14

^{D}**Prob. 2.8-14.** The L-shaped loading-platform frame in Fig. P2.8-14 is supported by a high-strength steel shear pin at C and by a tie-rod AB. Both the tie rod and the pin are to be sized with a factor of safety of $FS = 3.0$, the tie-rod with respect to tensile yielding and the pin with respect to shear failure. The strength properties of the rod and pin are: $\sigma_Y =$ 340 MPa and $\tau_U = 340$ MPa; the respective lengths are: $L_1 =$ 1.5 m and $L_2 = L_3 = 2.0$ m. (a) If the loading platform is to be able to handle loads W up to $W = 8$ kN, what is the required diameter, d_r, of the tie-rod (to the nearest millimeter)? (b) What is the required diameter, d_p, of the shear pin at C (to the nearest millimeter)?

^{D}**Prob. 2.8-15.** A three-bar, pin-jointed, planar truss supports a single horizontal load P at joint B. Joint C is free to move horizontally. The allowable stress in tension is $(\sigma_T)_{allow.} =$ 140 MPa, and the allowable stress in compression is $(\sigma_C)_{allow.} =$ -85 MPa. If the truss is to support a maximum load $P_{allow.} = 50$ kN, what are the required cross-sectional areas, A_i, of the three truss members?

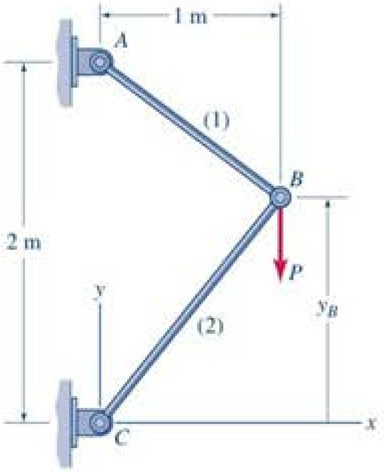


P2.8-15

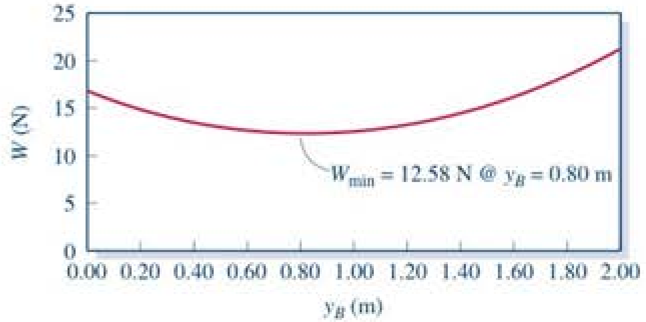
▼ COMPUTER-BASED DESIGN FOR AXIAL LOADS

For Problems 2.8-16 through 2.8-18 ***you are to develop a computer program to generate the required graph(s) that will enable you to choose the "optimum design." You may use a spreadsheet program or other mathematical application program (e.g., TK Solver or Mathcad), or you may write a program in a computer language (e.g., BASIC or FORTRAN).***

^{C}**Prob. 2.8-16.** The pin-jointed planar truss shown in Fig. P2.8-16*a* is to be made of two steel two-force members and support a single vertical load $P = 10$ kN at joint B. For the steel truss members, the allowable stress in tension is $(\sigma_T)_{allow} =$ 150 MPa, the allowable stress in compression is $(\sigma_C)_{allow} =$ -100 MPa, and the weight density is 77.0 kN/m^3. You are to consider truss designs for which joint B can be located at any point along the vertical line that is 1 m to the right of AC, with y_B varying from $y_B = 0$ to $y_B = 2$m. (a) Show that, if each member has the minimum cross-sectional area that meets the strength criteria stated above, the weight W of the truss can be expressed as a function of y_B, the position of joint B, by the function that is plotted in Fig. P2.8-16*b*. (b) What value of y_B gives the minimum-weight truss, and what is the weight of that truss?



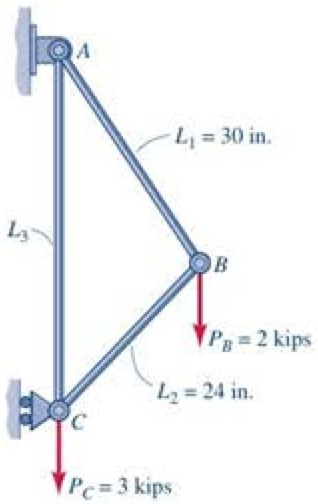
(*a*) A two-bar planar truss.



(*b*) Minimum-weight design for the two-bar planar truss.

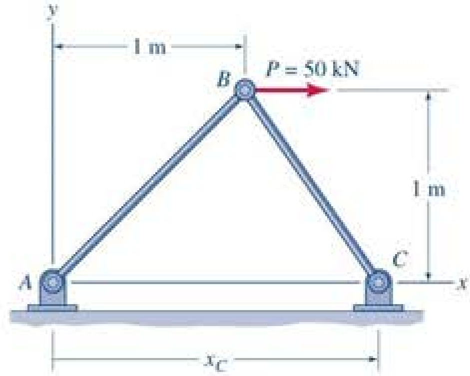
P2.8-16

[C]***Prob. 2.8-17.** The pin-jointed planar truss shown in Fig. P2.8-17 is to be made of three steel two-force members and is to support vertical loads P_B = 2 kips at joint *B* and P_C = 3 kips at joint *C*. The lengths of members *AB* and *BC* are L_1 = 30 in. and L_2 = 24 in., respectively. Joint *C* is free to move vertically. For the steel truss members, the allowable stress in tension is $(\sigma_T)_{allow}$ = 20 ksi, the allowable stress in compression is $(\sigma_C)_{allow}$ = −12 ksi, and the weight density is 0.284 lb/in^3. You are to consider truss designs for which the vertical member *AC* has lengths varying from L_3 = 18 in. to L_3 = 50 in. (a) Show that, if each member has the minimum cross-sectional area that meets the strength criteria stated above, the weight *W* of the truss can be expressed as a function of the length L_3 of member *AC* by a function that is similar to the one plotted in Fig. P2.8-16*b*. (Hint: Use the *law of cosines* to obtain expressions for the angle at joint *A* and the angle at joint *C*.) (b) What value of L_3 gives the minimum-weight truss, and what is the weight of that truss?



P2.8-17

[C]***Prob. 2.8-18.** The pin-jointed planar truss shown in Fig. P2.8-18 is to be made of two aluminum two-force members and support a single horizontal load *P* = 50 kN at joint *B*. For the aluminum truss members, the allowable stress in tension is $(\sigma_T)_{allow}$ = 200 MPa, the allowable stress in compression is $(\sigma_C)_{allow}$ = −130 MPa, and the weight density is 28.0 kN/m^3. You are to consider truss designs for which support *C* can be located at any point along the *x* axis, with x_C varying from x_C = 1 m to x_C = 2.4 m. (a) Show that, if each member has the minimum cross-sectional area that meets the strength criteria stated above, the weight *W* of the truss can be expressed as a function of x_C, the position of support *C*, by a function that is similar to the one plotted in Fig. 2.8-16*b*. (b) What value of x_C gives the minimum-weight truss, and what is the weight of that truss?

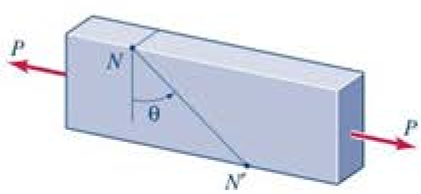


P2.8-18

▼ STRESSES ON INCLINED PLANES

In Problems 2.9-1 through 2.9-13, ***use free-body diagrams and equilibrium equations to solve for the required stresses.***

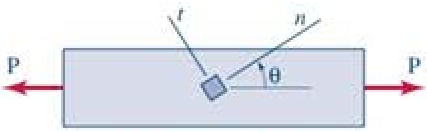
Prob. 2.9-1. The plane *NN'* makes an angle θ = 30° with respect to the cross section of the prismatic bar shown in Fig. P2.9-1. The dimensions of the rectangular cross section



P2.9-1

of the bar are 1 in. × 2 in. Under the action of an axial tensile load P, the normal stress on the NN' plane is $\sigma_n = 8$ ksi. (a) Determine the value of the axial load P; (b) determine the shear stress τ_{nt} on the NN' plane; and (c) determine the maximum shear stress in the bar. Use free-body diagrams and equilibrium equations to solve for the required stresses.

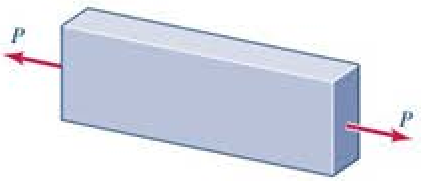
Prob. 2.9-2. The prismatic bar in Fig. P2.9-2 is subjected to an axial compressive load $P = -70$ kips. The cross-sectional area of the bar is 2.0 in^2. Determine the normal stress and the shear stress on the n face and on the t face of an element oriented at angle $\theta = 40°$. Use free-body diagrams and equilibrium equations to solve for the required stresses.



P2.9-2 and P2.9-3

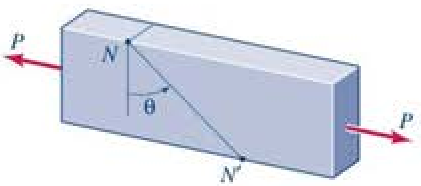
Prob. 2.9-3. A prismatic bar in tension has a cross section that measures 20 mm × 50 mm and supports a tensile load $P = 200$ kN, as illustrated in Fig. P2.9-3. Determine the normal and shear stresses on the n and t faces of an element oriented at angle $\theta = 30°$. Use free-body diagrams and equilibrium equations to solve for the required stresses.

^{D}Prob. 2.9-4. Determine the allowable tensile load P for the prismatic bar shown in Fig. P2.9-4 if the allowable tensile stress is $\sigma_{\text{allow}} = 135$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 100$ MPa. The cross-sectional dimensions of the bar are 12.7 mm × 50.8 mm.



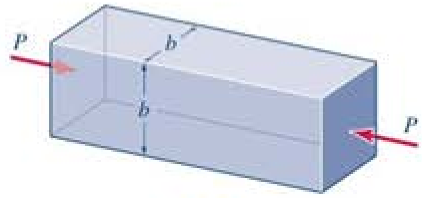
P2.9-4

Prob. 2.9-5. A bar with rectangular cross section is subjected to an axial tensile load P, as shown in Fig. P2.9-5. (a) Determine the angle, call it θ_{na}, of the plane NN' on which $\tau_{nt} = 2\sigma_n$, that is, the plane on which the magnitude of the shear stress is twice the magnitude of the normal stress. (b) Determine the angle, call it θ_{nb}, of the plane on which $\sigma_n = 2\tau_{nt}$. (Hint: You can get approximate answers from Fig. 2.34.)



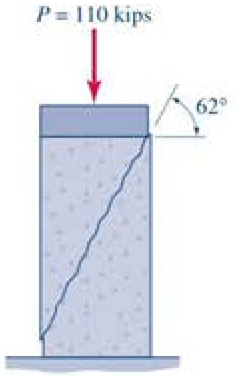
P2.9-5

^{D}Prob. 2.9-6. A brass bar with a square cross section of dimension b is subjected to a compressive load $P = 10$ kips, as shown in Fig. P2.9-6. If the allowable compressive stress for the brass is $\sigma_{\text{allow.}} = -12$ ksi, and the allowable shear stress is $\tau_{\text{allow.}} = 7$ ksi, what is the minimum value of the dimension b, to the nearest $\frac{1}{16}$ in.?



P2.9-6

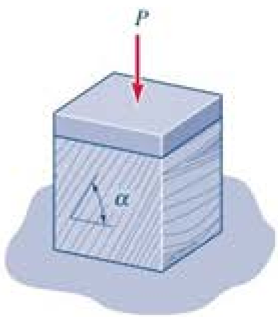
Prob. 2.9-7. A 6-in.-diameter concrete test cylinder is subjected to a compressive load $P = 110$ kips, as shown in Fig. P2.9-7. The cylinder fails along a plane that makes an angle of 62° to the horizontal. (a) Determine the (compressive) axial stress in the cylinder when it reaches the failure load. (b) Determine the normal stress, σ, and shear stress, τ, on the failure plane at failure.



P2.9-7

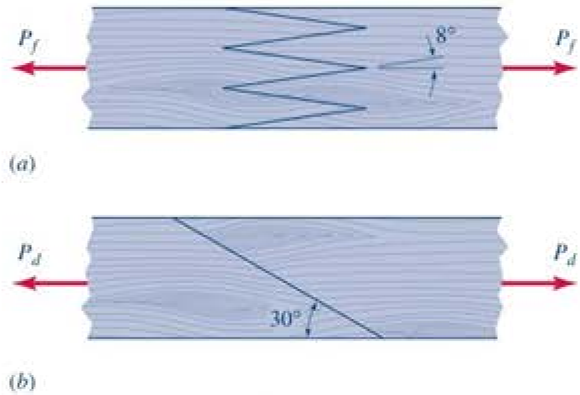
Prob. 2.9-8. A wood cube that has dimension b on each edge is tested in compression, as illustrated in Fig. P2.9-8. The direction of the grain of the wood is shown in the figure. Determine the normal stress σ_n and shear stress τ_{nt} on planes that are parallel to the grain of the wood.

$$P = 5\text{ kN},\quad b = 150\text{ mm},\quad \alpha = 55°$$



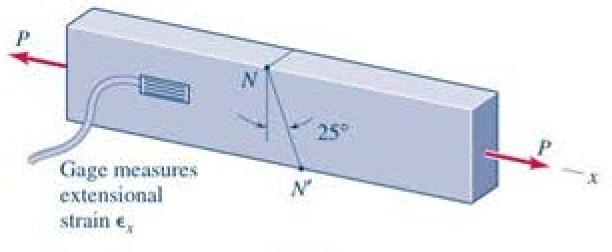
P2.9-8

[D]*__Prob. 2.9-9.__ Either a finger-joint splice, Fig. P2.9-9*a*, or a diagonal lap-joint splice, Fig. P2.9-9*b*, may be used to glue two wood strips together to form a longer tension member. Determine the ratio of allowable loads, $(P_f)_{\text{allow.}}/(P_d)_{\text{allow.}}$ for the following two glue-strength cases: (a) the glue is twice as strong in tension as it is in shear, that is $\tau_{\text{allow.}} = 0.5\sigma_{\text{allow.}}$, and (b) the glue is twice as strong in shear as it is in tension, that is $\tau_{\text{allow.}} = 2\sigma_{\text{allow.}}$. (Hint: For each of the above cases, determine $P_{\text{allow.}}$ in terms of $\sigma_{\text{allow.}}$, using the given glue strength ratios.)



P2.9-9

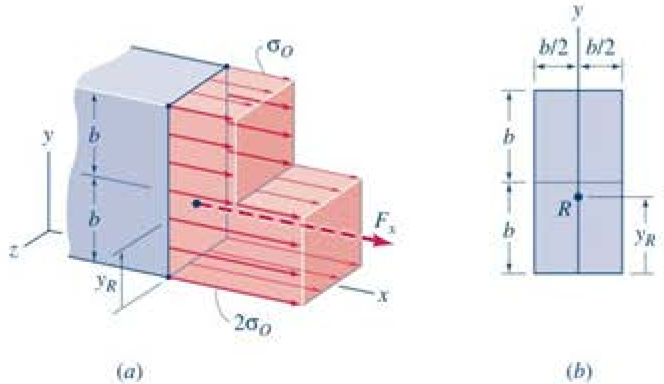
Prob. 2.9-10. At room temperature (70°F) and with no axial load ($P = 0$) the extensional strain of the prismatic bar (Fig. P2.9-10) in the axial direction is zero, that is, $\epsilon_x = 0$. Subsequently, the bar is heated to 120°F and a tensile load P is applied. The material properties for the bar are: $E = 10 \times 10^3$ ksi and $\alpha = 13 \times 10^{-6}$/°F, and the cross-sectional area of the bar is 1.8 in^2. For the latter load-temperature condition, the extensional strain is found to be $\epsilon_x = 900 \times 10^{-6} \frac{\text{in.}}{\text{in.}}$. (a) Determine the value of the axial tensile load P. (b) Determine the normal stress and the shear stress on the oblique plane NN'. (Note: The total strain is the sum of strain associated with normal stress σ_x (Eq. 2.14) and the strain due to change of temperature ΔT (Eq. 2.8).)



P2.9-10

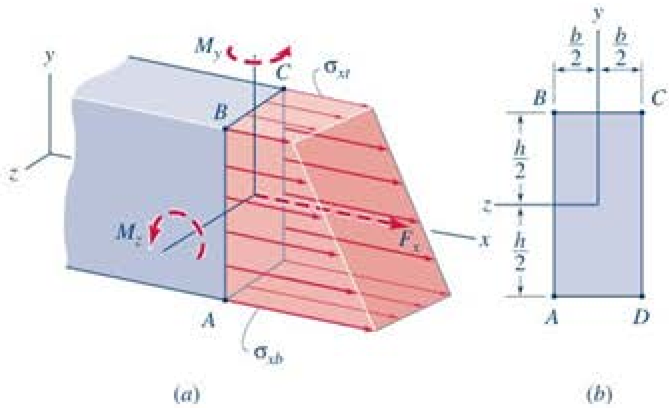
STRESS RESULTANTS

Prob. 2.12-1. The normal stress, σ_x, over the top half of the cross section of the rectangular bar in Fig. P2.12-1 is σ_0, while the normal stress acting on the bottom half of the cross section is $2\sigma_0$. (a) Determine the value of the resultant axial force, F_x. (b) Locate the point R in the cross section through which the resultant axial force, F_x, acts.



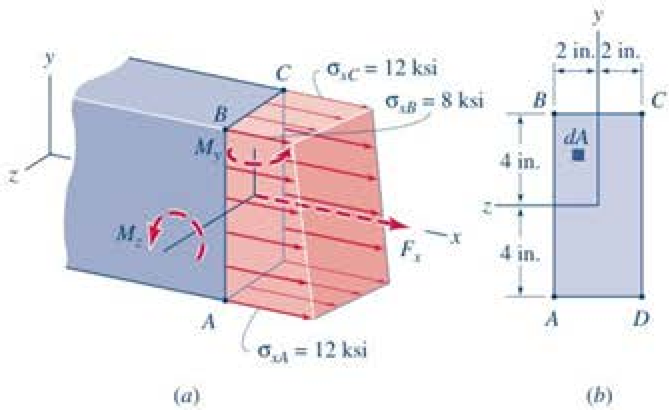
P2.12-1

Prob. 2.12-2 The normal stress on the rectangular cross section $ABCD$ in Fig. P2.12-2 varies linearly with respect to the y coordinate. That is, σ_x has the form $\sigma_x = a + by$, varying linearly from σ_{xb} at the bottom edge of the cross section to σ_{xt} at the top edge of the cross section. (a) Show that $M_y = 0$ for this symmetrical normal-stress distribution. (b) Determine an expression for the axial force F_x in terms of the stresses σ_{xb} and σ_{xt} and the dimensions of the cross section, width b and height h. (c) Determine an expression for the corresponding value of the bending moment M_z.



P2.12-2

Prob. 2.12-3. The normal stress on the rectangular cross section $ABCD$ in Fig. P2.12-3 varies linearly with respect to



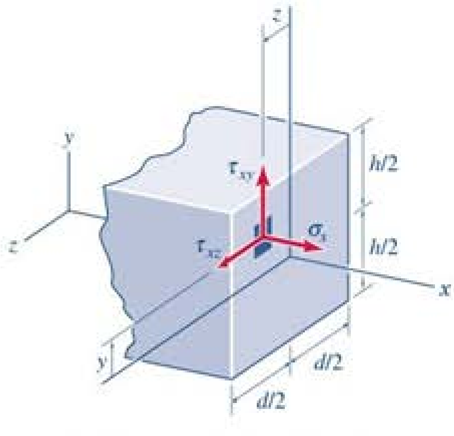
P2.12-3

position (y, z) in the cross section. That is, σ_x has the form $\sigma_x = a + by + cz$. The values of σ_x at corners A, B, and C are: $\sigma_{xA} = 12$ ksi, $\sigma_{xB} = 8$ ksi, and $\sigma_{xC} = 12$ ksi. (a) Determine the value of σ_{xD}, the normal stress at corner D. (b) Determine the axial force, F_x. (c) Determine the bending moment M_y.

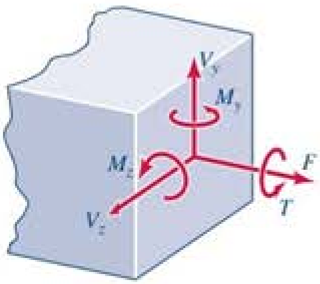
Prob. 2.12-4. The stress distribution on the cross section shown in Fig. P2.12-4a is given by

$$\sigma_x = a + by \quad ; \quad \tau_{xy} = c\left[\left(\frac{h}{2}\right)^2 - y^2\right] \quad ; \quad \tau_{xz} = 0$$

Determine expressions for the resultant forces F_x and V_y and the bending moment M_z in terms of stress-related quantities a, b, and c and the dimensions d and h of the cross section. (See Example 2.13.)



(a) The stresses on cross section x.



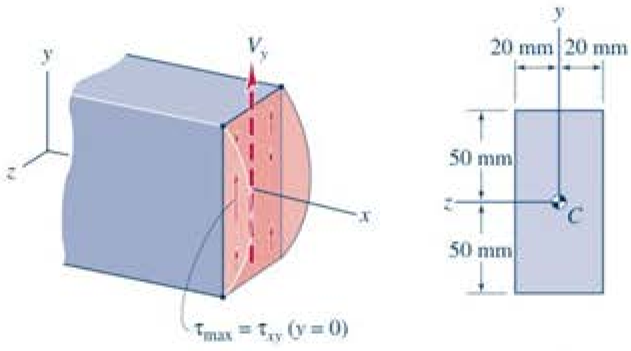
(b) The stress resultants at x.

P2.12-4

[D]Prob. 2.12-5. On a particular cross section of the rectangular bar shown in Fig. 2.12-5 there is shear stress whose distribution has the form

$$\tau_{xy} = \tau_{max}\left[1 - \left(\frac{y}{50}\right)^2\right]$$

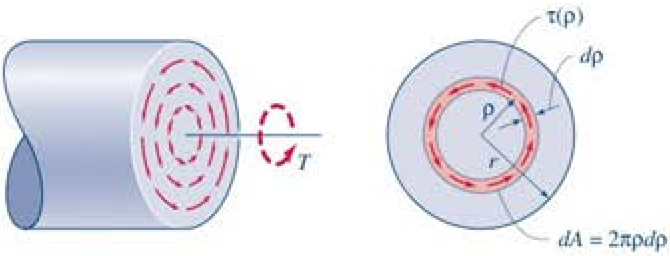
where y is measured in mm from the centroid of the cross section (Fig. P2.12-5b). If the shear stress τ_{xy} may not exceed $\tau_{allow.} = 50$ MPa, what is the maximum shear force V_y that may be applied to the bar at this cross section?



(a) (b)

P2.12-5

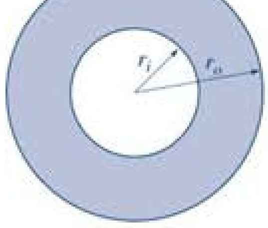
Prob. 2.12-6. On the cross section of a circular rod, the shear stress at a point acts in the circumferential direction at that point, as illustrated in Fig. P2.12-6. The shear stress magnitude varies linearly with distance from the center of the cross section, that is, $\tau = \frac{\tau_{max}\rho}{r}$. Using the ring-shaped area in Fig. P2.12-6b, determine the formula that relates τ_{max} and the resultant torque, T.



(a) (b)

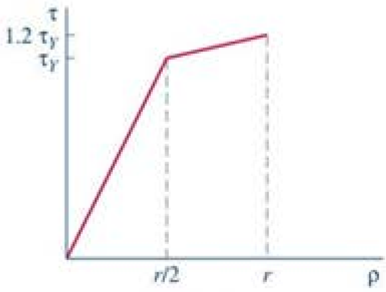
P2.12-6

Prob. 2.12-7. Determine the relationship between τ_{max} and T if, instead of acting on a solid circular bar, as in Fig. P2.12-6, the shear stress distribution $\tau = \frac{\tau_{max}\rho}{r_o}$ acts on a tubular cylinder with outer radius r_o and inner radius r_i. (The cross-sectional dimensions are shown in Fig. P2.12-7. See Prob. 2.12-6 for an illustration of the shear stress distribution on a circular cross section and for the definitions of T and ρ.)



P2.12-7

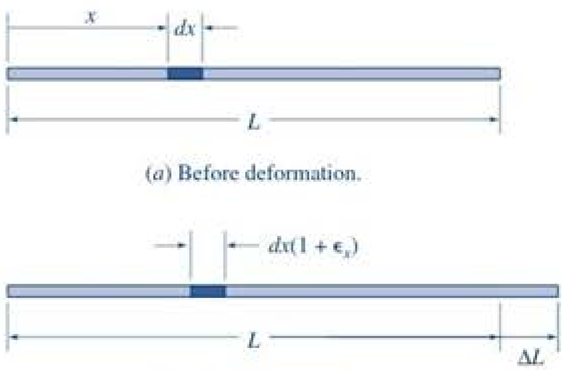
***Prob. 2.12-8.** If the magnitude of the shear stress on a solid, circular rod of radius r varies with radial position ρ as shown in Fig. P2.12-8, determine the formula that relates the resultant torque T to the maximum shear stress τ_Y. (See Prob. 2.12-6 for an illustration of the shear stress distribution on the cross section and for the definitions of T and ρ, and use the area shown in Fig. 2.12-6*b*.)



P2.12-8

▼ STRAIN-DEFORMATION EQUATIONS

Prob. 2.12-9. (a) Using Figs. P2.12-9 and the definition of extensional strain given in Eq. 2.35, show that the change in length, ΔL, of a thin wire whose original length is L is given by $\Delta L = \int_0^L \epsilon_x(x)\, dx$, where $\epsilon_x(x)$ is the extensional strain of the wire at x. (b) Determine the elongation of a 2-m-long wire if it has a coefficient of thermal expansion $\alpha = 20 \times 10^{-6}/°C$, and if the change in temperature along the wire is given by $\Delta T = 10x^2$ (°C).

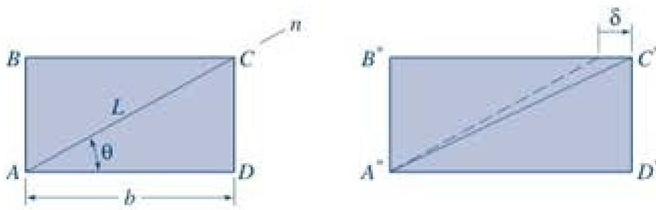


(*a*) Before deformation.

(*b*) After deformation.

P2.12-9

Prob. 2.12-10. The thin, rectangular plate $ABCD$ shown in Fig. P2.12-10*a* undergoes uniform stretching in the x direction, so that it is elongated by an amount δ. Determine an expression for the (uniform) extensional strain ϵ_n of the diagonal AC. Express your answer in terms of δ, L, and the angle θ. Assume that $\delta \ll L$ and see Appendix A.2 for relevant approximations.

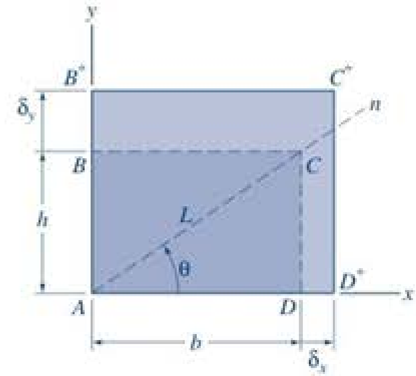


(*a*) Undeformed plate.

(*b*) Deformed plate.

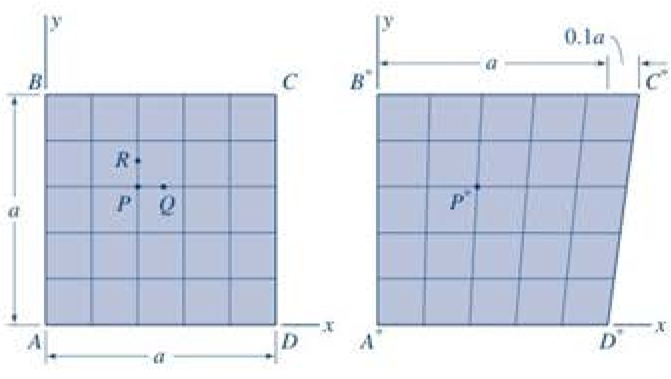
P2.12-10

Prob. 2.12-11. A rectangular plate $ABCD$ with base b and height h is uniformly stretched an amount δ_x in the x direction and δ_y in the y direction to become the enlarged rectangle $AB^*C^*D^*$ shown in Fig. P2.12-11. Determine an expression for the (uniform) extensional strain ϵ_n of the diagonal AC. Express your answer in terms of δ_x, δ_y, L, and θ, where $L = \sqrt{b^2 + h^2}$ and $\tan\theta = h/b$. Base your calculations on the small-displacement assumptions, that is, assume that $\delta_x \ll L$ and $\delta_y \ll L$. (See Appendix A.2 for relevant approximations.)



P2.12-11

Prob. 2.12-12. A thin, square plate $ABCD$ undergoes deformation in which no point in the plate moves in the y direction. Every horizontal line (except the bottom edge) is uniformly stretched as edge CD remains straight and rotates clockwise about D. Using the definition of extensional strain in Eq. 2.35, determine an expression for the extensional strain in the x direction, $\epsilon_x(x, y)$.



(*a*) Undeformed plate.

(*b*) Deformed plate.

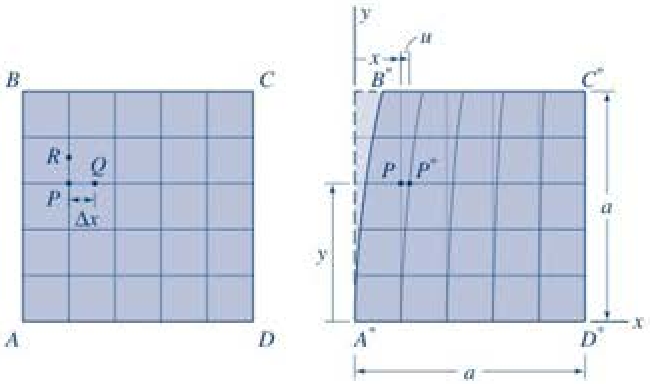
P2.12-12 and P2.12-13

***Prob. 2.12-13.** Using the definition of shear strain, Eq. 2.36, and using the "undeformed plate" and "deformed plate" sketches in Fig. P2.12-13, determine an expression for the shear strain γ_{xy} as a function of position in the plate, that is, $\gamma_{xy}(x, y)$.

Prob. 2.12-14. A thin, square plate *ABCD* undergoes deformation such that a typical point *P* with coordinates (x, y) moves horizontally an amount

$$u(x, y) = \overline{PP^*} = \frac{1}{100}(a - x)\left(\frac{y}{a}\right)^2$$

The undeformed and deformed plates are shown in Figs. P2.12-14*a* and P2.12-14*b*, respectively. Using the definition extensional strain in Eq. 2.35, determine an expression for $\epsilon_x(x, y)$, the extensional strain in the x direction.



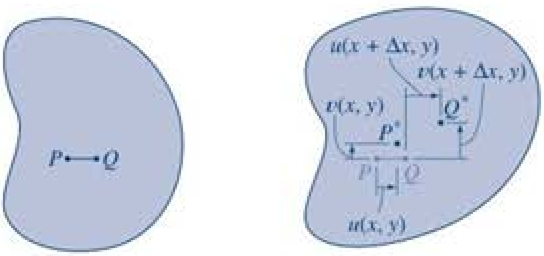
(*a*) Before deformation. (*b*) After deformation.

P2.12-14 and P2.12-15

***Prob. 2.12-15.** Using the definition of shear strain, Eq. 2.36, and using the "before deformation" and "after deformation" sketches in Fig. P2.12-15, determine an expression for the shear strain γ_{xy} as a function of position in the plate, that is, determine $\gamma_{xy}(x, y)$.

***Prob. 2.12-16.** A typical point *P* at coordinates (x, y) in a flat plate moves through *small* displacements $u(x, y)$ and $v(x, y)$ in the x direction and the y direction, respectively. Using the definition of extensional strain, Eq. 2.35, and using the "before deformation" and "after deformation" sketches in Fig. P2.12-16, show that the formula for the extensional strain in the x direction, $\epsilon_x(x, y)$, is the partial differential equation

$$\epsilon_x = \frac{\partial u}{\partial x}$$



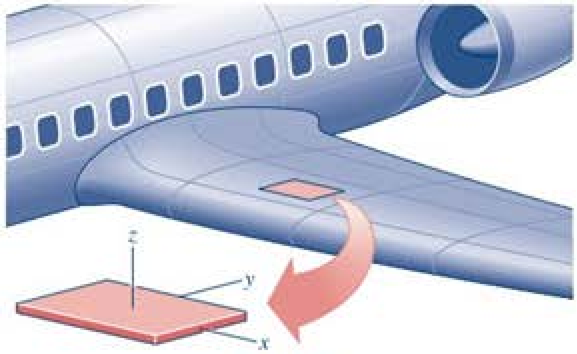
(*a*) Before deformation. (*b*) After deformation.

P2.12-16

▼ HOOKE'S LAW FOR ISOTROPIC MATERIALS; DILATATION

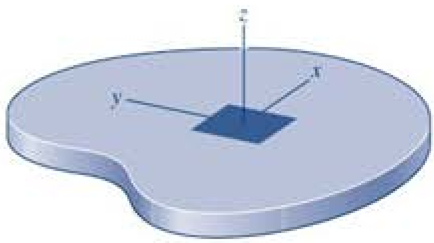
Prob. 2.13-1. When thin sheets of material, like the top "skin" of the airplane wing in Fig. P2.13-1, are subjected to stress, they are said to be in a state of *plane stress*, with $\sigma_z = \tau_{xz} = \tau_{yz} = 0$. Starting with Eqs. 2.38, with $\Delta T = 0$, show that for the case of plane stress Hooke's Law can be written as

$$\sigma_x = \frac{E}{1 - \nu^2}(\epsilon_x + \nu\epsilon_y), \qquad \sigma_y = \frac{E}{1 - \nu^2}(\epsilon_y + \nu\epsilon_x)$$



P2.13-1

Prob. 2.13-2. Figure P2.13-2 shows a small portion of a thin aluminum-alloy plate in plane stress ($\sigma_z = \tau_{xz} = \tau_{yz} = 0$). At a particular point in the plate $\epsilon_x = 600\mu$, $\epsilon_y = -200\mu$, and $\gamma_{xy} = 200\mu$. For the aluminum alloy, $E = 10 \times 10^3$ ksi and $\nu = 0.33$. Determine the stresses σ_x, σ_y, and τ_{xy} at this point in the plate. (Note: Start with Eqs. 2.38, not with Eqs. 2.40.)



P2.13-2

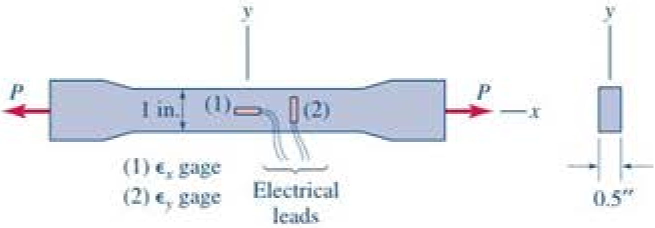
Prob. 2.13-3. Determine the state of strain that corresponds to the following three-dimensional state of stress at a certain point in a steel machine component:

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_y = 20 \text{ MPa}, \quad \sigma_z = 30 \text{ MPa}$$
$$\tau_{xy} = 20 \text{ MPa}, \quad \tau_{xz} = 15 \text{ MPa}, \quad \tau_{yz} = 10 \text{ MPa}$$

Use $E = 210$ GPa and $\nu = 0.30$ for the steel.

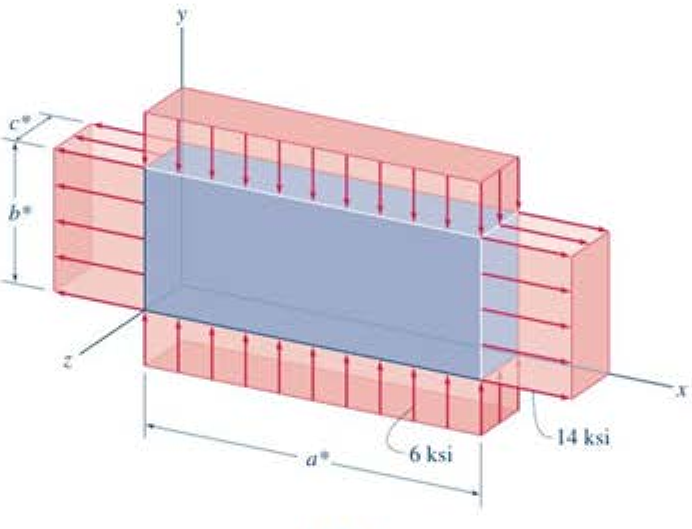
Prob. 2.13-4. The flat-bar plastic test specimen shown in Fig. P2.13-4 has a reduced-area "test section" that measures

0.5 in. × 1.0 in. Within the test section a strain gage oriented in the axial direction measures $\epsilon_x = 0.002\ \frac{\text{in.}}{\text{in.}}$, while a strain gage mounted in the transverse direction measures $\epsilon_y = -0.0008\ \frac{\text{in.}}{\text{in.}}$, when the load on the specimen is $P = 300$ lb. (a) Determine the values of the modulus of elasticity, E, and Poisson's ratio, ν. (b) Determine the value of the dilatation, ϵ_V, within the test section.



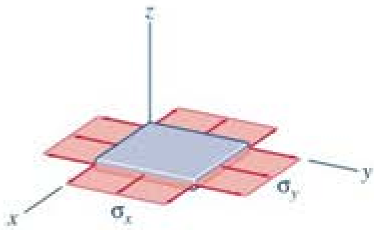
P2.13-4

Prob. 2.13-5. A titanium-alloy bar has the following original dimensions: $a = 10$ in., $b = 4$ in., and $c = 2$ in. The bar is subjected to stresses $\sigma_x = 14$ ksi and $\sigma_y = -6$ ksi, as indicated in Fig. P2.13-5. The remaining stresses—σ_z, τ_{xy}, τ_{xz}, and τ_{yz}—are all zero. Let $E = 16 \times 10^3$ ksi and $\nu = 0.33$ for the titanium alloy. (a) Determine the changes in the lengths: Δa, Δb, and Δc, where $a^* = a + \Delta a$, etc. (b) Determine the dilatation, ϵ_V.



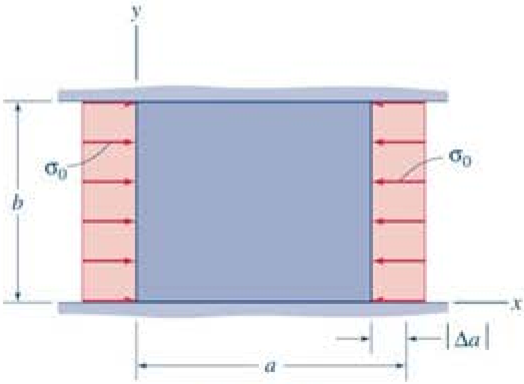
P2.13-5

Prob. 2.13-6. An aluminum-alloy plate is subjected to a biaxial state of stress, as illustrated in Fig. P2.13-6 ($\sigma_z = \tau_{xz} = \tau_{yz} = \tau_{xy} = 0$). For the aluminum alloy, $E = 72$ GPa and $\nu = 0.33$. Determine the stresses σ_x and σ_y if $\epsilon_x = 200\mu$, and $\epsilon_y = 140\mu$. (Note: Start with Eqs. 2.38, not with Eqs. 2.40.)



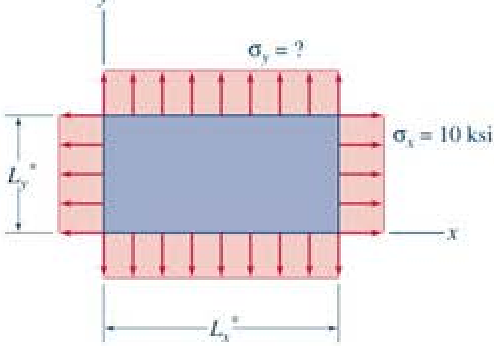
P2.13-6

Prob. 2.13-7. A block of linearly elastic material (E, ν) is compressed between two rigid, perfectly smooth surfaces by an applied stress $\sigma_x = -\sigma_0$, as depicted in Fig. P2.13-7. The only other nonzero stress is the stress σ_y induced by the restraining surfaces at $y = 0$ and $y = b$. (a) Determine the value of the restraining stress σ_y. (b) Determine Δa, the change in the x dimension of the block. (c) Determine the change Δt in the thickness t in the z direction.



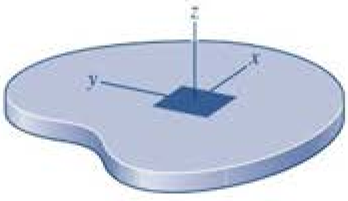
P2.13-7

Prob. 2.13-8. A thin, rectangular plate is subjected to a uniform biaxial state of stress (σ_x, σ_y). All other components of stress are zero. The initial dimensions of the plate are $L_x = 4$ in. and $L_y = 2$ in., but after the loading is applied, the dimensions are $L_x^* = 4.00176$ in., and $L_y^* = 2.00344$ in. If it is known that $\sigma_x = 10$ ksi and $E = 10 \times 10^3$ ksi, (a) what is the value of Poisson's ratio? (b) What is the value of σ_y?



P2.13-8

Prob. 2.13-9. At a point in a thin steel plate in plane stress ($\sigma_z = \tau_{xz} = \tau_{yz} = 0$), $\epsilon_x = 800\mu$, $\epsilon_y = -400\mu$, and $\gamma_{xy} = 200\mu$. For the steel plate, $E = 200$ GPa and $\nu = 0.30$. (a) Determine the extensional strain ϵ_z at this point. (b) Determine the stresses σ_x, σ_y and τ_{xy} at this point. (c) Determine the dilatation, ϵ_V, at this point.

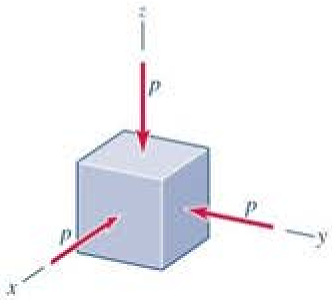


P2.13-9

*__Prob. 2.13-10.__ A block of linearly elastic material (E, ν) is placed under hydrostatic pressure: $\sigma_x = \sigma_y = \sigma_z = -p$; $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$, as shown in Fig. P2.13-10. (a) Determine an expression for the extensional strain ϵ_x ($= \epsilon_y = \epsilon_z$). (b) Determine an expression for the dilatation, ϵ_V. (c) The bulk modulus, k_b, of a material is defined as the ratio of the hydrostatic pressure, p, to the magnitude of the volume change per unit volume, $|\epsilon_V|$, that is,

$$k_b = \frac{p}{|\epsilon_V|}$$

Determine an expression for the bulk modulus of this block of linearly elastic material. Express your answer in terms of E and ν.



(Stresses on hidden faces not shown.)

P2.13-10

Prob. 2.14-1. You are to evaluate a new concept for an environmentally friendly building product, a laminated composite floor panel. This composite panel will use a new material consisting of a recycled polymer filled with recycled wood particles. This recycled material has an elastic modulus of 6 GPa and is produced in sheets 2 mm thick. These are laminated with thin, 0.5-mm-thick sheets of aluminum, $E_{Al} = 70$ GPa. The two different materials are firmly bonded by a strong adhesive to create the laminated composite panel. The final laminated composite panel contains 10 sheets of aluminum and 11 sheets of the recycled material in alternating layers.

Use the techniques discussed in Section 2.14 to calculate approximate values of elastic modulus in the plane of the laminated panel and through the thickness of the laminated panel. Before you begin your calculations, be sure to draw a simple schematic of the laminated composite structure, and use this to help you determine the volume fractions of each material.

Prob. 2.14-2. Consider a polymer matrix having $E_m = 2.8$ GPa, which is reinforced with $V_f = 0.2$ volume fraction of randomly oriented, short glass fibers having $E_f = 72$ GPa. (a) Calculate an approximate elastic modulus, E_c, for this composite material. (b) Would you expect the actual elastic modulus to be higher, or lower, than your approximation?