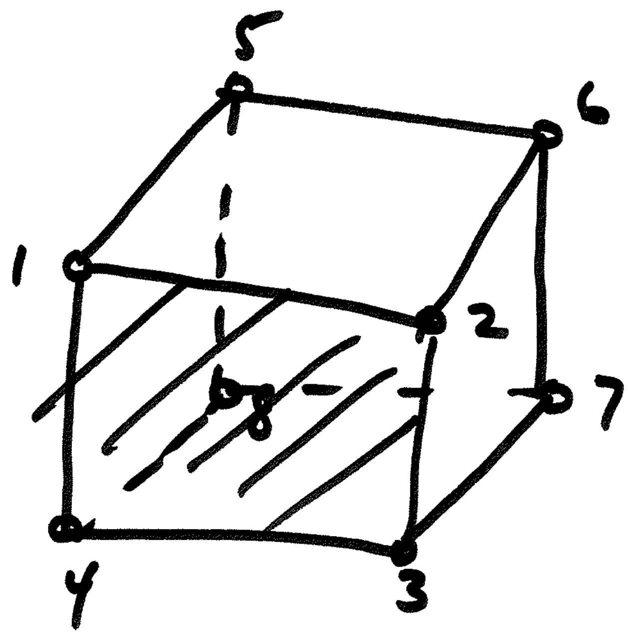
$$\sigma_{vm} = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2 + 6\tau_{xz}^2 + 6\tau_{yz}^2 \right]^{1/2}$$

$$\underset{\sim}{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

$\sigma_{vm} \geq \sigma_{yield}$

then yielding is predicted.

Refer to ABAQUS MANUAL
FOR FACE DEFINITIONS



207 , 1, 2, 3, 4, 5, 6, 7, 8

Applied Finite Element Analysis

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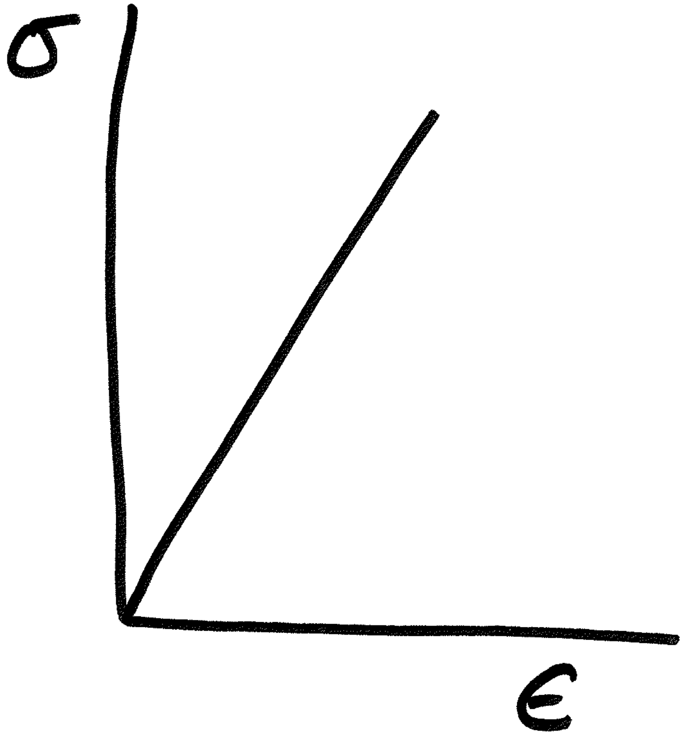
Linear Static Analysis

The most basic type of finite element analysis is Linear Static Analysis.

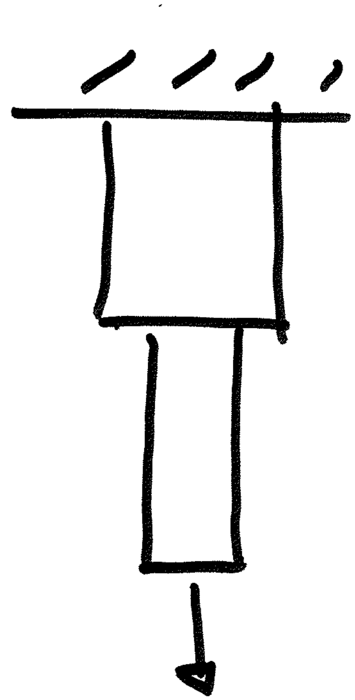
The term "Linear" means that both the material response and the structural response is assumed to be linear throughout the entire analysis.

This is a good assumption for many parts and components that do not undergo large deformation and have no yielding in the material.



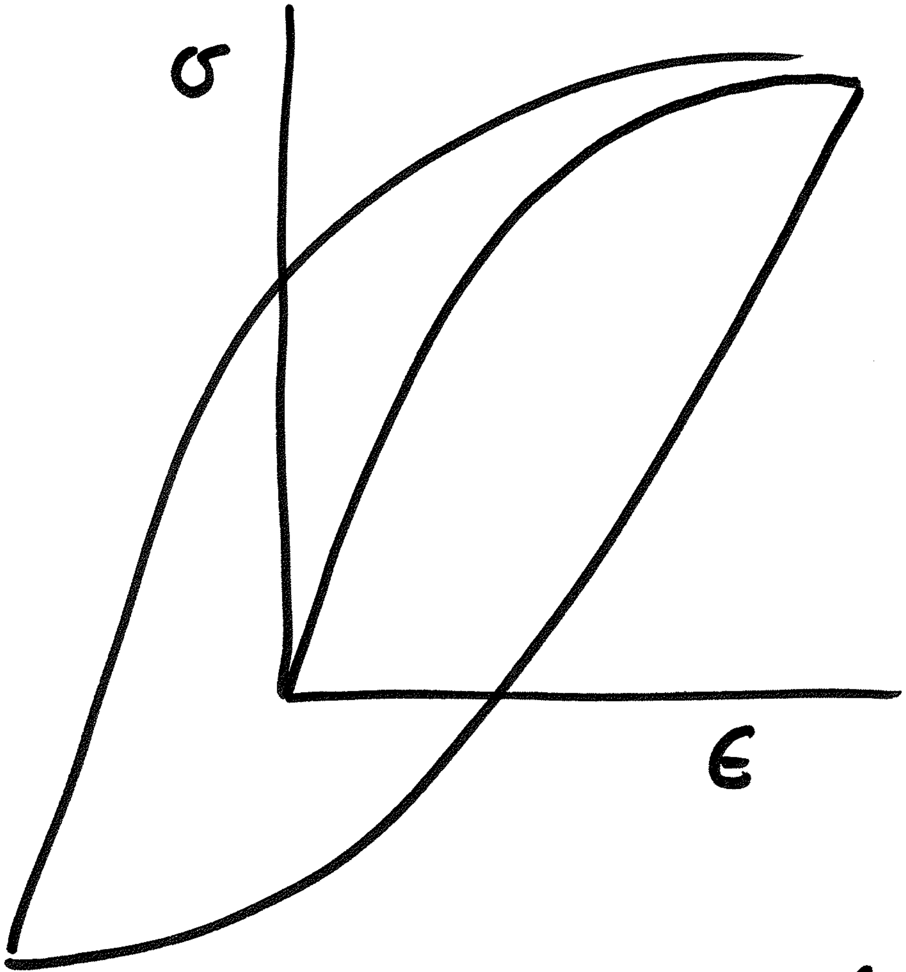
linear material

– i.e. linear elastic material

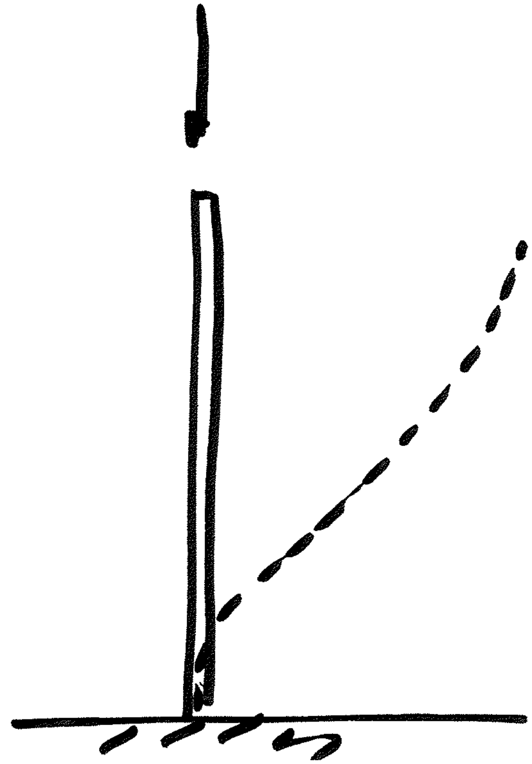


linear structure

i.e. small displacements



non-linear material



non-linear structure

Linearity in Mathematics

The mathematical principle of linearity can be defined as

$$m * (A + B) = m*A + m*B$$

where m is a scalar, and A and B could be vectors, tensors, or other quantities.

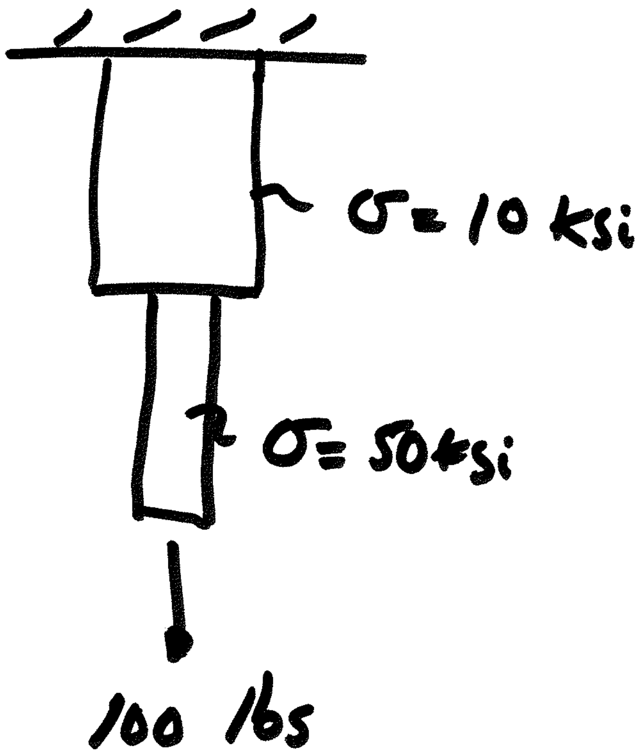
In FEA, we can interpret a linear analysis where m is the magnitude of the loads and A and B are the stresses in the structure.

Linearity in FEA

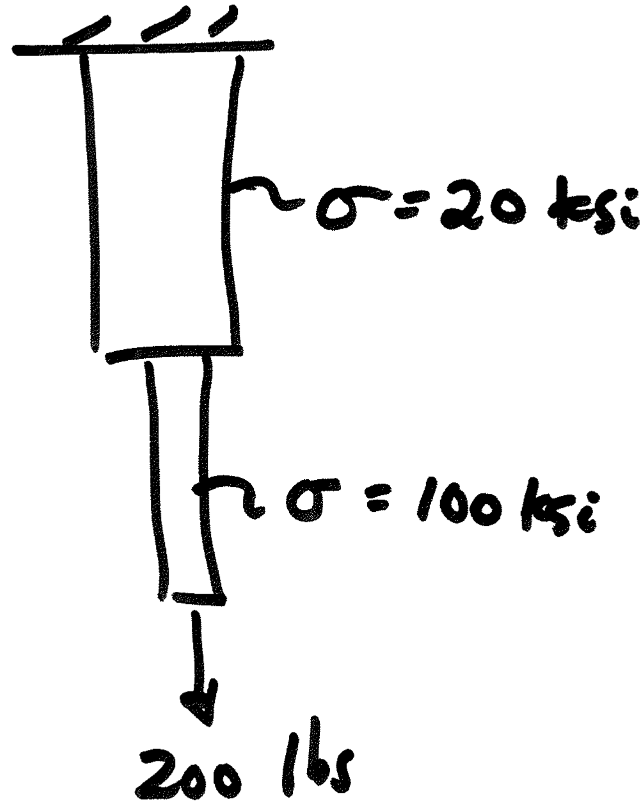
In FEA, we can interpret a linear analysis where m is the magnitude of the loads and A and B are the stresses in the structure.

If we double the load value on our structure, we will double the calculated stress. If we superimpose two loads on the structure, we can superimpose the stress states.

if:

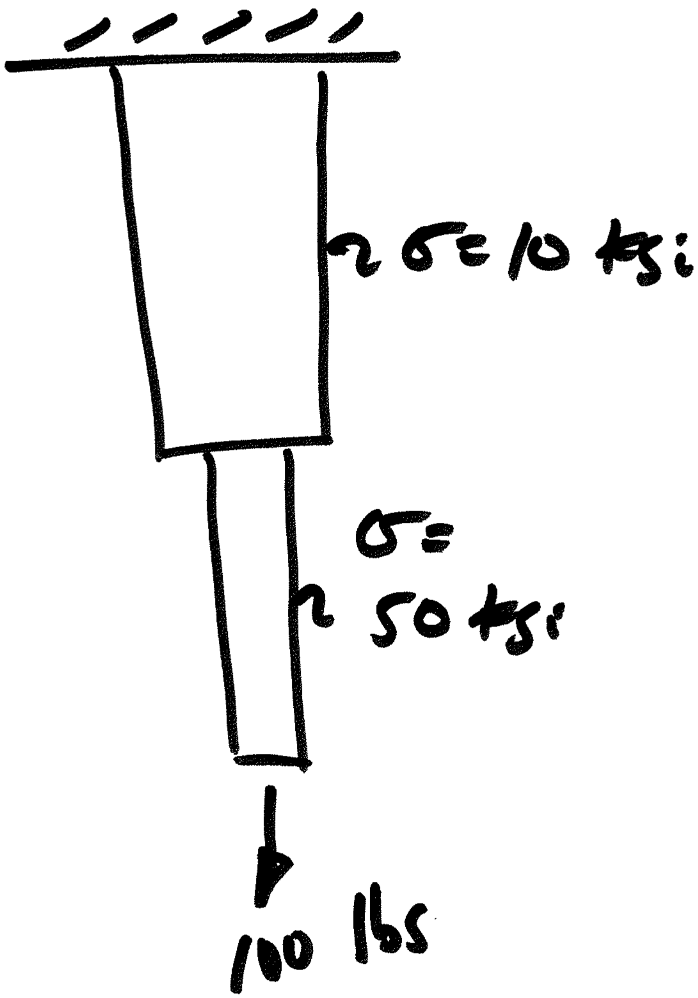


then:

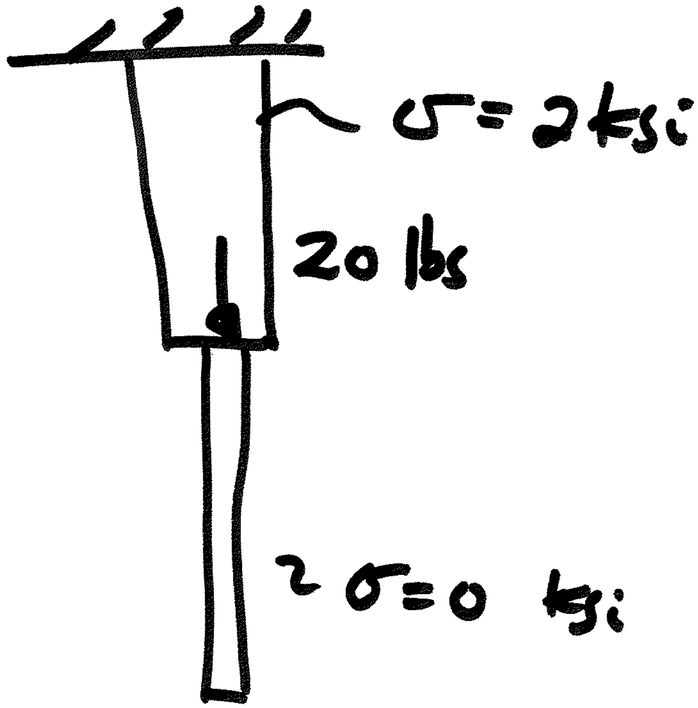


as long as the system is <u>linear</u>.

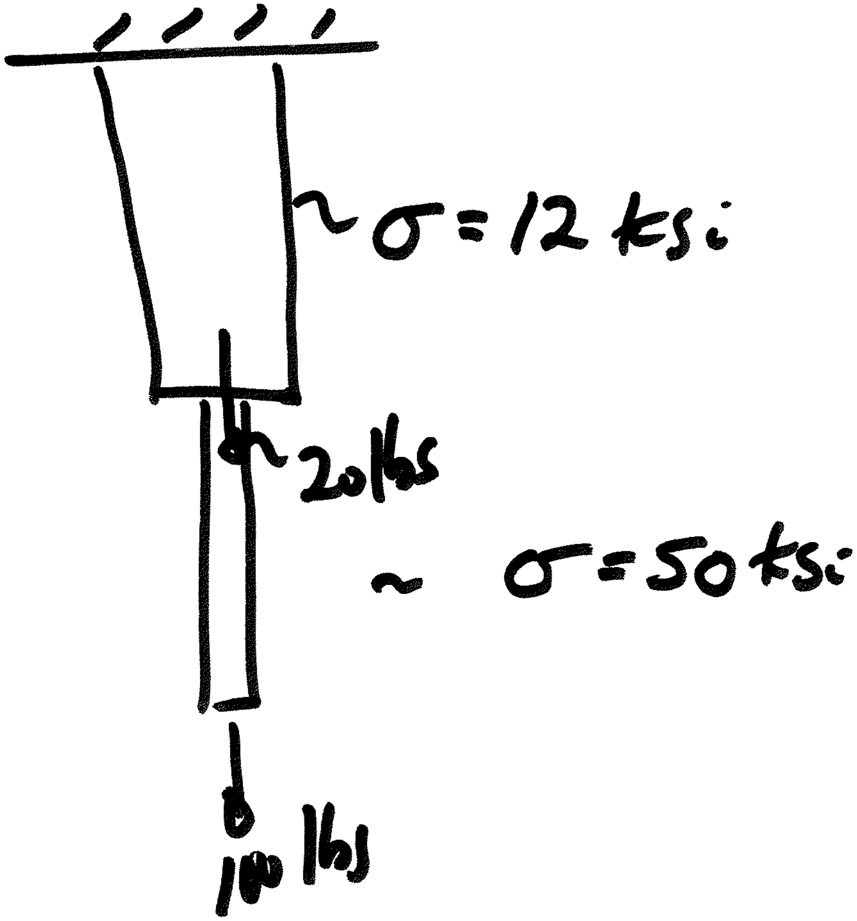
if



and



then



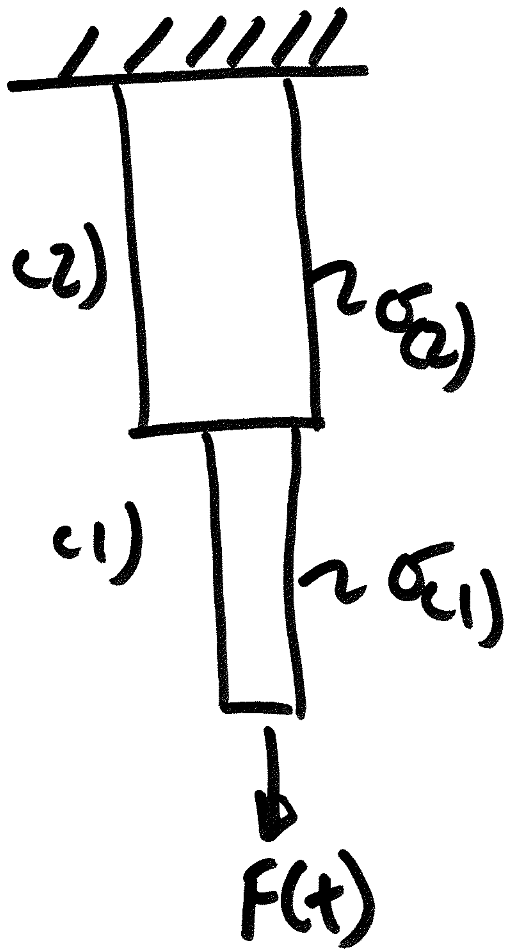
as long as the system remains linear.

Implications

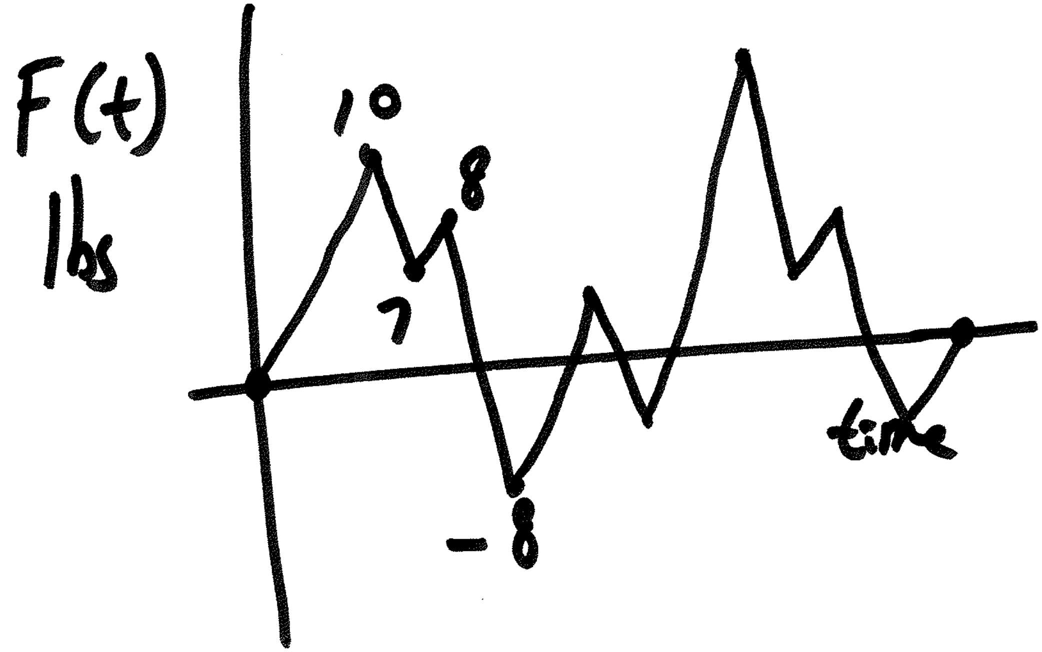
This means for linear static analysis, you can save a lot of computer time by doing separate load cases and superimposing the results.

For example, suppose we had a linear elastic system with one load and it changed with time: F(t).

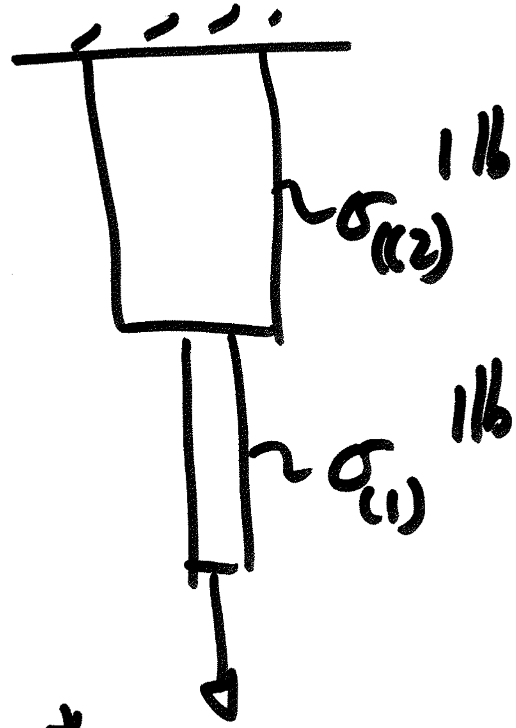
You can conduct an analysis with a unit load, then scale the stress results based on the scale factor in the load history.



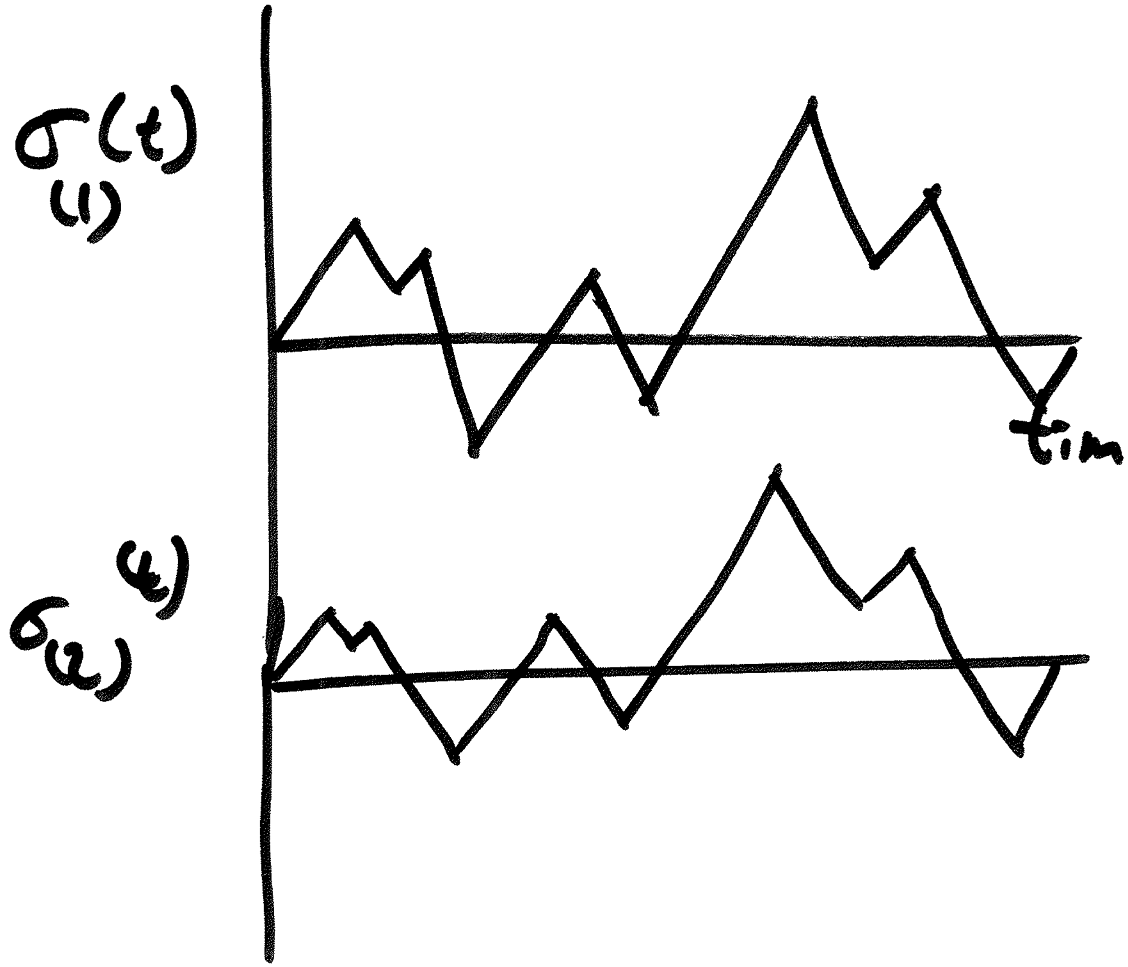
(2)
$\sigma_{(2)}$
(1)
$\sigma_{(1)}$
F(t)



F(t)
lbs
10
8
7
-8
time



$F^* = 1$ lb



$$\sigma_{(1)}(t) = \sigma_{(1)}^{1\,lb} * \frac{F(t)}{F^*}$$

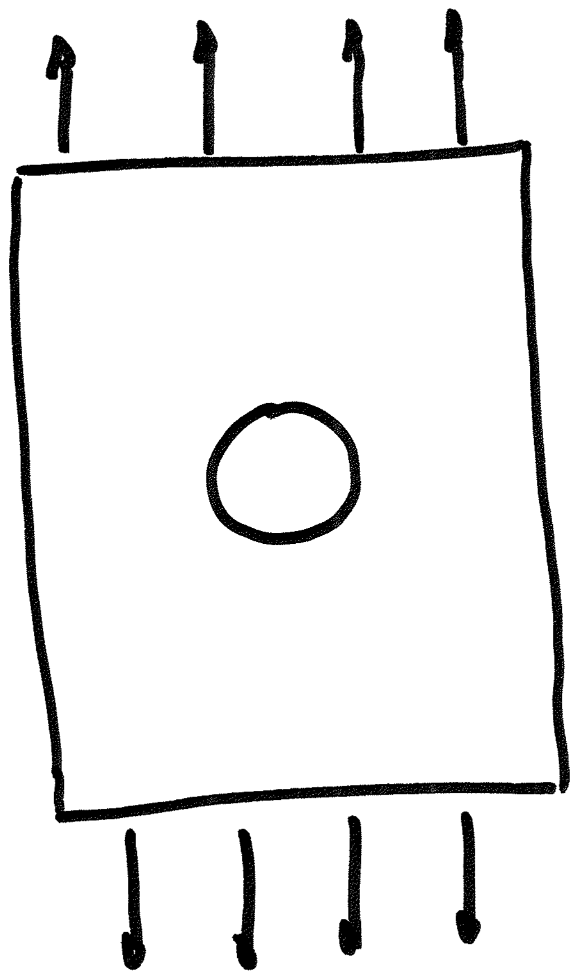
$$\sigma_{(2)}(t) = \sigma_{(2)}^{1\,lb} * \frac{F(t)}{F^*}$$

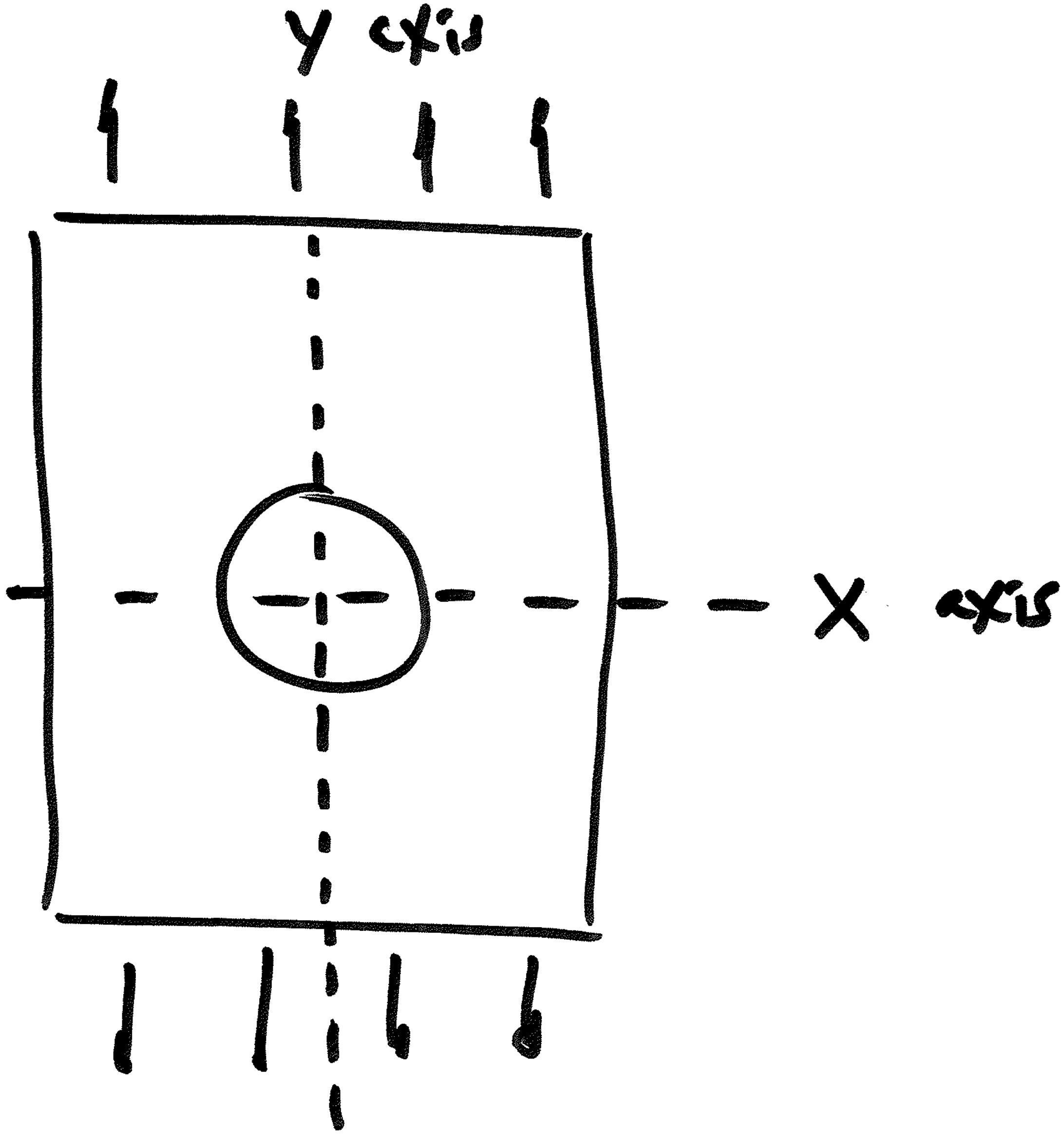
Symmetry in FEA

Problem symmetry can greatly reduce the complexity/size of the model. By problem symmetry, we mean not only the structure, but the loads, too.

Symmetry boundary conditions must be applied to the portion of the model you are building.

Suppose we wanted to model a plate that is in tension and it has a hole in it: the structure has two planes of symmetry.




y axis
X axis

BC's from Symmetry

If you carefully think about how the plate will deform, you will notice that there can not be any movement in the plate in the y-direction along the x-axis, and there can not be any movement in the plate along the x-direction along the y-axis.

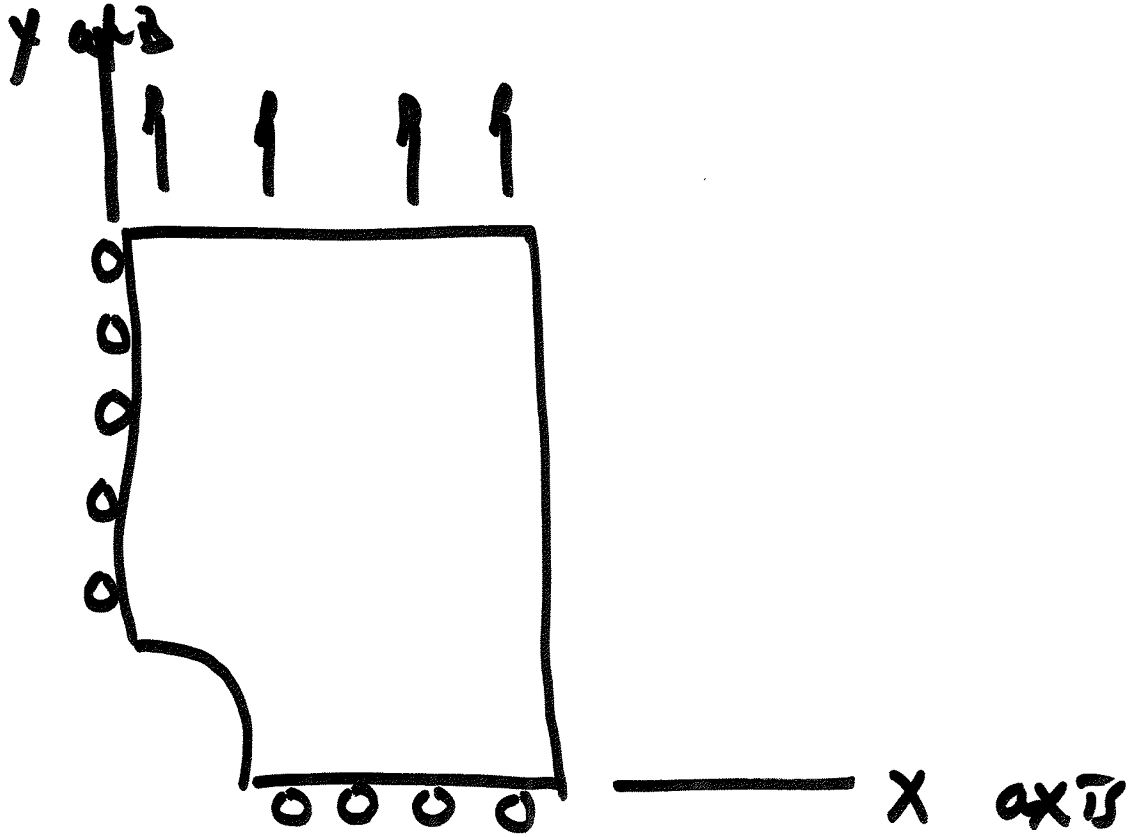
This is because the structure and the loads are symmetric about these lines.

Therefore, if we model one quadrant (one quarter) of the structure, we have all the information about the entire plate.

BC’s from Symmetry con’t

Consider the x-axis. We can not have any movement in the y-direction, but notice that because of Poisson’s ratio, we can still have movement in the x-direction.

Therefore we need to put “rollers” on the x-axis, meaning we only constrain the y displacement along this line.



Y axis
X axis

Cautions about symmetry

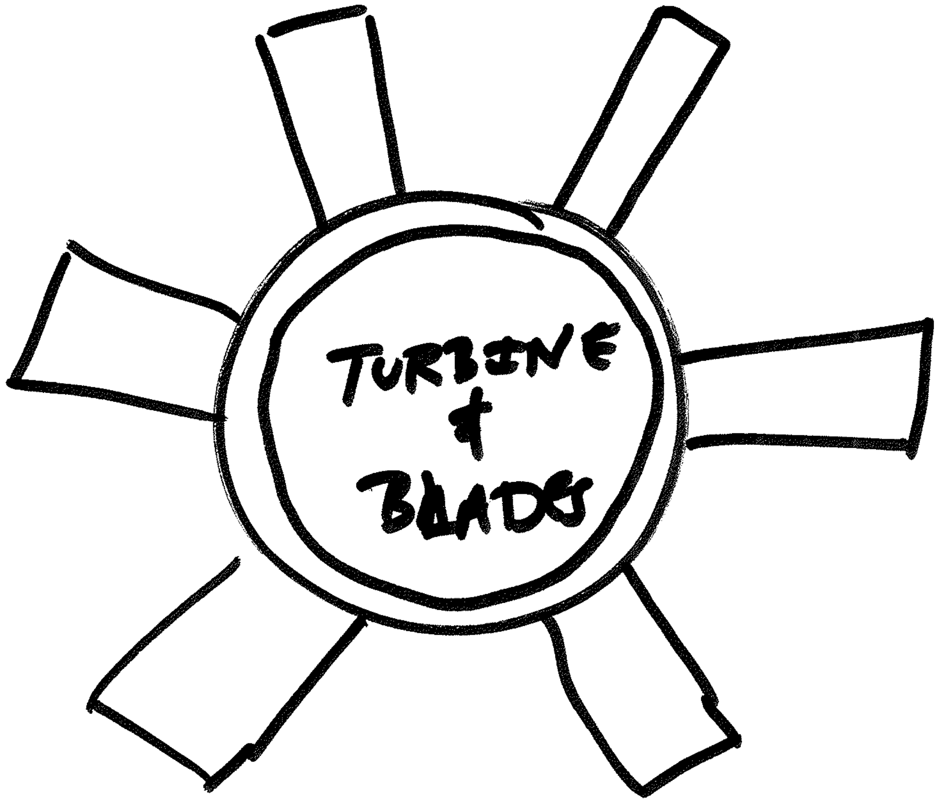
Be careful when interpreting resultant forces in symmetric problems. FEA will calculate the resultant force for only the portion of the component you have modeled.

You will have to account for the resultant forces for un-modeled structure yourself.

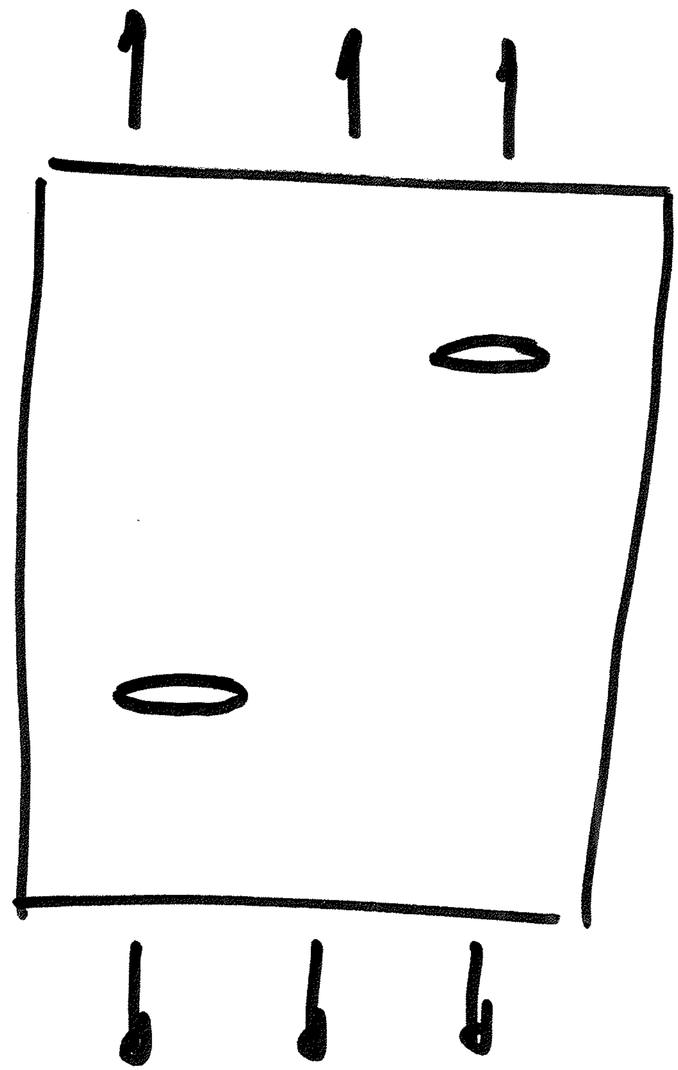
Cyclic Symmetry and Anti Symmetry

It is possible for some problems to account for Cyclic Symmetry and Anti Symmetry.

Be cautious. It is often easier for small components to just model the entire component in these cases.



CYCLIC SYMMETRY



ANTI SYMMETRY