2

STRAIN–LIFE

2.1 INTRODUCTION

The strain–life method is based on the observation that in many components the response of the material in critical locations (notches) is strain or deformation dependent. When load levels are low, stresses and strains are linearly related. Consequently, in this range, load-controlled and strain-controlled test results are equivalent. (Recall from Chapter 1 that stress–life data are generated from load-controlled tests.) At high load levels, in the low cycle fatigue (LCF) regime, the cyclic stress–strain response and the material behavior are best modeled under strain-controlled conditions.

Early fatigue research showed that damage is dependent on plastic deformation or strain. In the strain–life approach the plastic strain or deformation is directly measured and quantified. As discussed in Chapter 1, the stress–life approach does not account for plastic strain. At long lives, where plastic strain is negligible and stress and strain are easily related, the strain–life and stress–life approaches are essentially the same.

Although most engineering structures and components are designed such that the nominal loads remain elastic, stress concentrations often cause plastic strains to develop in the vicinity of notches. Due to the constraint imposed by the elastically stressed material surrounding the plastic zone, deformation at the notch root is considered strain-controlled. The strain–life method assumes that smooth specimens tested under strain-control can simulate fatigue damage at the notch root of an engineering component. Equivalent fatigue damage (and fatigue life) is assumed to occur in the material at the notch root and in the smooth

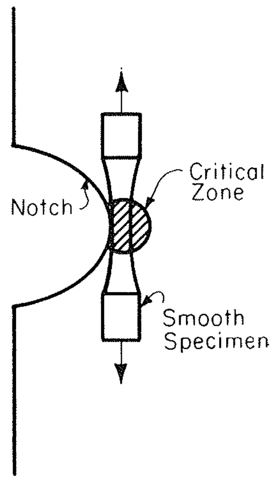

Figure 2.1 Equally stressed volume of material.

specimen when both are subjected to identical stress–strain histories. As seen in Fig. 2.1, the laboratory specimen models an equally stressed volume of material at the notch root.

Crack growth is not explicitly accounted for in the strain–life method. Rather, failure of the component is assumed to occur when the "equally stressed volume of material" fails. Because of this, strain–life methods are often considered "initiation" life estimates. For some applications the existence of a crack is an overly conservative criterion for component failure. In these situations, fracture mechanics methods may be employed to determine crack propagation life from an assumed initial crack size to a final crack length. Total lives are then reported as the sum of the initiation and propagation segments. The fracture mechanics approach is presented in Chapter 3.

The local strain–life approach has gained acceptance as a useful method of evaluating the fatigue life of a notched component. Both the American Society for Testing and Materials (ASTM) and the Society of Automotive Engineers (SAE) have recommended procedures and practices for conducting strain-controlled tests and using these data to predict fatigue lives [1–5].

Fatigue life predictions may be made using the strain–life approach with the following information:

1. Material properties obtained from smooth specimen strain-controlled laboratory fatigue data (cyclic stress–strain response and strain–life data)
2. Stress–strain history at the critical location (e.g., at a notch)
3. Techniques for identifying damaging events (cycle counting)
4. Methods to incorporate mean stress effects
5. Damage summation technique (e.g., Miner's rule)

In this chapter we review the necessary material behavior background needed for an understanding of the strain–life approach. Fundamental equations used in the strain–life method are presented, as well as modified equations to account for mean stress effects. Techniques to identify damaging events and to sum damage (items 3 and 5 above) are presented in Chapter 5. Fatigue analysis of notches using the strain–life approach is discussed in Chapter 4.

2.2 MATERIAL BEHAVIOR

2.2.1 Monotonic Stress–Strain Behavior

Basic definitions. A monotonic tension test of a smooth specimen is usually used to determine the engineering stress–strain behavior of a material where

$$S = \text{engineering stress} = \frac{P}{A_0} \tag{2.1}$$

$$e = \text{engineering strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0} \tag{2.2}$$

The following terms are shown in Fig. 2.2:

P = applied load

l_0 = original length

d_0 = original diameter

A_0 = original area

l = instantaneous length

d = instantaneous diameter

A = instantaneous area

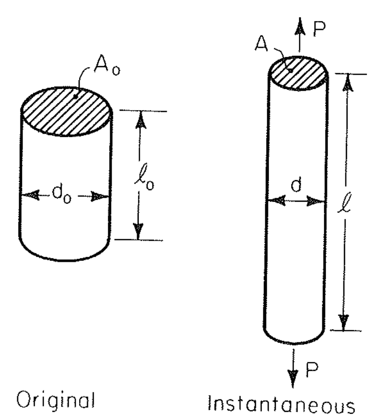

Figure 2.2 Original and deformed (instantaneous) configuration of test specimen.

In tension the true stress is larger than the engineering stress, due to changes in cross-sectional area during deformation.

$$\sigma = \text{true stress} = \frac{P}{A} \tag{2.3}$$

Similarly, until necking occurs in the specimen, true strain is smaller than engineering strain. *True* or *natural strain,* based on an instantaneous gage length l, is defined as

$$\epsilon = \text{true strain} = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} \tag{2.4}$$

Figure 2.3 compares the monotonic tension stress–strain curve using true stress and strain and the engineering values.

True and engineering stress–strain. True stress and strain can be related to engineering stress and strain. The instantaneous length is

$$l = l_0 + \Delta l \tag{2.5}$$

Combining Eqs. (2.4) and (2.5), the true strain is

$$\epsilon = \ln \frac{l_0 + \Delta l}{l_0} \tag{2.6}$$

$$= \ln\left(1 + \frac{\Delta l}{l_0}\right) \tag{2.7}$$

From Eq. (2.2) the true strain in terms of engineering strain is

$$\epsilon = \ln(1 + e) \tag{2.8}$$

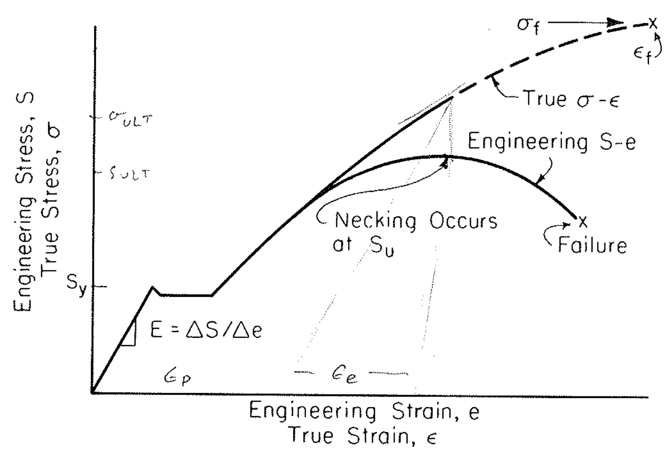

Figure 2.3 Comparison of engineering and true stress–strain.

Equation (2.8) is only valid up to necking. At necking the strain is no longer uniform throughout the gage length.

Assuming that the volume of the material remains constant during straining, we have

$$A_0 l_0 = Al = \text{constant} \tag{2.9}$$

Restating gives

$$\frac{A_0}{A} = \frac{l}{l_0} \tag{2.10}$$

True strain can be stated in terms of cross-sectional area

$$\epsilon = \ln\frac{l}{l_0} = \ln\frac{A_0}{A} \tag{2.11}$$

From Eq. (2.1),

$$P = SA_0 \tag{2.12}$$

and since

$$\sigma = \frac{P}{A} \tag{2.3}$$

true stress can be stated in terms of engineering stress:

$$\sigma = S\frac{A_0}{A} \tag{2.13}$$

Combining Eqs. (2.8) and (2.11) (valid only up to necking) gives us

$$\epsilon = \ln(1 + e) = \ln\frac{A_0}{A} \tag{2.14}$$

or

$$\frac{A_0}{A} = 1 + e \tag{2.15}$$

Therefore, true stress can be stated as a function of engineering stress and strain using Eqs. (2.13) and (2.15).

$$\sigma = S(1 + e) \tag{2.16}$$

This relation is valid only up to necking.

Stress–strain relationships. The total true strain ϵ_t in a tension test can be separated into elastic and plastic components:

1. *Linear elastic strain:* that portion of the strain which is recovered upon unloading, ϵ_e

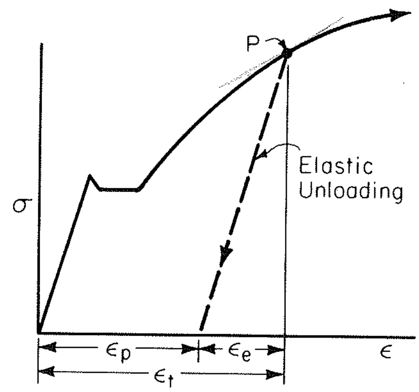

Figure 2.4 Elastic and plastic strain.

2. *Plastic strain (nonlinear):* that portion which cannot be recovered on unloading, ϵ_p (see Fig. 2.4)

Stated in equation form,

$$\epsilon_t = \epsilon_e + \epsilon_p \tag{2.17}$$

For most metals a log–log plot of true stress versus true plastic strain is modeled as a straight line. Consequently, this curve can be expressed using a power function

$$\sigma = K(\epsilon_p)^n \tag{2.18}$$

or

$$\epsilon_p = \left(\frac{\sigma}{K}\right)^{1/n} \tag{2.19}$$

where K is the *strength coefficient* and n is the *strain hardening exponent.*

At fracture two important quantities can be defined (see Fig. 2.3). These are true fracture strength and true fracture ductility. *True fracture strength,* σ_f, is the true stress at final fracture.

$$\sigma_f = \frac{P_f}{A_f} \tag{2.20}$$

where A_f is the area at fracture and P_f is the load at fracture.

True fracture ductility, ϵ_f, is the true strain at final fracture. This value can be defined in terms of the initial cross-sectional area and the area at fracture.

$$\epsilon_f = \ln\frac{A_0}{A_f} = \ln\frac{1}{1 - \mathrm{RA}} \tag{2.21}$$

$$\mathrm{RA} = \frac{A_0 - A_f}{A_0} = \text{reduction in area}$$

The strength coefficient, K, can be defined in terms of the true stress at fracture, σ_f, and the true strain at fracture, ϵ_f.

Substituting σ_f and ϵ_f into Eq. (2.18) yields

$$\sigma_f = K(\epsilon_f)^n \tag{2.22}$$

Rearranging gives

$$K = \frac{\sigma_f}{\epsilon_f^n} \tag{2.23}$$

We can also define plastic strain in terms of these quantities. Combining Eqs. (2.23) and (2.19), we have

$$\epsilon_p = \left(\frac{\sigma}{\sigma_f / \epsilon_f^n}\right)^{1/n} \tag{2.24}$$

$$= \left(\frac{\sigma \epsilon_f^n}{\sigma_f}\right)^{1/n} \tag{2.25}$$

$$= \epsilon_f \left(\frac{\sigma}{\sigma_f}\right)^{1/n} \tag{2.26}$$

The total strain can be expressed as

$$\epsilon_t = \epsilon_e + \epsilon_p \tag{2.17}$$

The elastic strain is defined as

$$\epsilon_e = \frac{\sigma}{E} \tag{2.27}$$

The expression for plastic strain is given in Eq. (2.19). Equation (2.17) may then be rewritten as

$$\epsilon_t = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \tag{2.28}$$

2.2.2 Cyclic Stress–Strain Behavior

Monotonic stress–strain curves have long been used to obtain design parameters for limiting stresses on engineering structures and components subjected to static loading. Similarly, cyclic stress–strain curves are useful for assessing the durability of structures and components subjected to repeated loading.

The response of a material subjected to cyclic inelastic loading is in the form of a hysteresis loop, as shown in Fig. 2.5. The total width of the loop is $\Delta\epsilon$ or the total strain range. The total height of the loop is $\Delta\sigma$ or the total stress range.

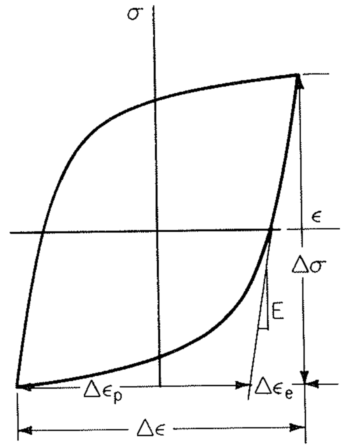

Figure 2.5 Hysteresis loop.

These can be stated in terms of amplitudes:

$$\epsilon_a = \frac{\Delta\epsilon}{2}$$

where ϵ_a is the strain amplitude and

$$\sigma_a = \frac{\Delta\sigma}{2}$$

where σ_a is the stress amplitude. The total strain is the sum of the elastic and plastic strain ranges,

$$\Delta\epsilon = \Delta\epsilon_e + \Delta\epsilon_p \tag{2.29}$$

or in terms of amplitudes,

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2} \tag{2.30}$$

Using Hooke's law, the elastic term may be replaced by $\Delta\sigma/E$.

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\epsilon_p}{2} \tag{2.31}$$

The area within the loop is the energy per unit volume dissipated during a cycle. It represents a measure of the plastic deformation work done on the material.

The *Bauschinger effect* [6] is usually observed in most metals. This effect is described graphically in Fig. 2.6. Shown in Fig. 2.6a is the material response of a bar loaded past the yield strength, σ_y, to some value, σ_{max}. In Fig. 2.6b, the material is unloaded and then loaded in compression to $-\sigma_{max}$. Notice that under compressive loading, inelastic (plastic) strains develop before $-\sigma_y$ is reached. This behavior is known as the Bauschinger effect.

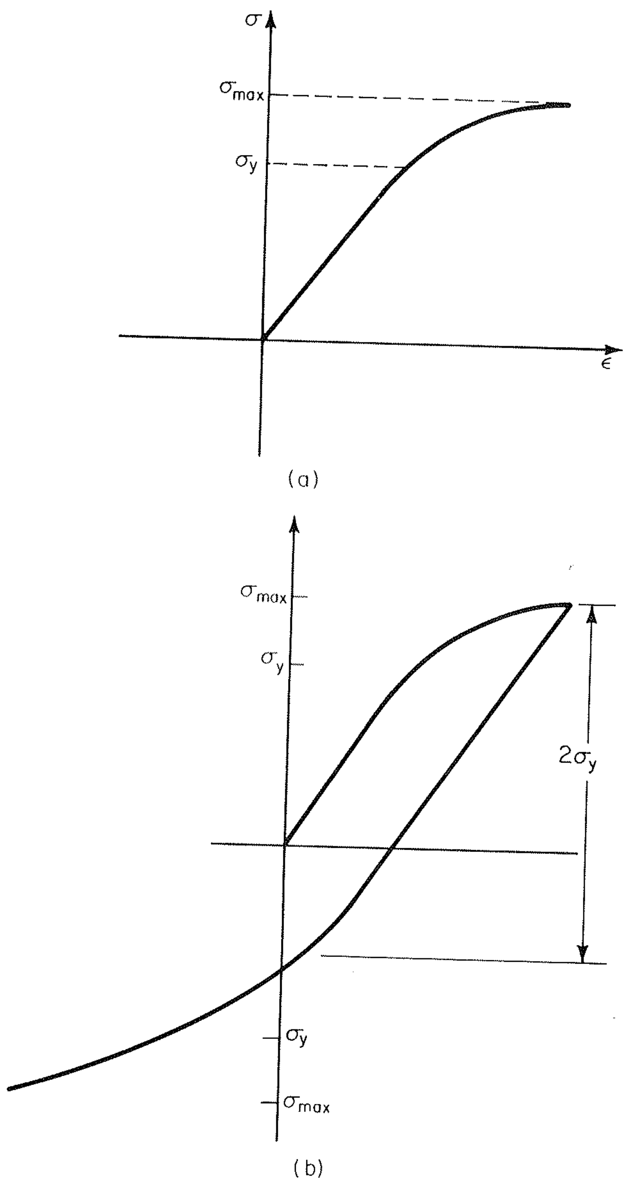

Figure 2.6 Bauschinger effect.

2.2.3 Transient Behavior: Cyclic Strain Hardening and Softening

The stress–strain response of metals is often drastically altered due to repeated loading. Depending on the initial conditions of a metal (i.e., quenched and

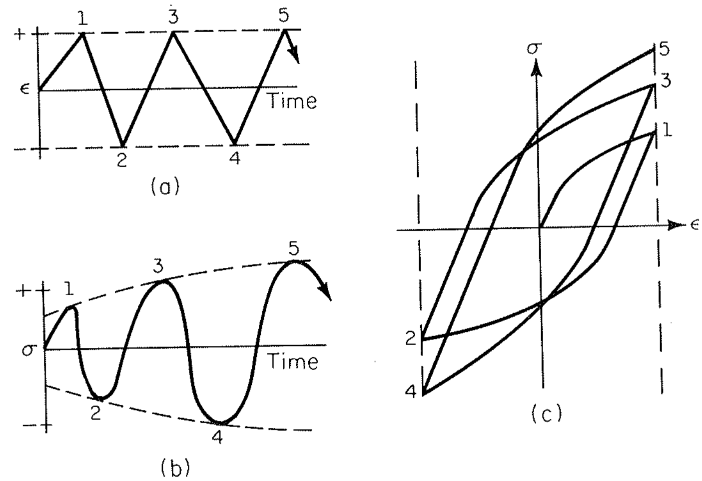

Figure 2.7 Cyclic hardening: (a) constant strain amplitude; (b) stress response (increasing stress level); (c) cyclic stress–strain response.

tempered, or annealed) and the test conditions, a metal may:

1. Cyclically harden
2. Cyclically soften
3. Be cyclically stable
4. Have mixed behavior (soften or harden depending on the strain range)

Figure 2.7b shows the stress response of a material loaded in strain-control. Figure 2.7c shows the hysteresis loops for the first two cycles. As seen, the maximum stress obtained increases with each cycle of strain. This is known as *strain hardening*. Conversely, if the maximum stress *decreases* with repeated straining, *strain softening* occurs as shown in Fig. 2.8.

The reason materials soften or harden appears to be related to the nature and stability of the dislocation substructure of the material [7]. Generally:

1. For a *soft material,* initially the dislocation density is low. The density rapidly increases due to cyclic plastic straining contributing to significant cyclic *strain hardening*.
2. For a *hard material* subsequent strain cycling causes a rearrangement of dislocations which offers less resistance to deformation and the material cyclically *softens*.

Manson [8] observed that the ratio of monotonic ultimate strength, σ_{ult}, to the 0.2% offset yield strength, σ_y, can be used to predict whether the material

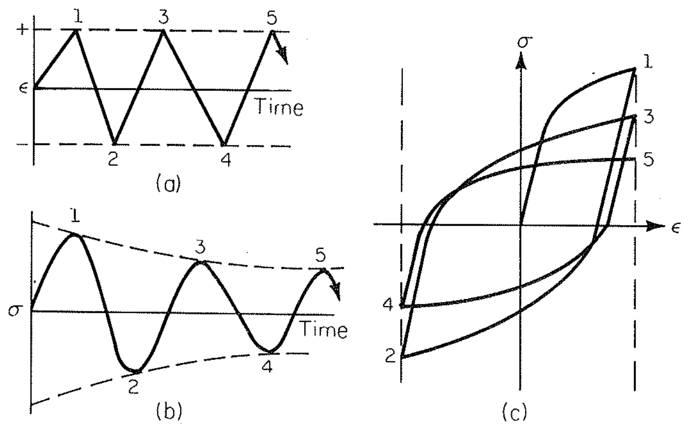

Figure 2.8 Cyclic softening: (a) constant strain amplitude; (b) stress response (decreasing stress level); (c) cyclic stress–strain response.

will soften or harden. If

$$\frac{\sigma_{\text{ult}}}{\sigma_y} > 1.4 \qquad \text{the material will cyclically harden}$$

$$\frac{\sigma_{\text{ult}}}{\sigma_y} < 1.2 \qquad \text{the material will cyclically soften}$$

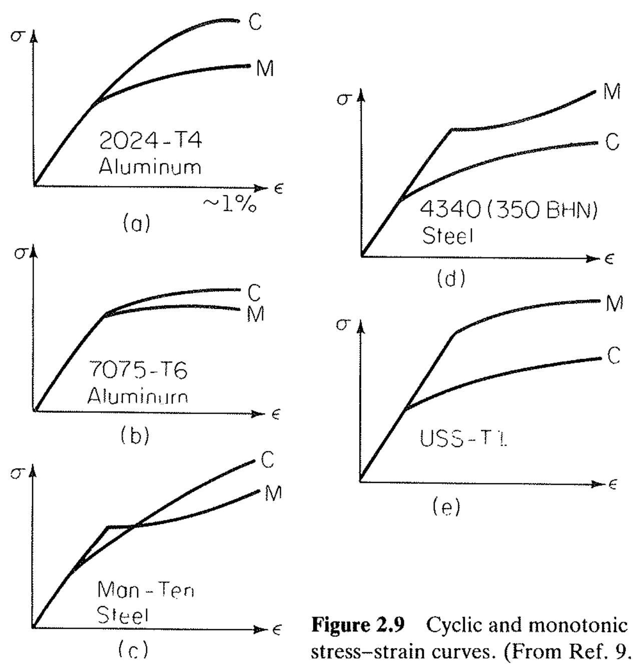

Figure 2.9 Cyclic and monotonic stress–strain curves. (From Ref. 9.)

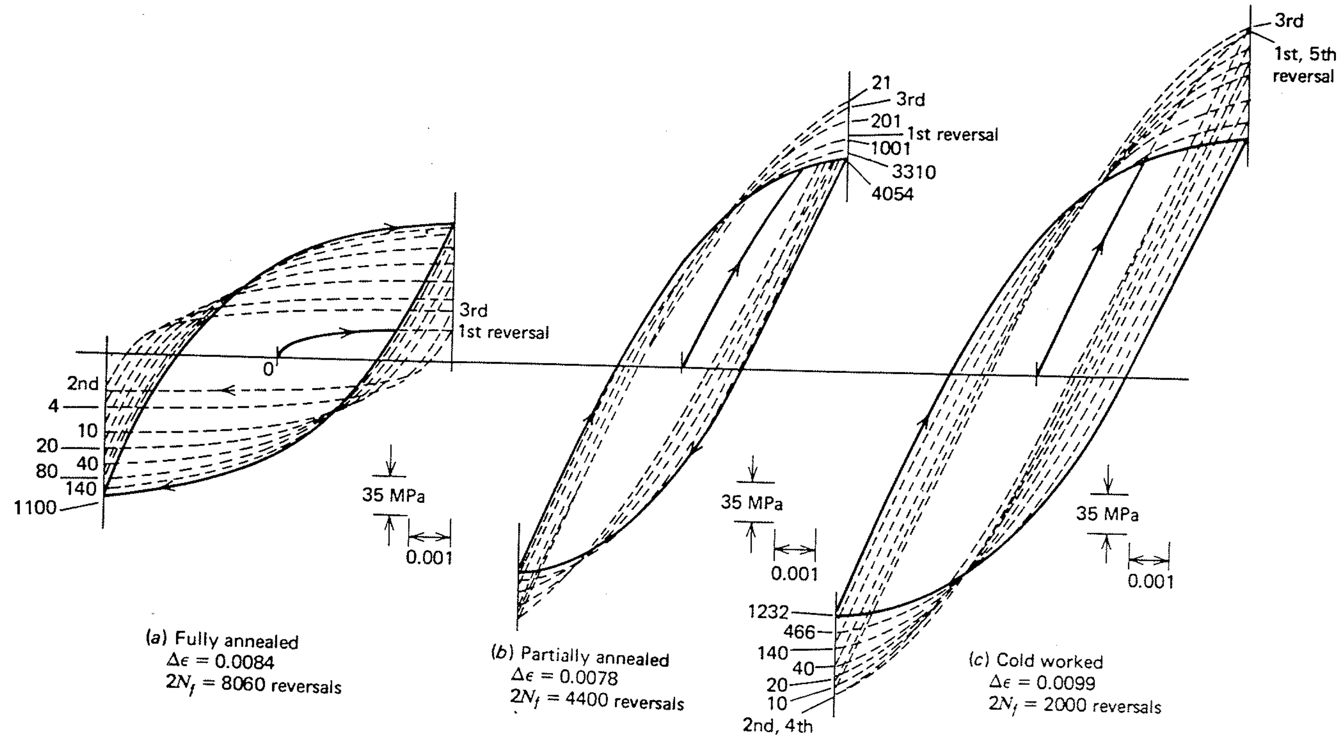

Figure 2.10 Hysteresis response of OFHC copper. (From Ref. 10.)

A large change in cyclic response is not expected for ratios between 1.2 and 1.4 and prediction is difficult. Also, the *monotonic* strain hardening exponent, n, can be used to predict the material's cyclic behavior. In general, if

$n > 0.20$ the material will cyclically harden

$n < 0.10$ the material will cyclically soften

Generally, transient behavior (strain hardening or softening) occurs only during the early fatigue life. After this, the material achieves a *cyclically stable condition.* This is usually achieved after approximately 20 to 40% of the fatigue life. Consequently, fatigue properties are usually specified at "half-life" (approximately 50% of the total fatigue life) when the material response is stabilized.

Figure 2.9 presents the cyclic and monotonic stress–strain curves for several materials. Figure 2.10 presents the hysteresis response of OFHC copper in three conditions.

A comparison between the monotonic and cyclic stress–strain curve provides a quantitative assessment of cyclically induced changes in mechanical behavior. As shown in Fig. 2.9e, a material that cyclically softens will have a cyclic yield strength lower than the monotonic. This points out the potential danger of using monotonic properties to predict cyclic strains. For example, monotonic properties may predict strains that are fully elastic, when in fact the material may experience large amounts of cyclic plastic strain.

2.2.4 Cyclic Stress–Strain Curve Determination

Cyclic stress–strain curves may be obtained from tests by several methods. Two of these are:

1. *Companion samples.* A series of companion samples are tested at various strain levels until the hysteresis loops become stabilized. The stable hysteresis loops are then superimposed and the tips of the loops are connected as shown in Fig. 2.11. This method is time consuming and requires many specimens.

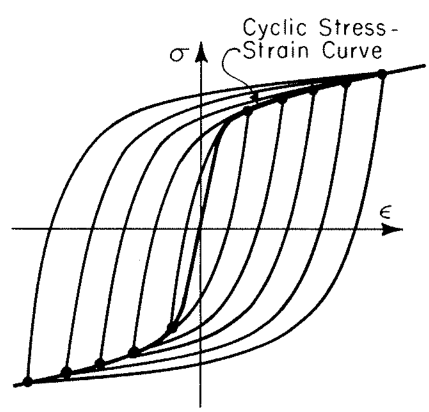

Figure 2.11 Cyclic stress–strain curve obtained by connecting tips of stabilized hysteresis loops.

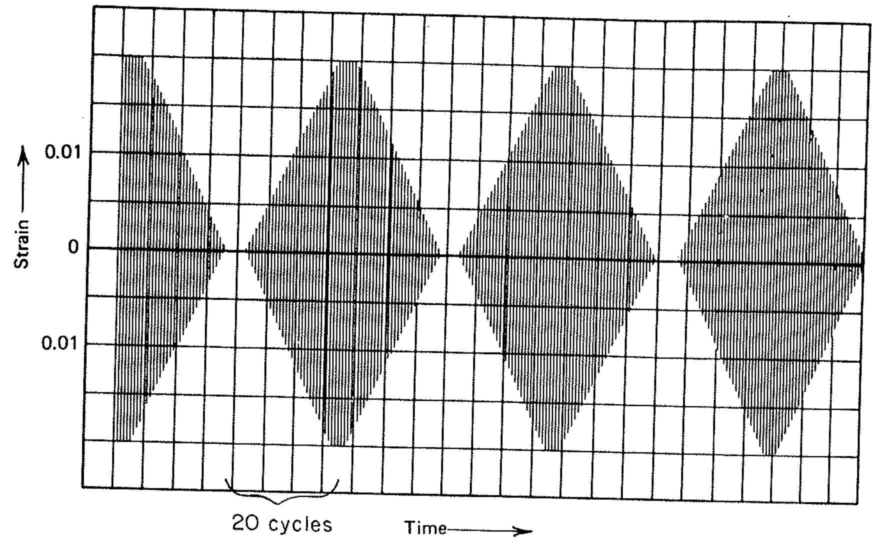

Figure 2.12 Incremental step test. (Data from Ref. 11.)

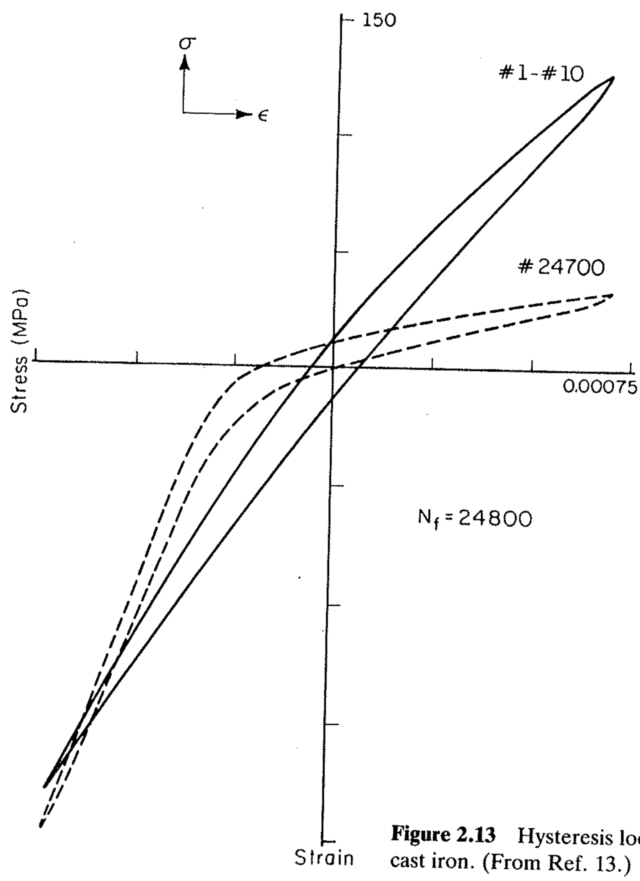

Figure 2.13 Hysteresis loops of gray cast iron. (From Ref. 13.)

2. *Incremental Step Test.* This method has become widely accepted, as it is very quick and produces good results. One specimen is subjected to a series of blocks of gradually increasing and decreasing strain amplitude. After a few blocks the material stabilizes. For example, for the test shown in Fig. 2.12, the loading block contains 20 cycles per half-block. The material response generally stabilizes after about three to four blocks and fails after approximately 20. The cyclic stress–strain curve can then be determined by connecting the tips of the stabilized hysteresis loops.

After the incremental step test, if the specimen is pulled to failure, the resulting stress–strain curve will be nearly identical to the one obtained by connecting the loop tips.

Knowing the cyclic stress–strain curve, use of *Massing's hypothesis* [12] allows the stabilized hysteresis loop to be estimated for a material that exhibits symmetric behavior in tension and compression. (The hysteresis loop of gray cast iron, for example, exhibits a different response in tension and compression, as shown in Fig. 2.13.)

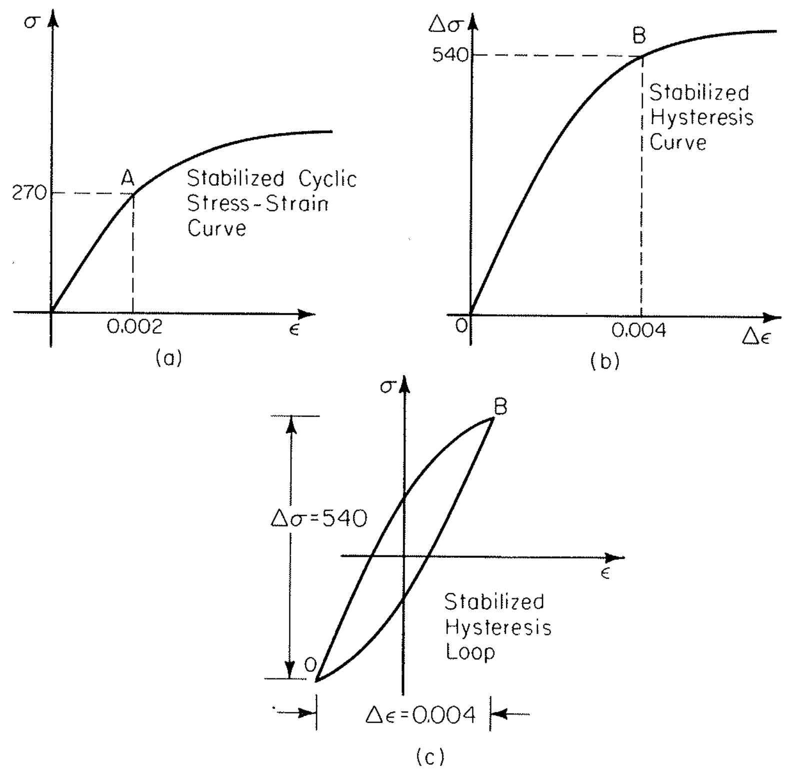

Figure 2.14 Development of stabilized hysteresis curve from cyclic stress–strain curve using Massing's hypothesis.

Massing's hypothesis states that the stabilized hysteresis loop may be obtained by doubling the cyclic stress–strain curve. By doubling the stress and strain value from the stabilized cyclic stress–strain curve, a corresponding point on the hysteresis loop is obtained as shown in Fig. 2.14. For example, by doubling the values corresponding to point A on the cyclic stress–strain curve in Fig. 2.14a, point *B* on the hysteresis loop (Fig. 2.14b) is obtained. Figure 2.14c shows the hysteresis loop for a fully reversed test. Note the location of point 0 on the hysteresis curve in Fig. 2.14b and c.

2.3 STRESS–PLASTIC STRAIN POWER LAW RELATION

Analogous to the monotonic stress–strain curve, a log–log plot of the completely reversed stabilized cyclic true stress versus true plastic strain can be approximated by a straight line as shown in Fig. 2.15.

Similar to the monotonic relationship, we can develop a power law function

$$\sigma = K'(\epsilon_p)^{n'} \tag{2.32}$$

where σ = cyclically stable stress amplitude
ϵ_p = cyclically stable plastic strain amplitude
K' = cyclic strength coefficient
n' = cyclic strain hardening exponent

For most metals the value of n' usually varies between 0.10 and 0.25, with an average value close to 0.15.

Rearranging Eq. (2.32) gives us

$$\epsilon_p = \left(\frac{\sigma}{K'}\right)^{1/n'} \tag{2.33}$$

The total strain is the sum of the elastic and plastic components. Using Eq. (2.33) and Hooke's law, the total strain can be written

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} \tag{2.34}$$

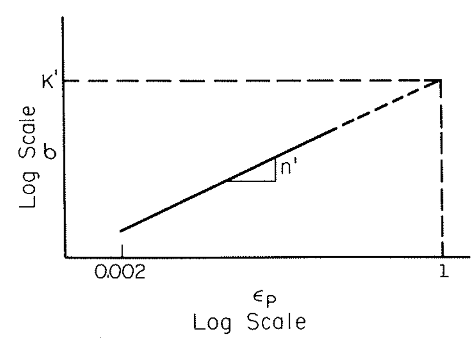

Figure 2.15 Log–log plot of true cyclic stress versus true cyclic plastic strain.

The equation of the hysteresis loop can be derived from the equation of the cyclic σ–ϵ curve [Eq. (2.34)] using Massing's hypothesis. Recall that Massing's hypothesis allows us to obtain the hysteresis loop by doubling the cyclic stress–strain curve as shown in Fig. 2.14. Given an arbitrary point, P_1, on the cyclic stress–strain curve, as shown in Fig. 2.16a, the corresponding values of stress and strain are σ_1 and ϵ_1, respectively. These values, σ_1 and ϵ_1, are related by the equation of the cyclic σ–ϵ curve. Equation (2.34) may be written

$$\epsilon_1 = \frac{\sigma_1}{E} + \left(\frac{\sigma_1}{K'}\right)^{1/n'} \tag{2.35}$$

From Massing's hypothesis a point corresponding to P_1 may be located on the hysteresis curve as shown in Fig. 2.16b. The coordinates of this point are $\Delta\sigma_1$ and $\Delta\epsilon_1$, where

$$\Delta\sigma_1 = 2\sigma_1$$
$$\Delta\epsilon_1 = 2\epsilon_1$$

Rearranging these equations, we obtain

$$\frac{\Delta\sigma_1}{2} = \sigma_1 \qquad \frac{\Delta\epsilon_1}{2} = \epsilon_1$$

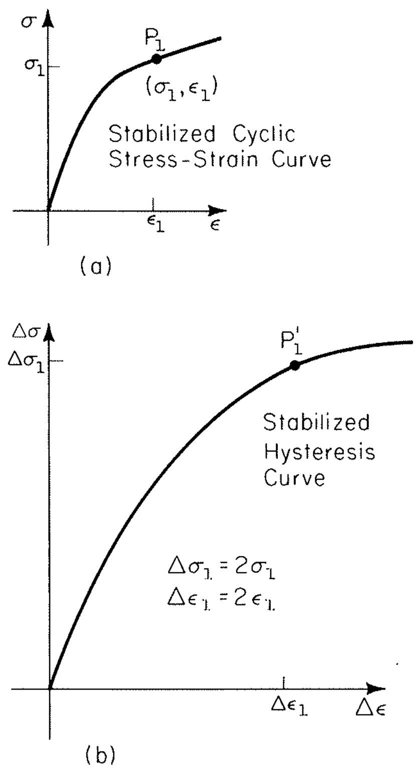

Figure 2.16 Stabilized cyclic stress–strain and hysteresis curves.

These can then be substituted into Eq. (2.35) to obtain the equation of the hysteresis loop:

$$\frac{\Delta\epsilon_1}{2} = \frac{\Delta\sigma_1}{2E} + \left(\frac{\Delta\sigma_1}{2K'}\right)^{1/n'}$$

Multiplying both sides by 2, the general hysteresis curve equation is

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \tag{2.36}$$

(Since this equation was derived for an arbitrary point, P_1, the subscripts may be omitted.)

The relationship between the hysteresis stress–strain curve and the cyclic stress–strain curve can be made clearer with the following example.

Example 2.1

Consider a test specimen with the following material properties:

E = modulus of elasticity = 30×10^3 ksi
n' = cyclic strain hardening exponent = 0.202
K' = cyclic strength coefficient = 174.6 ksi

The specimen is subjected to a fully reversed cyclic strain with a strain range, $\Delta\epsilon$, of 0.04. Determine the stress–strain response of the material.

Solution Figure E2.1a shows the strain history. On the initial application of strain (point 1) the material response follows the cyclic stress–strain curve [Eq. (2.34)]:

$$\epsilon_1 = \frac{\sigma_1}{E} + \left(\frac{\sigma_1}{K'}\right)^{1/n'}$$

Substituting in the material properties and a strain value of 0.02 gives

$$0.02 = \frac{\sigma_1}{30 \times 10^3 \text{ ksi}} + \left(\frac{\sigma_1}{174.6 \text{ ksi}}\right)^{1/0.202}$$

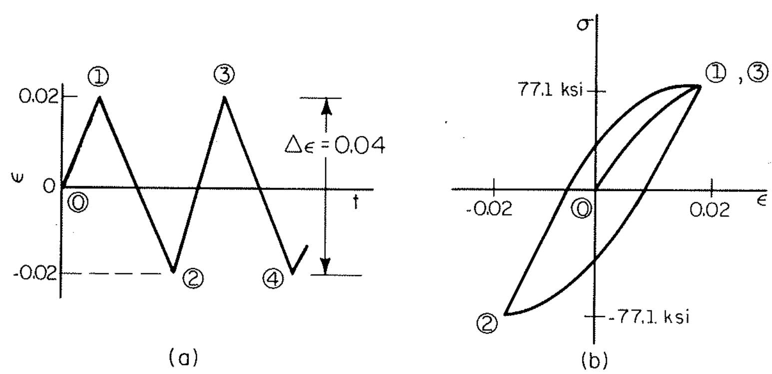

Figure E2.1 (a) Strain history; (b) stress–strain response.

The stress value at point 1 may be determined by solving this equation by iteration. As shown in Fig. E2.1b, this yields

$$\sigma_1 = 77.1 \text{ ksi}$$

The cyclic stress–strain curve is used only for the initial application of strain. On all successive strain reversals, the material response is modeled using the hysteresis curve [Eq. (2.36)]:

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

Substituting in the material properties and a change in strain, $\Delta\epsilon$, of 0.04 gives

$$0.04 = \frac{\Delta\sigma}{30 \times 10^3} + 2\left(\frac{\Delta\sigma}{2 \times 174.6}\right)^{1/0.202}$$

The change in stress can be obtained by using an iterative method:

$$\Delta\sigma = 154.2 \text{ ksi}$$

The stress and strain values corresponding to point 2 can now be determined by subtracting the changes in stress and strain ($\Delta\sigma$, $\Delta\epsilon$) from the values at point 1 (σ_1, ϵ_1).

$$\begin{aligned} \epsilon_2 &= \epsilon_1 - \Delta\epsilon \\ &= 0.02 - 0.04 \\ &= -0.02 \\ \sigma_2 &= \sigma_1 - \Delta\sigma \\ &= (77.1 - 154.2) \text{ ksi} \\ &= -77.1 \text{ ksi} \end{aligned}$$

(*Note:* The actual algebraic sign of the changes in stress and strain must be accounted for by considering the sign of the change in applied strain.)

The stress and strain values corresponding to point 3 can be determined by again using the hysteresis curve. The solution would show that the material response would return to point 1. As expected, the material response forms a closed hysteresis loop and all successive strain cycles would follow this loop.

Two points need to be considered concerning Example 2.1. First is the response of the material on the initial application of load or strain. In the solution of the problem it was assumed that the material response would follow the cyclic stress–strain curve. There is justification to an alternative approach which says that the response would follow the monotonic stress–strain curve. This argument states that the material response of a virgin material follows the monotonic stress–strain curve. Most fatigue life predictions are not greatly affected by which approach is used.

The second point that needs to be considered is that this analysis assumes that the material exhibits cyclically stable response from the initial loading. An exact analysis would account for the cyclically hardening or softening characteristics of the material. This type of analysis is very difficult and time consuming. It also requires that material properties be available to model this transient behavior. In general, there appears to be no significant effects on life predictions when this transient behavior is ignored.

The recommended procedure is to use the cyclic stress–strain curve to model material behavior on the initial load cycle and cyclically stable material properties during the entire analysis.

2.4 STRAIN–LIFE CURVE

In 1910, Basquin [14] observed that stress–life (S–N) data could be plotted linearly on a log–log scale. Using the true stress amplitude, the plot may be linearized by

$$\frac{\Delta\sigma}{2} = \sigma_f'(2N_f)^b \tag{2.37}$$

where $\frac{\Delta\sigma}{2}$ = true stress amplitude

$2N_f$ = reversals to failure (1 rev = $\frac{1}{2}$ cycle)

σ_f' = fatigue strength coefficient

b = fatigue strength exponent (Basquin's exponent)

σ_f' and b are fatigue properties of the material. The fatigue strength coefficient, σ_f', is approximately equal to the true fracture strength, σ_f. The fatigue strength exponent, b, will usually vary between -0.05 and -0.12.

Coffin [15] and Manson [16], working independently in the 1950s, found that plastic strain–life ($\epsilon_p - N$) data could also be linearized on log–log coordinates. Again, plastic strain can be related by a power law function

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f'(2N_f)^c \tag{2.38}$$

where $\frac{\Delta\epsilon_p}{2}$ = plastic strain amplitude

$2N_f$ = reversals to failure

ϵ_f' = fatigue ductility coefficient

c = fatigue ductility exponent

ϵ_f' and c are also fatigue properties of the material. The fatigue ductility coefficient, ϵ_f', is approximately equal to true fracture ductility, ϵ_f. The fatigue ductility exponent, c, varies between -0.5 and -0.7.

An expression may now be developed that relates total strain range to life to failure. As discussed with reference to Eq. (2.30), the total strain is the sum of the elastic and plastic strains. In terms of strain amplitude [repeating Eq. (2.30)],

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2} \tag{2.30}$$

The elastic term can be written as

$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\sigma}{2E} \tag{2.39}$$

Using Eq. (2.37) we can now state this in terms of life to failure:

$$\frac{\Delta\epsilon_e}{2} = \frac{\sigma_f'}{E}(2N_f)^b \tag{2.40}$$

From Eq. (2.38) the plastic term is

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f'(2N_f)^c \tag{2.38}$$

Using Eq. (2.30), the total strain can now be rewritten using Eqs. (2.38) and (2.40):

$$\frac{\Delta\epsilon}{2} = \underbrace{\frac{\sigma_f'}{E}(2N_f)^b}_{\text{elastic}} + \underbrace{\epsilon_f'(2N_f)^c}_{\text{plastic}} \tag{2.41}$$

Equation (2.41) is the basis of the strain–life method and is termed the *strain–life relation.*

Equation (2.41) can be explained graphically. Recalling that the elastic and plastic relations are both straight lines on a log–log plot, the total strain amplitude, $\Delta\epsilon/2$, can be plotted simply by summing the elastic and plastic values as shown in Fig. 2.17. At large strain amplitudes the strain–life curve approaches the plastic line, and at low amplitudes, the curve approaches the elastic line.

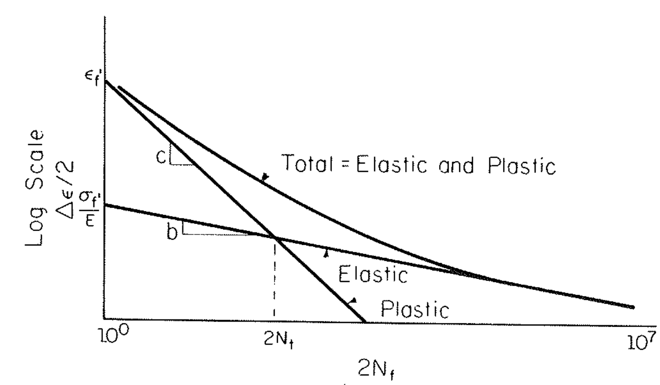

Figure 2.17 Strain–life curve.

In Fig. 2.17, the transition fatigue life, $2N_t$, represents the life at which the elastic and plastic curves intersect. Note that this is the life at which the stabilized hysteresis loop has equal elastic and plastic strain components. By equating elastic and plastic terms the following expression is derived for the transition life:

$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\epsilon_p}{2}$$

$$\frac{\sigma_f'}{E}(2N_f)^b = \epsilon_f'(2N_f)^c \qquad \text{at } N_f = N_t$$

$$2N_t = \left(\frac{\epsilon_f' E}{\sigma_f'}\right)^{1/(b-c)} \tag{2.42}$$

A schematic representation of the shape of the hysteresis loop at different lives is shown in Fig. 2.18 in relationship to the transition life. As seen, at shorter lives more plastic strain is present and the loop is wider. At long lives the loop is narrower, representing less plastic strain.

As shown in Fig. 2.19, the transition life of steel decreases with increasing hardness.

As the ultimate strength of the material increases, the transition life decreases, and elastic strains dominate for a greater portion of the life range.

Figure 2.20 presents the strain–life curves for a medium carbon steel in two different heat treated conditions. The material in a normalized (soft) ductile condition has a transition life of 90,000 cycles, while the material in a quenched (high strength) condition has a transition life of 15 cycles. As shown, for a given strain the high strength material (quenched) provides longer fatigue lives in the high cycle regime. At short lives or high strains the ductile (normalized) material exhibits better fatigue resistance.

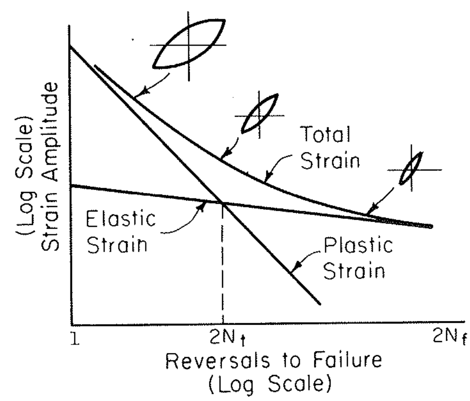

Figure 2.18 Shape of the hysteresis curve in relation to the strain–life curve. (From Ref. 17.)

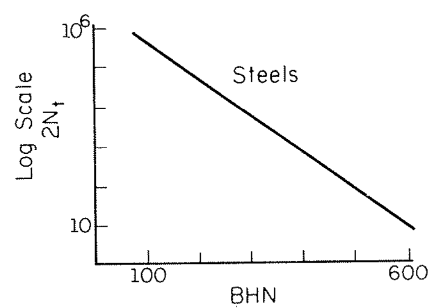

Figure 2.19 Relationship between transition life and hardness for steels. (From Ref. 17.)

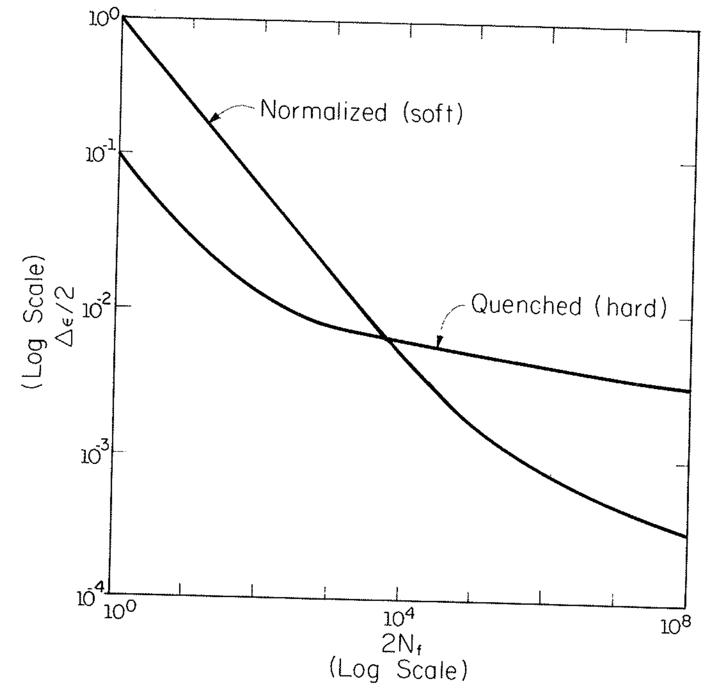

Figure 2.20 Strain–life curves for a medium carbon steel in a quenched and normalized condition.

The optimum material would be one that has *both* high ductility and high strength. Unfortunately, there is usually a trade-off between these two properties and a compromise must be made for the expected load or strain conditions being considered.

Note that life to failure may be defined in several ways. These include:

1. Separation of specimen
2. Development of given crack length (often 1.0 mm)
3. Loss of specified load carrying capability (often 10 or 50% load drop)

Specimen separation is the most common failure criteria for uniaxial loading. However, in many cases, there is not a large difference in life between these criteria.

The strain–life equation as stated in Eq. (2.41) does not predict the endurance limit behavior seen in some metals. When the endurance limit behavior is significant, the methods described in Chapter 1 should be used.

Before continuing it is worthwhile to consider the "factor of 2" problem found in the strain–life analysis. There are three cases where it is very easy to lose track of a factor of 2 and cause errors in a strain–life fatigue analysis. All of these cases have been discussed earlier, but to reemphasize, they are:

1. *Cycles versus reversal.* The strain–life approach measures life in terms of reversals ($2N$), whereas the stress–life method uses cycles (N). A reversal is one-half of a full cycle.
2. *Amplitude versus range.* The strain–life approach uses both strain range, $\Delta\epsilon$, and amplitude, ϵ_a, which differ by a factor of 2 (i.e., $\Delta\epsilon/2 = \epsilon_a$).
3. *Cyclic σ–ϵ curve versus hysteresis curve.* Massing's hypothesis states that the hysteresis curve can be modeled as twice the cyclic stress–strain curve.

Although the preceding points may seem trivial to some, it is the experience of the authors that the "factor of 2" phantom can cause extreme hardship for the unwary.

2.5 DETERMINATION OF FATIGUE PROPERTIES

The strain–life equation [Eq. (2.41)] requires four empirical constants (b, c, σ_f', ϵ_f'). Several points must be considered in attempting to obtain these constants from fatigue data.

1. Not all materials may be represented by the four-parameter strain–life equation. (Examples of these are some high strength aluminum alloys and titanium alloys.)
2. The four fatigue constants may represent a curve fit to a limited number of data points. The values of these constants may be changed if more data points are included in the curve fit.
3. The fatigue constants are determined from a set of data points over a given range. Gross errors may occur when extrapolating fatigue life estimates outside this range.
4. The use of power law relationships in Eqs. (2.32), (2.37), and (2.38) is strictly a matter of mathematical convenience and is not based on a physical phenomenon.

From Eqs. (2.34) and (2.41) the following properties may be related:

$$K' = \frac{\sigma_f'}{(\epsilon_f')^{n'}} \qquad (2.43)$$

$$n' = \frac{b}{c} \qquad (2.44)$$

Although these relationships may be useful, K' and n' are usually obtained from a curve fit of the cyclic stress–strain data using Eq. (2.32). Due to the approximate nature of the curve fits, values obtained from Eqs. (2.32), (2.43), and (2.44) may not be equal.

Fatigue properties may be approximated from monotonic properties. Currently, due to the available data, these techniques are no longer used extensively. Nevertheless, the following approximate methods may be useful.

Fatigue strength coefficient, σ_f'. A fairly good approximation is

$$\sigma_f' \approx \sigma_f \qquad \text{(corrected for necking)} \tag{2.45}$$

For steels with hardnesses below 500 BHN:

$$\sigma_f \approx S_u + 50\ \text{ksi} \tag{2.46}$$

Fatigue strength exponent, b. b varies from -0.05 to -0.12 for most metals with an average of -0.085. (Note that this corresponds to the approximate slope of the S–N curve discussed in Section 1.2.)

Fatigue ductility coefficient, ϵ_f'. A fairly good approximation is

$$\epsilon_f' \approx \epsilon_f \tag{2.47}$$

$$\text{where } \epsilon_f = \ln \frac{1}{1 - \text{RA}}$$

where RA is the reduction in area.

Fatigue ductility exponent, c. c is not as well defined as the other parameters. A rule-of-thumb approach must be followed rather than an empirical equation.

Coffin found c to be about -0.5.

Manson found c to be about -0.6.

Morrow found that c varied between -0.5 and -0.7.

Fairly ductile metals (where: $\epsilon_f \approx 1$) have average values of $c = -0.6$. For strong metals (where: $\epsilon_f \approx 0.5$) a value of $c = -0.5$ is probably more reasonable.

Example 2.2

Given below are the monotonic and cyclic strain–life data for smooth steel specimens. Determine the cyclic stress–strain and strain–life constants (K', n', σ_f', b, ϵ_f', c) for this material.

Monotonic data

$$S_y = 158\ \text{ksi} \qquad E = 28.4 \times 10^3\ \text{ksi}$$

$$S_u = 168\ \text{ksi} \qquad \sigma_f = 228\ \text{ksi}$$

$$\%\,\text{RA} = 52 \qquad \epsilon_f = 0.734$$

Smooth Specimen-Cyclic Data

Total Strain Amplitude, $\Delta\epsilon/2$	Stress Amplitude, $\Delta\sigma/2$ (ksi)	Plastic Strain Amplitude, $\Delta\epsilon_p/2$[a]	Reversals to Failure, $2N_f$
0.0393	162.5	0.0336	50
0.0393	162	0.0336	68
0.02925	155	0.0238	122
0.01975	143.5	0.0147	256
0.0196	143.5	0.0145	350
0.01375	136.5	0.00894	488
0.00980	130.5	0.00521	1,364
0.00980	126.5	0.00534	1,386
0.00655	121	0.00229	3,540
0.00630	119	0.00211	3,590
0.00460	114	0.00059	9,100
0.00360	106	0.00000	35,200
0.00295	84.5	0.00000	140,000

[a] $\dfrac{\Delta\epsilon_p}{2} = \dfrac{\Delta\epsilon}{2} - \dfrac{\Delta\epsilon_e}{2} = \dfrac{\Delta\epsilon}{2} - \dfrac{\Delta\sigma}{2E}$.

Solution Determine the fatigue strength coefficient, σ_f', and the fatigue strength exponent, b, by fitting a power law relationship to the stress amplitude, $\Delta\sigma/2$, versus reversals to failure, $2N_f$, data.

$$\frac{\Delta\sigma}{2} = \sigma_f'(2N_f)^b$$

Determine the fatigue ductility coefficient, ϵ_f', and the fatigue ductility exponent, c, by fitting a power law relationship to the plastic strain amplitude, $\Delta\epsilon_p/2$, versus reversals to failure, $2N_f$, data.

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f'(2N_f)^c$$

The curve fits to the strain–life data are shown in Fig. E2.2. The resulting cyclic

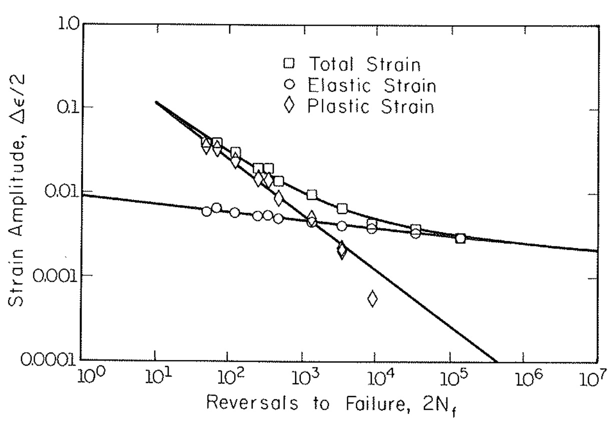

Figure E2.2 Strain–life curve.

properties are

$$\sigma_f' = 222 \text{ ksi} \qquad b = -0.076$$

$$\epsilon_f' = 0.811 \qquad c = -0.732$$

Determine the cyclic strength coefficient, K', and the cyclic strain hardening exponent, n'. These can be found using two different procedures. First, these terms can be found by fitting a power law relationship to the stress amplitude, $\Delta\sigma/2$, versus plastic strain amplitude, $\Delta\epsilon_p/2$, data.

$$\sigma = K'(\epsilon_p)^{n'}$$

If this is done, the resulting values are

$$K' = 216 \text{ ksi} \qquad n' = 0.094$$

These terms may also be determined using the relationships stated in Eqs. (2.43) and (2.44).

$$K' = \frac{\sigma_f'}{(\epsilon_f')^{n'}}$$

$$n' = \frac{b}{c}$$

Using these relationships, the resulting values are

$$K' = 227 \text{ ksi} \qquad n' = 0.104$$

Note the difference between the predictions for K' and n' found using the two different methods. In general, the first procedure (curve fit) is the preferred method.

For comparison purposes the following table lists the strain–life constants determined from strain–life data and the values found using the approximations discussed in Section 2.5 [Eqs. (2.45) and (2.47)]. Note the difference between these values.

Value	Determined from Strain–Life Data	Determined Using Approximations
σ_f'	222	228
b	−0.076	−0.085
ϵ_f'	0.811	0.734
c	−0.732	−0.6

2.6 MEAN STRESS EFFECTS

Cyclic fatigue properties of a material are obtained from completely reversed, constant amplitude strain-controlled tests. Components seldom experience this type of loading, as some mean stress or mean strain is usually present. The effect

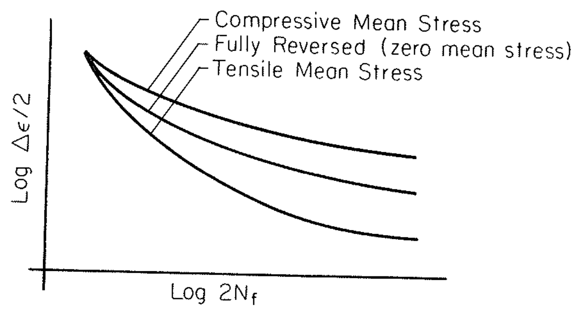

Figure 2.21 Effect of mean stress on strain–life curve.

of mean strain is, for the most part, negligible on the fatigue life of a component. Mean stresses, on the other hand, may have a significant effect on the fatigue life.

Mean stress effects are seen predominantly at longer lives. They can either increase the fatigue life with a nominally compressive load or decrease it with a nominally tensile value, as shown schematically in Fig. 2.21.

At high strain amplitudes (0.5% to 1% or above), where plastic strains are significant, mean stress relaxation occurs and the mean stress tends toward zero (see Fig. 2.22.) Note that this is not cyclic softening. Mean stress relaxation can occur in materials that are cyclically stable.

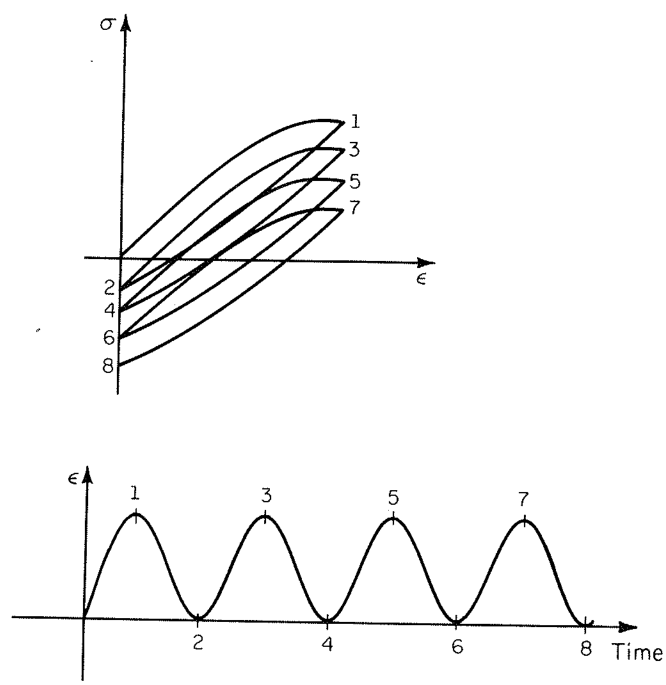

Figure 2.22 Mean stress relaxation.

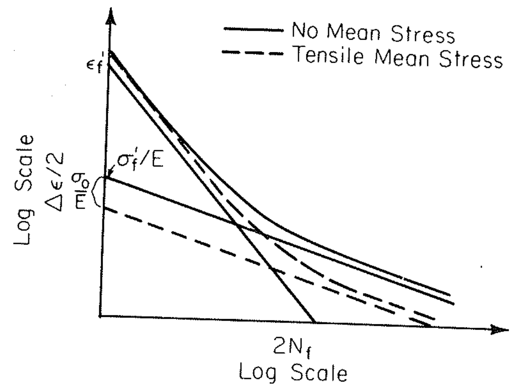

Figure 2.23 Morrow's mean stress correction to the strain–life curve for a tensile mean.

Modifications to the strain–life equation have been made to account for mean stress effects. Morrow [18] suggested that the mean stress effect could be taken into account by modifying the elastic term in the strain–life equation [Eq. (2.41)] by the mean stress, σ_0.

$$\frac{\Delta\epsilon_e}{2} = \frac{\Delta\sigma}{2E} = \frac{\sigma_f' - \sigma_0}{E}(2N_f)^b \tag{2.48}$$

The strain–life equation, accounting for mean stresses is, then,

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f' - \sigma_0}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{2.49}$$

This is shown graphically in Fig. 2.23. The predictions made with this equation are consistent with the observations that mean stress effects are significant at low values of plastic strain, where the elastic strain dominates. They also reflect the trend that mean stresses have little effect at shorter lives, where plastic strains are large.

Equation (2.49), though, incorrectly predicts that the ratio of elastic to plastic strain is dependent on mean stress. This is clearly not true, as demonstrated in Fig. 2.24. The two smaller hysteresis loops have the same strain range and the same ratio of elastic to plastic strain, while they have vastly different mean stresses.

Manson and Halford [19] modified both the elastic and plastic terms of the strain–life equation to maintain the independence of the elastic–plastic strain ratio from mean stress. This equation,

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f' - \sigma_0}{E}(2N_f)^b + \epsilon_f'\left(\frac{\sigma_f' - \sigma_0}{\sigma_f'}\right)^{c/b}(2N_f)^c \tag{2.50}$$

is shown graphically in Fig. 2.25. (Note that the transition life remains constant.) This equation tends to predict too much mean stress effect at short lives or where plastic strains dominate. At high plastic strains, mean stress relaxation occurs.

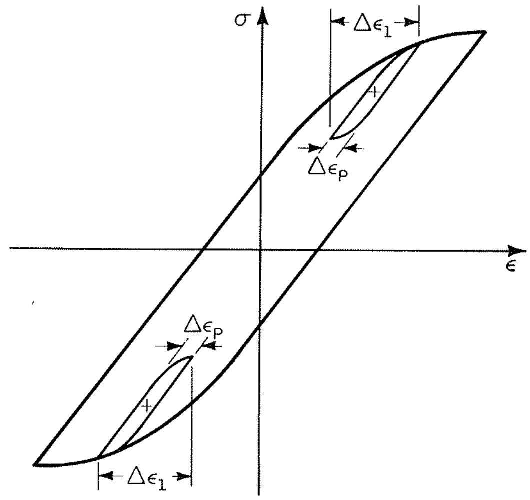

Figure 2.24 Independence of elastic/plastic strain ratio from mean stress. (Note that in this figure the plastic strain portion of the small hysteresis loops has been exaggerated for clarity.)

Although Eq. (2.49) violates the constitutive relationship, it generally does a better job predicting mean stress effects.

Smith, Watson, and Topper (SWT) [20] have proposed another equation to account for mean stress effects. Recalling Eq. (2.37), for completely reversed loading

$$\sigma_{max} = \frac{\Delta\sigma}{2} = \sigma_f'(2N_f)^b \tag{2.51}$$

and multiplying the strain–life equation by this term, results in

$$\sigma_{max}\frac{\Delta\epsilon}{2} = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\epsilon_f'(2N_f)^{b+c} \tag{2.52}$$

For application of this equation, the term σ_{max} is evaluated as

$$\sigma_{max} = \frac{\Delta\sigma}{2} + \sigma_0 \tag{2.53}$$

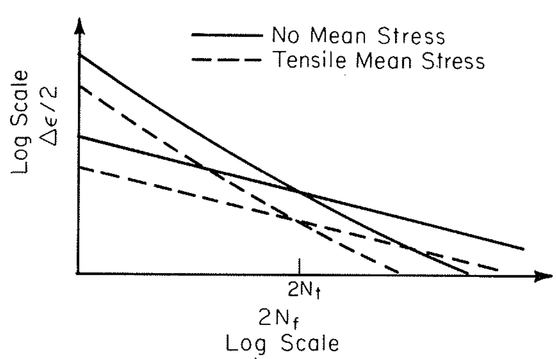

Figure 2.25 Mean stress correction for independence of elastic/plastic strain ratio from mean stress.

Since this equation is in the general form

$$\sqrt{\sigma_{max}\,\Delta\epsilon} \propto N_f \tag{2.54}$$

it becomes undefined when σ_{max} is negative. The physical interpretation of this approach assumes that no fatigue damage occurs when $\sigma_{max} < 0$.

The mean stress equations above are empirically based. Therefore, care must be taken when they are used outside the ranges from which they were developed.

2.7 IMPORTANT CONCEPTS

- Cyclic response of a material often differs from monotonic response. For instance, a soft material usually strain hardens and a hard material usually strain softens.
- Fatigue properties, σ_f', ϵ_f', b, c, n', and K', are curve fit to the experimental data and must be used for the range of life from which they were determined.
- Tensile mean stresses are detrimental to fatigue life, while compressive mean stresses are beneficial.
- At high strain levels, cyclic plastic strains tend to cause the mean stresses to relax to zero.
- Three cases where it is easy to lose a factor of 2 are:
  a. Cycles versus reversals
  b. Amplitude versus range
  c. Cyclic stress–strain curve versus hysteresis curve

2.8 IMPORTANT EQUATIONS

Power Relationship between Stress and Plastic Strain

$$\sigma = K(\epsilon_p)^n \tag{2.18}$$

Strain Hardening Exponent

$$n = \text{slope of } \log\sigma \text{ vs. } \log\epsilon_p \quad \text{or} \quad n = \ln(1 + e \text{ at necking})$$

Strength Coefficient

$$K = \frac{\sigma_f}{\epsilon_f^n} \tag{2.23}$$

Total Strain = Elastic Strain + Plastic Strain

$$\epsilon_t = \epsilon_e + \epsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n} \tag{2.17}$$

Strain and Stress Amplitude

$$\epsilon_a = \frac{\Delta\epsilon}{2}$$

$$\sigma_a = \frac{\Delta\sigma}{2}$$

$$\Delta\epsilon = \Delta\epsilon_e + \Delta\epsilon_p \tag{2.29}$$

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\epsilon_p}{2} \tag{2.31}$$

Cyclic Stress–Plastic Strain Relationship

$$\sigma = K'(\epsilon_p)^{n'} \tag{2.32}$$

Cyclic Stress–Total Strain Relationship

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} \tag{2.34}$$

Hysteresis Curve

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \tag{2.36}$$

Strain–Life Relationship

$$\frac{\Delta\epsilon}{2} = \underbrace{\frac{\sigma_f'}{E}(2N_f)^b}_{\text{elastic}} + \underbrace{\epsilon_f'(2N_f)^c}_{\text{plastic}} \tag{2.41}$$

Transition Life

$$2N_t = \left(\frac{\epsilon_f' E}{\sigma_f'}\right)^{1/(b-c)} \tag{2.42}$$

Cyclic Strength Coefficient

$$K' = \frac{\sigma_f'}{(\epsilon_f')^{n'}} \tag{2.43}$$

Cyclic Strain Hardening Exponent

$$n' = \frac{b}{c} \tag{2.44}$$

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PROBLEMS

2.1 2.16 2.4 2.24 2.12 2.35 2.14 2.38

SECTION 2.2.1

2.1. During the initial stages of a tension test of an engineering material, the engineering stress and strain (S, e) were determined to be 62.2 ksi and 0.0098. Later, after some plastic deformation had occurred, the engineering stress and strain were determined to be 90.8 ksi and 0.0898. Calculate the true stress and strain (σ, ϵ) values at these two points. Discuss the relationship between the engineering and true values as plastic strain increases.

2.2. A cylindrical bar of structural steel with an initial diameter of 50 mm is loaded in tension. (*Note:* The general behavior of structural steel in a tension test is shown in Fig. 2.3.) The following deflection measurements are made over a 250-mm gage