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FEATURE

# Stress-Based Uniaxial Fatigue Analysis Using Methods Described in FKM-Guideline

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**Abstract** The process of prevention of failure from structural fatigue is a process that should take place during the early development and design phases of a structure. In the ground vehicle industry, for example, the durability specifications of a new product are directly interweaved with the desired performance characteristics, materials selection, manufacturing methods, and safety characteristics of the vehicle. In the field of fatigue and durability analysis of materials, three main techniques have emerged: nominal stress-based analysis, local strain-based analysis, and fracture mechanics analysis. Each of these methods has their own strengths and domain of applicability—for example, if an initial crack or flaw size is known to exist in a structure, a fracture mechanics approach can give a meaningful estimate of the number of cycles it takes to propagate the initial flaw to failure. The development of the local strain-based fatigue analysis approach has been used to great success in the automotive industry, particularly for the analysis of measured strain time histories gathered during proving ground testing or customer usage. However, the strain life approach is dependent on specific material properties data and the ability to measure (or calculate) a local strain history. Historically, the stress-based fatigue analysis approach was developed first—and is sometimes considered an "old" approach—but the stress-based fatigue analysis methods have been continued to be developed. The major strengths of this approach include the ability to give both quantitative and qualitative estimates of fatigue life with minimal estimates on stress levels and material properties, thus making the stress-based approach very relevant in the early design phase of structures where uncertainties regarding material selection, manufacturing processes, and final design specifications may cause numerous design iterations. This article explains the FKM-Guideline approach to stress-based uniaxial fatigue analysis. The Forschungskuratorium Maschinenbau (FKM) was developed in 1994 in Germany and has since continued to be updated. The guideline was developed for the use of the mechanical engineering community involved in the design of machine components, welded joints, and related areas. It is our desire to make the failure prevention and design community aware of these guidelines through a thorough explanation of the method and the application of the method to detailed examples.

**Keywords** Structural fatigue · Durability analysis · Failure prevention · Stress-based fatigue analysis · Surface finish effect

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**List of symbols**

| | |
|---|---|
| $A$ | Fatigue parameter |
| $a_d$ | Constant in the size correction formula |
| $a_R$ | Roughness constant |
| $a_P$ | Peterson's material constant |
| $a_N$ | Neuber's material constant |

| | |
|---|---|
| $a_{SS}$ | Siebel and Stieler material parameter |
| $a_G$ | Material constant in the $K_t/K_f$ ratio |
| $a_M$ | Material parameter in determining the mean stress sensitivity factor |
| $B$ | Width of a plate |
| $b$ | Slope (height-to-base ratio) of an $S$–$N$ curve in the HCF regime |
| $b_M$ | Material parameter in determining the mean stress sensitivity factor |
| $b_W$ | Width of a rectangular section |
| $b_{nw}$ | Net width of a plate |
| $b_G$ | Material constant in the $K_t/K_f$ ratio |
| $C_R$ | Reliability correction factor |
| $C_D$ | Size correction factor |
| $C_{u,T}$ | Temperature correction factor for ultimate strength |
| $C_\sigma$ | Stress correction factor for normal stress |
| $C_\tau$ | Stress correction factor for shear stress |
| $C_{b,L}$ | Load correction factor for bending |
| $C_{t,L}$ | Load correction factor for torsion |
| $C_{E,T}$ | Temperature correction factor for the endurance limit |
| $C_{\sigma,E}$ | Endurance limit factor for normal stress |
| $C_S$ | Surface treatment factor |
| $C_{\sigma,R}$ | Roughness correction factor for normal stress |
| $C_{\tau,R}$ | Roughness correction factor for shear stress |
| $COV_S$ | Coefficient of variations |
| $D$ | Diameter of a shaft |
| $D_{PM}$ | Critical damage value in the linear damage rule |
| $d$ | Net diameter of a notched shaft |
| $d_{eff}$ | Effective diameter of a cross section |
| $d_{eff,min}$ | Minimum effective diameter of a cross section |
| $G$ | Stress gradient along a local $x$-axis |
| $\bar{G}$ | Relative stress gradient |
| $\bar{G}_\sigma(r)$ | Relative normal stress gradient for plate or shaft based on notch radius |
| $\bar{G}_\sigma(d)$ | Relative normal stress gradient for plate or shaft based on component net diameter or width at notch |
| $\bar{G}_\tau(r)$ | Relative shear stress gradient for plate or shaft based on notch radius |
| $\bar{G}_\tau(d)$ | Relative shear stress gradient for plate or shaft based on component net diameter or width at notch |
| HB | Brinell hardness |
| $2h_T$ | Height of a rectangular section |
| $K_{ax,f}$ | Fatigue notch factor for a shaft under axial loading |
| $K_{ax,t}$ | Elastic stress concentration factor for a shaft under axial loading |
| $K_{b,f}$ | Fatigue notch factor for a shaft under bending |
| $K_{b,t}$ | Elastic stress concentration factor for a shaft under bending |
| $K_f$ | Fatigue notch factor or the fatigue strength reduction factor |
| $K_{i,f}$ | Fatigue notch factor for a superimposed notch |
| $K_{s,f}$ | Fatigue notch factor for a shaft under shear |
| $K_{s,t}$ | Elastic stress concentration factor for a shaft under shear |
| $K_t$ | Elastic stress concentration factor |
| $K_{t,f}$ | Fatigue notch factor for a shaft under torsion |
| $K_{t,t}$ | Elastic stress concentration factor for a shaft under torsion |
| $K_{x,f}$ | Fatigue notch factor for a plate under normal stress in $x$-axis |
| $K_{y,f}$ | Fatigue notch factor for a plate under normal stress in $y$-axis |
| $K_{\tau_{xy},f}$ | Fatigue notch factor for a plate under shear |
| $K_{x,t}$ | Elastic stress concentration factor for a plate under normal stress in $x$-axis |
| $K_{y,t}$ | Elastic stress concentration factor for a plate under normal stress in $y$-axis |
| $K_{\tau_{xy},t}$ | Elastic stress concentration factor for a plate under shear stress |
| $k$ | Slope factor (negative base-to-height ratio) of an $S$–$N$ curve in the HCF regime |
| $M_i$ | Initial yielding moment |
| $M_o$ | Fully plastic yielding moment |
| $M_\sigma$ | Mean stress sensitivity factor in normal stress |
| $N$ | Number of cycles to a specific crack initiation length |
| $2N$ | Number of reversals to a specific crack initiation length |
| $N_E$ | Endurance cycle limit |
| $N_{f,i}$ | Number of cycles to failure at the specific stress event |
| $n_K$ | $K_t/K_f$ ratio or the supporting factor |

| | |
|---|---|
| $n_{K,\sigma}(r)$ | $K_t/K_f$ ratio for a shaft under normal stress based on the notch radius |
| $n_{K,\sigma}(d)$ | $K_t/K_f$ ratio for a shaft under normal stress based on component net diameter or width at notch |
| $n_{K,\sigma,x}(r)$ | $K_t/K_f$ ratio for a plate under normal stress in *x*-axis based on notch radius |
| $n_{K,\sigma,y}(r)$ | $K_t/K_f$ ratio for a plate under normal stress in *y*-axis based on notch radius |
| $n_{K,\tau}(r)$ | $K_t/K_f$ ratio for a plate or shaft under shear stress based on notch radius |
| $n_{K,\tau}(d)$ | $K_t/K_f$ ratio for a shaft under shear stress based on component net diameter or width at notch |
| $n_i$ | Number of stress cycles |
| $O$ | Surface area of the section of a component |
| $q$ | Notch sensitivity factor |
| $R_r$ | Reliability value |
| $R$ | Stress ratio = ratio of minimum stress to maximum stress |
| $R_Z$ | Average roughness value of the surface based on German DIN system |
| $r$ | Notch root radius |
| $r_{max}$ | Larger of the superimposed notch radii |
| $S$ | Nominal stress |
| $S_C$ | Nominal stress of a notched component |
| $S_a$ | Stress amplitude |
| $S_m$ | Mean stress |
| $S_{max}$ | Maximum stress |
| $S_{min}$ | Minimum stress |
| $S_{\sigma,a}$ | Normal stress amplitude in a stress cycle |
| $S_{\sigma,m}$ | Mean normal stress in a stress cycle |
| $S_{\sigma,max}$ | Maximum normal stresses in a stress cycle |
| $S_{\sigma,min}$ | Minimum normal stresses in a stress cycle |
| $S_{\sigma,ar}$ | Equivalent fully reversed normal stress amplitude |
| $S_{\sigma,E}$ | Endurance limit for normal stress at $10^6$ cycles |
| $S_{\tau,E}$ | Endurance limit for shear stress at $10^6$ cycles |
| $S_E$ | Endurance limit at $10^6$ cycles |
| $S_{N,E}$ | Nominal endurance limit of a notched component |
| $S_{S,E,Smooth}$ | Nominal endurance limit of a smooth component at $10^6$ cycles |
| $S_{S,E,Notched}$ | Nominal endurance limit of a notched component at $10^6$ cycles |
| $S_{S,\sigma,E}$ | Endurance limit of a smooth, polish component under fully reversed normal stress |
| $S_{\sigma,FL}$ | Fatigue limit in normal stress at $10^8$ cycles |
| $S_{S,\tau,E}$ | Endurance limit of a smooth, polish component under fully reversed shear stress |
| $S_{S,\tau,u}$ | Ultimate strength of a notched, shell-shaped component for shear stress |
| $S_{S,\tau_{xy},E}$ | Endurance limit of a notched, shell-shaped component under fully reversed shear stress |
| $S_{\tau,FL}$ | Fatigue limit in shear at $10^8$ cycles |
| $S_{S,ax,E}$ | Endurance limit of a notched, rod-shaped component under fully reversed axial loading |
| $S_{S,ax,u}$ | Ultimate strength of a notched, rod-shaped component in axial loading |
| $S_{S,b,E}$ | Endurance limit of a notched, rod-shaped component under fully reversed bending loading |
| $S_{S,b,u}$ | Ultimate strength of a notched, rod-shaped component in bending |
| $S_{S,s,E}$ | Endurance limit of a notched, rod-shaped component under fully reversed shear loading |
| $S_{S,s,u}$ | Ultimate strength of a notched, rod-shaped component in shear |
| $S_{S,t,E}$ | Endurance limit of a notched, rod-shaped component under fully reversed torsion loading |
| $S_{S,t,u}$ | Ultimate strength of a notched, rod-shaped component in torsion |
| $S_{S,x,E}$ | Endurance limit of a notched, shell-shaped component under fully reversed normal stress in *x*-axis |
| $S_{S,x,u}$ | Ultimate strength of a notched, shell-shaped component for normal stress in *x*-axis |
| $S_{S,y,E}$ | Endurance limit of a notched, shell-shaped component under fully reversed normal stress in *y*-axis |
| $S_{S,y,u}$ | Ultimate strength of a notched, shell-shaped component for normal stress in *y*-axis |
| $S_{t,u}$ | Ultimate tensile strength with R97.5 |

| | |
|---|---|
| $S_{t,u,min}$ | Minimum ultimate tensile strength |
| $S_{t,u,std}$ | Mean ultimate tensile strength of a standard material test specimen |
| $S_{t,y}$ | Tensile yield strength with R97.5 |
| $S_{t,y,max}$ | Maximum tensile yield strength |
| $S'_f$ | Fatigue strength coefficient |
| $S'_{\sigma,f}$ | Fatigue strength coefficient in normal stress |
| $T$ | Temperature in degrees Celsius |
| $t_c$ | Coating layer thickness in μm |
| $V$ | Volume of the section of a component |
| $\sigma^e$ | Fictitious or pseudo-stress |
| $\sigma^e(x)$ | Pseudo-stress distribution along $x$ |
| $\sigma^e_E$ | Pseudo-endurance limit |
| $\sigma^e_{max}$ | Maximum pseudo-stress at $x = 0$ |
| $\Phi(z)$ | Standard normal density function |
| $\varphi = 1/(4\sqrt{t/r} + 2)$ | Parameter to calculate relative stress gradient |
| $\gamma_W$ | Mean stress fitting parameter in Walker's mean stress formula |

## Introduction

Stress-based uniaxial fatigue analysis is introduced in this article. The stress may either refer to the nominal stress of a component or the local pseudo-stress at a stress concentration area. Depending on loading and specimen configuration, the nominal stress ($S$) can be calculated using a traditional stress formula applied to elastic net cross-sectional properties, while the local fictitious or pseudo-stress ($\sigma^e$) can be computed either by the product of the nominal stress and the elastic stress concentration factor ($K_t$) or by a linear elastic finite element analysis. In general, the nominal stress approach is preferable for a shaft or shell-shaped component with a well-defined notched geometry. It should be noted that shell-shaped refers to a tubular or hollow cross section. Alternatively, the local pseudo-stress approach is recommended if the stress is directly determined by an elastic finite element analysis when there is no well-defined notched geometry or the elastic stress concentration factor is not known.

Uniaxial fatigue analysis is used to estimate the fatigue life of a component under cyclic loading when the crack is initiated due to a uniaxial state of stress. The fatigue life of a component refers to the fatigue initiation life defined as the number of cycles ($N$) or reversals ($2N$) to a specific crack initiation length of the component under cyclic stress controlled tests. Note that one cycle consists of two reversals. Cyclic stresses are typically described either by stress amplitude ($S_a$) and mean stress ($S_m$) or by maximum stress ($S_{max}$) and minimum stress ($S_{min}$), as shown in Fig. 1. Since $S_a$ is the primary factor affecting $N$, it is often chosen as the controlled or independent parameter in fatigue testing, and consequently, $N$ is the dependent variable on $S_a$.

The choice of the dependent and independent variables places an important role in performing a linear regression analysis to define the stress and life relation. As the stress amplitude becomes larger, the fatigue life is expected to be shorter. The stress and life relation (namely, the constant amplitude $S$–$N$ curve) can be generated by fatigue testing material specimens or real components at various load/stress levels. For this type of fatigue testing, the mean stress is usually held as a constant, and is commonly equal to zero.

The constant amplitude $S$–$N$ curve is often plotted by a straight line on log–log coordinates, representing fatigue data in the high cycle fatigue (HCF) regime where fatigue damage is due to little plastic deformation. In German, the constant amplitude $S$–$N$ curve is often named as the "Wöhler curve" to honor Mr. Wöhler for his contribution to the first fatigue study in the world. The term $S$–$N$ curve is used as the abbreviation of a constant amplitude $S$–$N$ curve. Depending on the test objective, the $S$–$N$ curve can represent the material or the component. Also, depending on the definition of stress, the real component $S$–$N$ curve can be categorized as the nominal $S$–$N$ curve or the pseudo $\sigma^e$-$N$ curve.

In general, an $S$–$N$ curve can be constructed as a piecewise–continuous curve consisting of two distinct linear regimes when plotted on log–log coordinates. For the typical $S$–$N$ curve of a component made of steel, as schematically illustrated in Fig. 2, there is one inclined linear segment for the HCF regime and one horizontal asymptote for the fatigue limit.

The parameters used to define the inclined linear segment of an $S$–$N$ curve are termed as the fatigue properties. The slope of an $S$–$N$ curve in the HCF regime can be

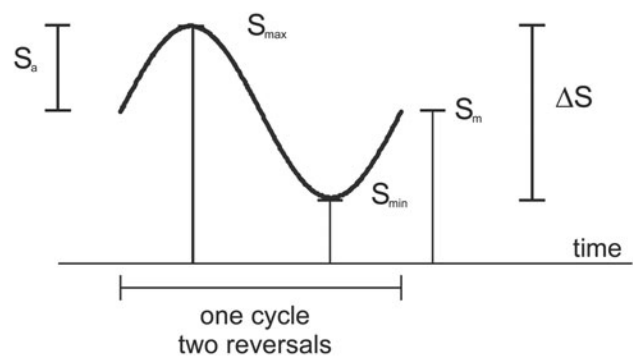



**Fig. 1** Definition of stress terms associated with constant amplitude loading [16]

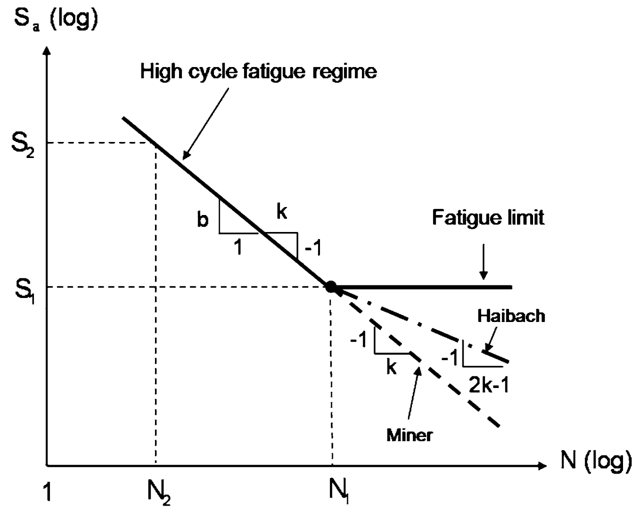



**Fig. 2** Schematic constant amplitude *S–N* curve of a component made of steels [16]

denoted as $b$ (the height-to-base ratio) or as $k$ (the negative base-to-height ratio). The parameter $k$ is termed as the slope factor. The two slopes are related in the following expression:

$$k = -\frac{1}{b}. \qquad \text{(Eq 1)}$$

Any two *S–N* data points ($S_1$,$N_1$) and ($S_2$,$N_2$) in the HCF regime can be related by the slope $b$ or the slope factor $k$ in the following equation:

$$\frac{N_2}{N_1} = \left(\frac{S_1}{S_2}\right)^k = \left(\frac{S_1}{S_2}\right)^{-1/b}. \qquad \text{(Eq 2)}$$

Equation 2 also means any data point ($S_2$,$N_2$) can be obtained by a reference point ($S_1$,$N_1$) and a given $b$ or $k$. The *S–N* curve is commonly expressed as

$$NS_a^k = A \qquad \text{(Eq 3)}$$

or

$$S_a = S'_{\mathrm{f}}(2N)^b, \qquad \text{(Eq 4)}$$

where $A$ is the fatigue parameter and $S'_{\mathrm{f}}$ is the fatigue strength coefficient defined as the fatigue strength at one reversal.

The fatigue limit of components made of steels and cast irons can be defined as the fully reversed stress amplitude at which the fatigue initiation life becomes infinite or when fatigue initiation failure does not occur. The fatigue limit can be interpreted from the physical perspective of the fatigue damage phenomenon under constant amplitude loading. Due to cyclic operating stresses, a microcrack will nucleate within a grain of material and grow to the size of about the order of a grain width until the grain boundary barrier impedes its growth. If the grain barrier is not strong enough, the microcrack will eventually propagate to a macrocrack and may lead to final failure. However, if the grain barrier is very strong, the microcrack will be arrested and become a non-propagating crack. The minimum stress amplitude to overcome the crack growth barrier for further crack propagation is referred to as the fatigue limit.

The fatigue limit might be negatively influenced by other factors such as periodic overloads, elevated temperatures, or corrosion. When Miner's rule [20] is applied in variable amplitude loading, the stress cycles with amplitudes below the fatigue limit could become damaging if some of the subsequent stress amplitudes exceed the original fatigue limit. It is believed that the increase in crack driving force due to periodic overloads will overcome the original grain barrier strength and help the crack to propagate until failure. Therefore, two methods such as the Miner rule and the Miner–Haibach model [10], as shown in Fig. 2, were proposed to include the effect of periodic overloads on the stress cycle behavior below the original fatigue limit. The Miner rule extends the *S–N* curve with the same slope factor $k$ to approach zero stress amplitude, while the Miner–Haibach model extends the original *S–N* curve below the fatigue limit to the zero stress amplitude with a flatter slope factor $2k - 1$. Stanzl et al. [31] concluded that good agreement is found for measured and calculated results according to the Miner–Haibach model.

For components made of aluminum alloys and austenitic steels, the fatigue limit does not exist and fatigue testing must be terminated at a specified large number of cycles. This non-failure stress amplitude is often referred to as the endurance limit, which needs not be the fatigue limit. However, in this article the endurance limit is defined as the fully reversed stress amplitude at $10^6$ cycles (the endurance cycle limit $N_{\mathrm{E}}$) for all materials. Through many years of experience and testing, empirical relationships that relate the data among ultimate tensile strengths and endurance limits at $10^6$ cycles have been developed. These relationships are not scientifically based but are simple and useful engineering tools for generating the synthetic component stress-life curves for various materials. The detailed procedures to generate the synthetic nominal *S–N* and the pseudo-$\sigma^{\mathrm{e}}$–$N$ curves for fatigue designs are the focus of this report, and will be addressed in the following sections. The techniques used to conduct *S–N* testing and perform data analysis for fatigue properties are beyond the scope of this discussion, and can be found elsewhere [2, 15]. It is also worth mentioning that the materials presented here have been drawn heavily from FKM-Guideline [11].

## Ultimate Tensile Strength of a Component

The mean ultimate tensile strength of a standard material specimen can be determined by averaging the static test results of several smooth (i.e., unnotched), polished round test specimens of 7.5 mm diameter. If the test data are not available, the mean ultimate tensile strength of a standard test specimen can be estimated by a hardness value. There has been a strong correlation between hardness and mean ultimate tensile strength of standard material test specimens. Several models have been proposed to estimate mean ultimate tensile strength from hardness. Lee and Song [14] reviewed most of them and concluded that Mitchell's equation [21] provides the best results for both steels and aluminum alloys. Mitchell's equation can be found below

$$S_{\mathrm{t,u,std}} = 3.45\,\mathrm{HB}, \quad \text{(Eq 5)}$$

where $S_{\mathrm{t,u,std}}$ is the mean ultimate tensile strength (in MPa) of a standard material test specimen and HB is the Brinell hardness. Figure 3 shows a plot of mean tensile strength versus Brinell hardness for steels, irons, and aluminum alloys and also includes a dashed line representing Eq 5.

It has been found that the surface treatment/roughness and the local notch geometry have little effect on the ultimate tensile strength of a notched component. However, the size of the real component has some degree of influence on the strength of the component. Therefore, based on the estimated or measured mean ultimate tensile strength of a "standard" material specimen, the ultimate tensile strength of a real component ($S_{\mathrm{t,u}}$) with a survival rate (reliability) of $R_{\mathrm{r}}$ is estimated as follows:

$$S_{\mathrm{t,u}} = C_{\mathrm{D}} C_{\mathrm{R}} S_{\mathrm{t,u,std}}, \quad \text{(Eq 6)}$$

where $C_{\mathrm{R}}$ is the reliability correction factor and $C_{\mathrm{D}}$ is the size correction factor.

If a component exposed to an elevated temperature condition is subjected to various loading modes, the ultimate strength values of a notched, rod-shaped component in axial ($S_{\mathrm{S,ax,u}}$), bending ($S_{\mathrm{S,b,u}}$), shear ($S_{\mathrm{S,s,u}}$), and torsion ($S_{\mathrm{S,t,u}}$) can be estimated as follows:

$$S_{\mathrm{S,ax,u}} = C_{\sigma} C_{\mathrm{u,T}} S_{\mathrm{t,u}} \quad \text{(Eq 7)}$$

$$S_{\mathrm{S,b,u}} = C_{\mathrm{b,L}} C_{\sigma} C_{\mathrm{u,T}} S_{\mathrm{t,u}} \quad \text{(Eq 8)}$$

$$S_{\mathrm{S,s,u}} = C_{\tau} C_{\mathrm{u,T}} S_{\mathrm{t,u}} \quad \text{(Eq 9)}$$

$$S_{\mathrm{S,t,u}} = C_{\mathrm{t,L}} C_{\tau} C_{\mathrm{u,T}} S_{\mathrm{t,u}}. \quad \text{(Eq 10)}$$

Similarly, the ultimate strength values of a notched, shell-shaped component for normal stresses in $x$ and $y$ directions ($S_{\mathrm{S},x,\mathrm{u}}$ and $S_{\mathrm{S},y,\mathrm{u}}$) and for shear stress ($S_{\mathrm{S},\tau,\mathrm{u}}$) can be determined as follows:

$$S_{\mathrm{S},x,\mathrm{u}} = C_{\sigma} C_{\mathrm{u,T}} S_{\mathrm{t,u}} \quad \text{(Eq 11)}$$

$$S_{\mathrm{S,y,u}} = C_{\sigma} C_{\mathrm{u,T}} S_{\mathrm{t,u}} \quad \text{(Eq 12)}$$

$$S_{\mathrm{S},\tau,\mathrm{u}} = C_{\tau} C_{\mathrm{u,T}} S_{\mathrm{t,u}}, \quad \text{(Eq 13)}$$

where $C_{\mathrm{u,T}}$ is the temperature correction factor, $C_{\sigma}$ and $C_{\tau}$ are the stress correction factors for normal and shear stresses, respectively, $C_{\mathrm{b,L}}$ and $C_{\mathrm{t,L}}$ are the load correction factors in bending and torsion, respectively.

Figure 4 shows the schematic effects of these corrections factors on the component ultimate tensile strength and its endurance limit. These correction factors will be discussed in the following sections.

## Reliability Correction Factor

If test data are not available, a statistical analysis cannot be performed to account for variability of the ultimate tensile strength. In the absence of the statistical analysis, the suggested correction factors for various reliability levels are given in Table 1. These values were derived under the assumptions of a normally distributed ultimate tensile

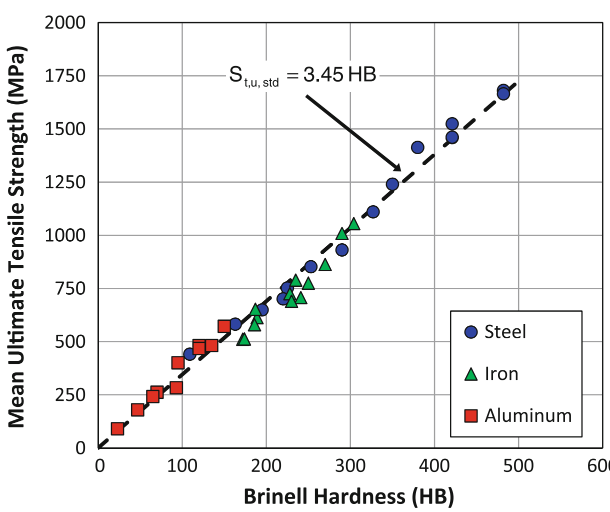



**Fig. 3** Mean ultimate tensile strength versus Brinell hardness (experimental data taken from [33])

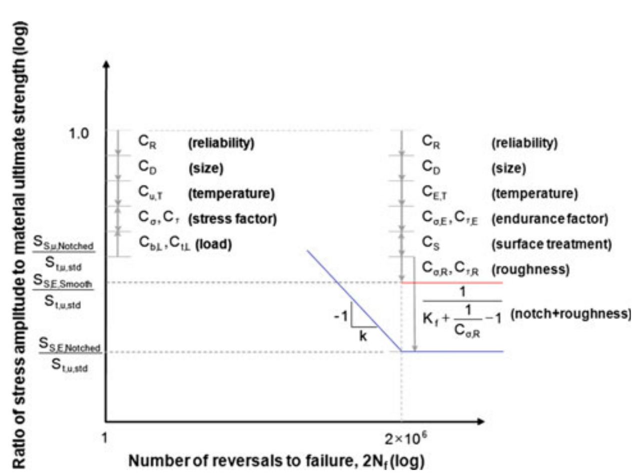



**Fig. 4** Correction factors for the component ultimate strength and the endurance limit [16]

**Table 1** Reliability correction factors, $C_R$

| Reliability | $C_R$ |
|---|---|
| 0.5 | 1.000 |
| 0.90 | 0.897 |
| 0.95 | 0.868 |
| 0.975 | 0.843 |
| 0.99 | 0.814 |
| 0.999 | 0.753 |
| 0.9999 | 0.702 |
| 0.99999 | 0.659 |

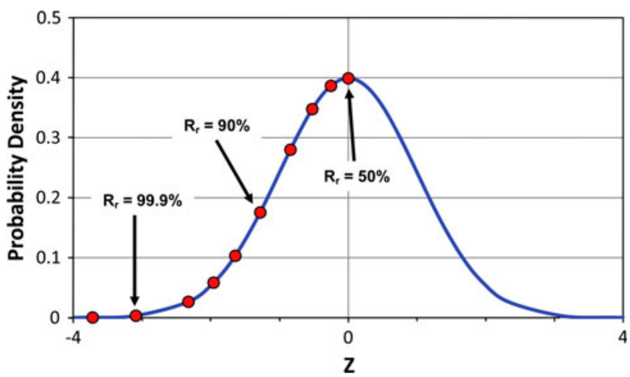



**Fig. 5** Standard normal density function

strength and the coefficient of variations ($COV_S$) of 8%. The coefficient of variations is defined as the standard deviation of a data set divided by the mean of a data set. The derivation of these $C_R$ values can be obtained using the following equation:

$$C_R = 1 - \left|\Phi^{-1}(1 - R_r)\right| COV_S, \quad \text{(Eq 14)}$$

where $R_r$ is the reliability value and $\Phi(z)$ is the standard normal density function. $\Phi(z)$ and $z$ are defined by the following equations:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} e^{-1/2(z)^2} \quad \text{(Eq 15)}$$

$$z = \Phi^{-1}(1 - R_r). \quad \text{(Eq 16)}$$

Figure 5 shows a plot of Eq 15 (standard normal density function). The points on the curve represent different levels of reliability.

Please note that FKM-Guideline [11] specifies that the ultimate strength of a component for a design should be based on the probability of a 97.5% survival rate, meaning a corresponding $C_R$ value of 0.843.

## Size Correction Factor

The size correction factor ($C_D$) is used to account for the fact that the strength of a component reduces as the size increases with respect to that of the standard material test specimen (a diameter of 7.5 mm). This is due to the increasing possibility of a weak link with increasing material volume. Based on FKM-Guideline [11], the size correction factor, dependent on the cross-sectional size and the type of material can be obtained as follows:

**Table 2** Constants used to estimate the size correction factors (adopted from FKM-Guideline)

| Material type | $d_{eff,min}$, mm | $a_d$ | Case of $d_{eff}$ |
|---|---|---|---|
| Plain carbon steel | 40 | 0.15 | Case 2 |
| Fine grained steel | 70 | 0.2 | Case 2 |
| Steel, quenched, and tempered | 16 | 0.3 | Case 2 |
| Steel, normalized | 16 | 0.1 | Case 2 |
| Steel, case hardened | 16 | 0.5 | Case 1 |
| Nitriding steel, quenched, and tempered | 40 | 0.25 | Case 1 |
| Forging steel, quenched, and tempered | 250 | 0.2 | Case 1 |
| Forging steel, normalized | 250 | 0 | Case 1 |
| Steel casting | 100 | 0.15 | Case 2 |
| Steel casting, quenched, and tempered | 200 | 0.15 | Case 1 |
| Ductile irons | 60 | 0.15 | Case 1 |
| Malleable cast iron | 15 | 0.15 | Case 1 |

- For wrought aluminum alloys

$$C_D = 1.0. \quad \text{(Eq 17)}$$

- For cast aluminum alloys

$$C_D = 1.0 \quad \text{for } d_{eff} \leq 12\,\text{mm}. \quad \text{(Eq 18)}$$

$$C_D = 1.1(d_{eff}/7.5\,\text{mm})^{-0.2} \quad \text{for } 12\,\text{mm} < d_{eff} < 150\,\text{mm}. \quad \text{(Eq 19)}$$

$$C_D = 0.6 \quad \text{for } d_{eff} \geq 150\,\text{mm}. \quad \text{(Eq 20)}$$

- For gray cast irons

$$C_D = 1.207 \quad \text{for } d_{eff} \leq 7.5\,\text{mm}. \quad \text{(Eq 21)}$$

$$C_D = 1.207(d_{eff}/7.5\text{mm})^{-0.1922} \quad \text{for } d_{eff} > 7.5\,\text{mm}. \quad \text{(Eq 22)}$$

- For all steels, steel castings, ductile irons, and malleable cast iron

$$C_D = 1.0 \quad \text{for } d_{eff} \leq d_{eff,min}. \quad \text{(Eq 23)}$$

$$C_D = \frac{1 - 0.7686 \cdot a_d \cdot \log(d_{eff}/7.5\text{mm})}{1 - 0.7686 \cdot a_d \cdot \log\left(d_{eff,min}/7.5\text{mm}\right)} \quad \text{(Eq 24)}$$

for $d_{eff} > d_{eff,min}$,

where $d_{eff}$ is the effective diameter of a cross section, $d_{eff,min}$ and $a_d$ are the constants tabulated in Table 2.

Depending on the type of material as listed in Table 2, two cases are required to be distinguished to determine $d_{\mathrm{eff}}$. In Case 1, $d_{\mathrm{eff}}$ is defined by

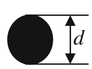
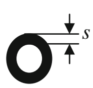
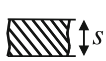
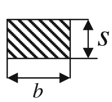
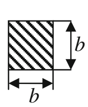
$$d_{\mathrm{eff}} = \frac{4V}{O}, \qquad \text{(Eq 25)}$$

where $V$ and $O$ are the volume and surface area of the section of the component of interest, respectively. In Case 2, $d_{\mathrm{eff}}$ is equal to the diameter or wall thickness of the component, and applies to all components made of aluminum alloys. Examples of $d_{\mathrm{eff}}$ calculation are illustrated in Table 3.

### Temperature Correction Factor for Ultimate and Yield Strengths

The temperature factor ($C_{\mathrm{u,T}}$) is used to take into account the ultimate and yield strength reductions in the field of elevated temperatures. FKM-Guideline [11] specifies these temperature effects for various materials as follows:

**Table 3** Calculation of the effective diameter $d_{\mathrm{eff}}$ (Adopted from FKM-Guideline)

| Cross section | $d_{\mathrm{eff}}$ Case 1 | $d_{\mathrm{eff}}$ Case 2 |
|---|---|---|
| Solid rod with round cross section  | $d$ | $d$ |
| Tubular rod with round cross section  | $2s$ | $s$ |
| Infinitely wide thin wall (tubes with large inner diameter)  | $2s$ | $s$ |
| Solid rod with rectangular cross section  | $\frac{2bs}{b+s}$ | $s$ |
| Solid rod with square cross section  | $b$ | $b$ |

- For age-hardening (or heat treatable) aluminum alloys where $T > 50$ °C

$$C_{\mathrm{u,T}} = 1 - 4.5 \times 10^{-3}(T - 50) \geq 0.1. \qquad \text{(Eq 26)}$$

- For non-age-hardening aluminum alloys where $T > 100$ °C

$$C_{\mathrm{u,T}} = 1 - 4.5 \times 10^{-3}(T - 100) \geq 0.1. \qquad \text{(Eq 27)}$$

- For fine grained steel where $T > 60$ °C

$$C_{\mathrm{u,T}} = 1 - 1.2 \times 10^{-3}T. \qquad \text{(Eq 28)}$$

- For steel castings where $T > 100$ °C

$$C_{\mathrm{u,T}} = 1 - 1.5 \times 10^{-3}(T - 100). \qquad \text{(Eq 29)}$$

- For all other steels where $T > 100$ °C

$$C_{\mathrm{u,T}} = 1 - 1.7 \times 10^{-3}(T - 100). \qquad \text{(Eq 30)}$$

- For ductile irons where $T > 100$ °C

$$C_{\mathrm{u,T}} = 1 - 2.4 \times 10^{-3}T. \qquad \text{(Eq 31)}$$

The temperature used in Eq 26–31 must be given in degrees Celsius. It should be noted that the analysis provided in FKM-Guideline is applicable in the following temperature ranges:

- $-40 < T < 500$ °C for steels
- $-25 < T < 500$ °C for cast irons
- $-25 < T < 200$ °C for aluminums.

### Stress Correction Factor

The stress correction factor is used to correlate the different material strengths in compression or shear with respect to that in tension, and can be found in Table 4. Note that $C_\sigma = 1.0$ for tension.

**Table 4** Stress correction factors $C_\sigma$ and $C_\tau$ in compression and in shear (adopted from FKM-Guideline)

| Materials | $C_\sigma$ | $C_\tau$ |
|---|---|---|
| Case hardening steel | 1 | $1/\sqrt{3} = 0.577$ |
| Stainless steel | 1 | 0.577 |
| Forging steel | 1 | 0.577 |
| Steel casting | 1 | 0.577 |
| Other types of steel | 1 | 0.577 |
| Ductile irons | 1.3 | 0.65 |
| Malleable cast iron | 1.5 | 0.75 |
| Gray cast iron | 2.5 | 0.85 |
| Aluminum alloys | 1 | 0.577 |
| Cast aluminum alloys | 1.5 | 0.75 |

Note that $1/\sqrt{3} = 0.577$ is based on the von Mises yield criterion

Load Correction Factor

The stress gradient of a component in bending or torsion can be taken into account by the load correction factor, also termed as the "section factor" or the "plastic notch factor" in FKM. It should be noted that this factor is not applicable for shear loading or unnotched components (only for notched components subjected to torsion or bending loading).The load correction factor is defined as the ratio of the nominal stress at global yielding to the nominal stress at the initial notch yielding. Alternatively, the load correction factors in Table 5 are derived from the ratio of fully plastic yielding force, moment, or torque to the initial yielding force, moment, or torque. For example, a component has the tensile yield strength ($S_{t,y}$), a rectangular section with a width of $b_w$, and a height of $2h_T$. Its initial yielding moment is calculated as $M_i = 2/3 b_W h_T^2 S_{t,y}$, and the fully plastic yielding moment is $M_o = b_W h_T^2 S_{t,y}$. Thus, the corresponding load-modifying factor for bending is found to be $M_o/M_i = 1.5$.

The load correction factor also depends on the type of material according to FKM-Guideline [11]. For surface hardened components, the load factors are not applicable ($C_{b,L} = C_{t,L} = 1.0$). For highly ductile austenitic steels in a solution annealed condition, $C_{b,L}$ and $C_{t,L}$ follow the values in Table 5. Also, for other steels, steel castings, ductile irons, and aluminum alloys

$$C_{b,L} = \text{minimum of } \left(\sqrt{S_{t,y,max}/S_{t,y}}; C_{b,L}\right) \tag{Eq 32}$$

$$C_{t,L} = \text{minimum of } \left(\sqrt{S_{t,y,max}/S_{t,y}}; C_{t,L}\right), \tag{Eq 33}$$

where $S_{t,y}$ is the tensile yield strength in MPa with a reliability of 97.5%, and $S_{t,y,max}$ is the maximum tensile yield strength in MPa, given in Table 6.

**Table 5** Load correction factors $C_{b,L}$ and $C_{t,L}$ (adopted from FKM-Guideline)

| Cross section | $C_{b,L}$ | $C_{t,L}$ |
|---|---|---|
| Rectangle | 1.5 | … |
| Circle | 1.7 | 1.33 |
| Tubular | 1.27 | 1 |

**Table 6** Maximum tensile yield strength $S_{t,y,max}$ for various materials (adopted from FKM-Guideline)

| Type of material | Steels, steel castings | Ductile irons | Aluminum alloys |
|---|---|---|---|
| $S_{t,y,max}$, MPa | 1050 | 320 | 250 |

**Component Endurance Limit Under Fully Reversed Loading**

The endurance limit is defined as the stress amplitude for fully reversed loading at an endurance cycle limit ($N_E = 10^6$ cycles). Since R is defined as the ratio of minimum stress to maximum stress, fully reversed loading is also termed as $R = -1$ loading. Even though the endurance limit is occasionally expressed in terms of "range" in some references, it is worth noting that the endurance limit in this article is clearly defined in "amplitude".

With the probability of survival ($R_r$), the endurance limit for a smooth, polished component at an elevated temperature condition and under fully reversed normal ($S_{S,\sigma,E}$) or shear stress ($S_{S,\tau,E}$) can be estimated from $S_{t,u}$ which has already taken into account the factors for size and reliability

$$S_{S,\sigma,E} = C_{\sigma,E} C_{E,T} S_{t,u} \tag{Eq 34}$$

$$S_{S,\tau,E} = C_\tau S_{S,\sigma,E}, \tag{Eq 35}$$

where $C_{E,T}$ is the temperature correction factor for the endurance limit, $C_{\sigma,E}$ is the endurance limit factor for normal stress, and $C_\tau$ is the shear stress correction factor. The shear stress correction factor used for estimating the material fatigue strength in shear is the same factor used for ultimate strength (see Table 4). Alternatively, if the material fatigue strength is known (fatigue strength of standard polished fatigue sample under fully reversed normal stress), it can be substituted for Eq 34. However, reliability and size correction factors should be applied to the material fatigue strength if applicable.

It has been found that the endurance limit of a notched component is affected by residual stress and surface hardened layers resulting from surface treatment. The endurance limit is also affected by the high stress concentration/stress gradient due to surface roughness and local geometrical changes. These effects have been empirically quantified by FKM-Guideline [11]. For example, the endurance limits of a notched, rod-shaped component under fully reversed loading in axial ($S_{S,ax,E}$), bending ($S_{S,b,E}$), shear ($S_{S,s,E}$), and torsion ($S_{S,t,E}$) can be obtained as follows:

$$S_{S,ax,E} = \frac{C_S S_{S,\sigma,E}}{K_{ax,f} + \frac{1}{C_{\sigma,R}} - 1} \tag{Eq 36}$$

$$S_{S,b,E} = \frac{C_S S_{S,\sigma,E}}{K_{b,f} + \frac{1}{C_{\sigma,R}} - 1} \tag{Eq 37}$$

$$S_{S,s,E} = \frac{C_S S_{S,\tau,E}}{K_{s,f} + \frac{1}{C_{\tau,R}} - 1} \tag{Eq 38}$$

$$S_{S,t,E} = \frac{C_S S_{S,\tau,E}}{K_{t,f} + \frac{1}{C_{\tau,R}} - 1}. \tag{Eq 39}$$

Similarly, the endurance limit values of a notched, shell-shaped component under fully reversed normal stresses in $x$ and $y$ directions and under shear stress can be obtained as follows:

$$S_{S,x,E} = \frac{C_S S_{S,\sigma,E}}{K_{x,f} + \frac{1}{C_{\sigma,R}} - 1} \quad \text{(Eq 40)}$$

$$S_{S,y,E} = \frac{C_S S_{S,\sigma,E}}{K_{y,f} + \frac{1}{C_{\sigma,R}} - 1} \quad \text{(Eq 41)}$$

$$S_{S,\tau_{xy},E} = \frac{C_S S_{S,\tau,E}}{K_{\tau_{xy},f} + \frac{1}{C_{\tau,R}} - 1}, \quad \text{(Eq 42)}$$

where $C_S$ is the surface treatment factor, $C_{\sigma,R}$ and $C_{\tau,R}$ are the roughness correction factors for normal and shear stresses, respectively, $K_{ax,f}$, $K_{b,f}$, $K_{s,f}$, $K_{t,f}$, $K_{x,f}$, $K_{y,f}$, and $K_{\tau_{xy},f}$ are the fatigue notch factors for various loading modes. Note that the endurance limit of a smooth component can be calculated using the above equations with $K_f = 1$.

Figure 4 shows the schematic effects of these corrections factors on the endurance limits of smooth and notched components. The correction factors for temperature, endurance limit for normal stress, surface treatment, surface roughness, and the fatigue notch factor are discussed in the following sections.

### Temperature Correction Factor

It has been observed that at an elevated temperature, the component fatigue strength reduces with increasing temperature. The temperature reduction factor for the endurance limit ($C_{E,T}$) is different from the factor applied to the ultimate tensile strength ($C_{u,T}$). Depending on the type of materials, FKM-Guideline [11] specifies the temperature correction factors as follows:

- For aluminum alloys where $T > 50$ °C

$$C_{E,T} = 1 - 1.2 \times 10^{-3}(T - 50)^2. \quad \text{(Eq 43)}$$

- For fine grained steel where $T > 60$ °C

$$C_{E,T} = 1 - 10^{-3}T. \quad \text{(Eq 44)}$$

- For steel castings where $T > 100$ °C

$$C_{E,T} = 1 - 1.2 \times 10^{-3}(T - 100). \quad \text{(Eq 45)}$$

- For all other steels where $T > 100$ °C

$$C_{E,T} = 1 - 1.4 \times 10^{-3}(T - 100). \quad \text{(Eq 46)}$$

- For ductile irons where $T > 100$ °C

$$C_{E,T} = 1 - 1.6\left(10^{-3} \cdot T\right)^2. \quad \text{(Eq 47)}$$

- For malleable cast iron where $T > 100$ °C

$$C_{E,T} = 1 - 1.3\left(10^{-3} \cdot T\right)^2. \quad \text{(Eq 48)}$$

- For gray cast iron where $T > 100$ °C

$$C_{E,T} = 1 - 1.0\left(10^{-3} \cdot T\right)^2. \quad \text{(Eq 49)}$$

Note that the temperature must be given in degrees Celsius.

### Endurance Limit Factor

The endurance limit factor ($C_{\sigma,E}$) for normal stress is an empirical factor used to estimate the endurance limit based on the ultimate tensile strength of a component with a 97.5% reliability, and can be found in Table 7.

### Surface Treatment Factor

The surface treatment factor, $C_S$, which takes into account the effect of a treated surface layer on the fatigue strength of a component, is defined as the ratio of the endurance limit of a surface layer to that of the core material. $C_S$ depends on whether the crack origin is expected to be located at the surface or in the core. According to FKM-Guideline [11], the upper and lower limits of the surface treatment factors for steel and cast iron materials are tabulated in Table 8. The decision of which value to use within the ranges given in Table 8 is left up to the user. There can be large amounts of variation in the material properties produced by a given surface treatment. As a result, one should be very cautious when choosing a value for the surface correction factor. There are two sets of values in the table. The values in the parenthesis are for components with a diameter of 8–15 mm and the other set of values are for components with a diameter of 30–40 mm. It is also stated in the guidelines that the surface treatment correction factors for cast irons can be applied to aluminum alloys, providing that the surface treatment can be performed on the aluminum component.

**Table 7** Endurance limit factors for various materials (adopted from FKM-Guideline)

| Material type | $C_{\sigma,E}$ |
|---|---|
| Case hardening steel | 0.40 |
| Stainless steel | 0.40 |
| Forging steel | 0.40 |
| Steel casting | 0.34 |
| Other types of steel | 0.45 |
| Ductile iron | 0.34 |
| Malleable cast iron | 0.30 |
| Gray cast iron | 0.30 |
| Wrought aluminum alloys | 0.30 |
| Cast aluminum alloys | 0.30 |

**Table 8** Surface treatment factors for various materials (adopted from FKM-Guideline)

| Surface treatment | Unnotched components | Notched components |
|---|---|---|
| *Steel* | | |
| Chemo-thermal treatment | | |
| *Nitriding* depth of case 0.1–0.4 mm, surface hardness 655–926 HB | 1.10–1.15 (1.15-1.25) | 1.30–2.00 (1.90–3.00) |
| *Case hardening* depth of case 0.2–0.8 mm, surface hardness 628–701 HB | 1.10–1.50 (1.20–2.00) | 1.20–2.00 (1.50–2.50) |
| *Carbo-nitriding* depth of case 0.2–0.8 mm, surface hardness 628–701 HB | (1.80) | |
| Mechanical treatment | | |
| Cold rolling | 1.10–1.25 (1.20–1.40) | 1.30–1.80 (1.50–2.20) |
| Shot peening | 1.10–1.20 (1.10–1.30) | 1.10–1.50 (1.40–2.50) |
| Thermal treatment | | |
| *Inductive hardening*, *flame hardening* depth of case 0.9–1.5 mm, surface hardness 495–722 HB | 1.20–1.50 (1.30–1.60) | 1.50–2.50 (1.60–2.8) |
| *Cast iron materials* | | |
| Nitriding | 1.10 (1.15) | 1.3 (1.9) |
| Case hardening | 1.1 (1.2) | 1.2 (1.5) |
| Cold rolling | 1.1 (1.2) | 1.3 (1.5) |
| Shot peening | 1.1 (1.1) | 1.1 (1.4) |
| Inductive hardening, flame hardening | 1.2 (1.3) | 1.5 (1.6) |

Nitriding is done to increase surface hardness, improve fatigue life, and improve wear resistance. Nitriding is a case-hardening heat treatment that introduces nitrogen into the surface of a ferrous alloy. The hardened surface is achieved by holding the component at a subcritical temperature of around 565 °C (1050°F) while exposing it to nitrogenous environment. The component can be exposed to the nitrogen in a gas form or in the form of a salt bath. Note that the quenching is not necessary to achieve the hard case.

Carbo-nitriding is more of a carburizing process than a nitriding process. Gas carburizing, also referred to as case carburizing, is a process in which the component is heated up in a carbon-rich environment allowing the carbon to diffuse into the material. Carburizing temperatures can vary depending on the application and desired case depth. However, carburizing is typically done around 925 °C (1700°F). When carbo-nitriding, small amounts of nitrogen are added to the gas carburizing atmosphere allowing the nitrogen and carbon to diffuse into the component simultaneously. Carbo-nitriding of steels is done at temperatures around 870 °C (1600°F) which is slightly lower than the normal carburizing temperature. The added nitrogen increases the hardenability of the component surface. Note the heating and cooling (air cooling or quenching) time of the component subjected to carburizing or carbo-nitriding produces varying degrees of case depth and hardness.

Inductive or induction hardening is a process that uses electromagnetic induction to heat the component. This process can be applied to the surfaces of carbon and alloy steels, cast and ductile irons and some stainless steels, followed by an appropriate quenching method. Heating and cooling duration and temperatures will affect the material properties and depth of the hardened layer. Flame hardening is a similar process, except a high temperature flame is used to heat the component instead. Induction heating allows for greater control of material properties by heating the material more uniformly than flame heating. It should be noted that case hardening is more of a general term for the thermal and chemo-thermal surface treatments in Table 8, including carburizing which is not listed in the table [6].

Both cold rolling and shot peening use local plastic deformation at the surface to induce compressive residual surface stresses. During shot peening small objects are shot at the component to plastically deform the surface. On the other hand, the cold rolling process utilizes a narrow roller to work the surface. The compressive residual stresses at the surface improve the fatigue life of a component.

FKM-Guideline does not specify whether or not the surface treatment correction factors can be used for components with rectangular cross sections. However, limited experimental data have shown that shot-peened components with rectangular cross sections have similar increases in endurance limit compared to components with round cross sections [19]. Figure 6 shows a plot of experimental endurance limit for surface-treated components versus experimental endurance limit for components without surface treatment. This figure also has two lines representing the range of factors given in Table 8 for shot-peened components. In this figure, one of the three red data points is for a component with a rectangular cross section and the other two are for round cross sections. In addition, experimental data have shown correction factors used for shot-peened components could be applicable to shot-cleaned components [18]. Shot cleaning is similar to shot peening, but is done to clean the surface of a component with the additional benefit of compressive residual surface stresses (connecting rods are a good example of components that are shot-cleaned). In general, shot peening results in greater and more uniform residual stress distribution at the surface. It can be seen in Fig. 6 that the points for shot-cleaned components fall within the ranges suggested by the guidelines for shot-peened components.

$C_S$ can also account for the effect of a surface coating such as electrolytically formed anodic coatings on the

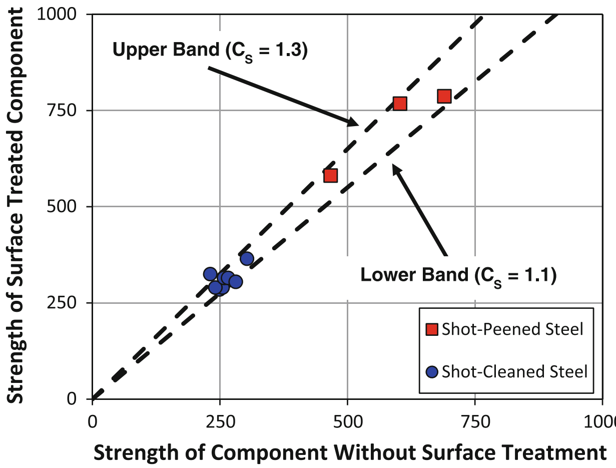



**Fig. 6** Experimental data showing accuracy of surface treatment correction factors for shot-peened components (experimental data taken from [18, 19])

endurance limit of a component made of aluminum alloys, and is specified as follows:

$$C_S = 1 - 0.271 \cdot \log(t_c), \quad \text{(Eq 50)}$$

where $t_c$ is the coating layer thickness in micrometers. The surface coatings on aluminum alloys reduce the endurance limit, but the surface treatments performed on steel and cast iron materials increase the endurance limit. It should be noted that surface treatments have different effects in the low cycle fatigue (LCF) region.

## Roughness Correction Factor

Surface roughness or irregularity acts as a stress concentration and results in a reduction in fatigue strength in the HCF region. The roughness correction factors $C_{\sigma,R}$ and $C_{\tau,R}$ account for the effect of surface roughness on the component endurance limit for normal and shear stresses, respectively. According to FKM-Guideline [11], the two roughness correction factors under normal and shear stresses are defined as follows:

$$C_{\sigma,R} = 1 - a_R \log(R_Z) \log\left(2S_{t,u}/S_{t,u,\min}\right) \quad \text{(Eq 51)}$$

and

$$C_{\tau,R} = 1 - C_\tau a_R \log(R_Z) \log\left(2S_{t,u}/S_{t,u,\min}\right), \quad \text{(Eq 52)}$$

where $a_R$ is a roughness constant and $S_{t,u,min}$ is the minimum ultimate tensile strength in MPa listed in Table 9. $R_z$ is defined as the average summation of the five highest peaks and the five lowest valleys

$$R_{z(\mathrm{DIN})} = \frac{1}{n}\left(\sum_{i=1}^{n} p_i + \sum_{i=1}^{n} v_i\right). \quad \text{(Eq 53)}$$

**Table 9** $a_R$ and $S_{t,u,min}$ for various materials (adopted from FKM-Guideline)

| Materials | $a_R$ | $S_{t,u,min}$, MPa |
|---|---|---|
| Steel | 0.22 | 400 |
| Steel castings | 0.20 | 400 |
| Ductile iron | 0.16 | 400 |
| Malleable cast iron | 0.12 | 350 |
| Gray cast iron | 0.06 | 100 |
| Wrought aluminum alloys | 0.22 | 133 |
| Cast aluminum alloys | 0.20 | 133 |

A roughness value of 200 μm applies for a rolling skin, a forging skin, and the skin of cast irons. For steels, the roughness value of a ground surface varies from 1 to 12 μm, and the value of a finished surface ranges from 6.3 to 100 μm.

It is important to use roughness of 200 μm when calculating a roughness factor for parts with rolled, forged, or cast surfaces even if measured values are available. Figure 7 shows a plot of predicted endurance limit (at $10^6$ cycles) versus experimental endurance limit (at $10^6$ cycles) for components having an as-forged surface. There are two sets of predictions in this figure, one using a roughness of 200 μm and one set using measured roughness values (measured values were around 50 μm). Figure 7 also has a 45° line representing perfect agreement between predicted and experimental values. It can be seen that predictions made using the measured roughness values have less agreement with experimental data.

## Fatigue Notch Factor

It was once believed that at the same crack initiation life near the endurance cycle limit of $10^6$ cycles, the pseudo-surface stress ($\sigma_E^e$) at the stress concentration location of a notched component would be identical to the surface stress of a smooth component ($S_{S,E,Smooth}$). As this belief provides $\sigma_E^e = S_{S,E,Smooth}$ and $\sigma_E^e = K_t \cdot S_{S,E,Notched}$, where $K_t$ and $S_{S,E,Notched}$ are the elastic stress concentration factor and the nominal stress of a notched component, respectively, one can conclude that $S_{S,E,Notched}$ is smaller than $S_{S,E,Smooth}$ by a factor of $K_t$. However, Tryon and Dey [34] presented a study revealing the effect of fatigue strength reduction for Ti–6Al–4V in the HCF regime in Fig. 8. The test has indicated at the same endurance cycle, the presence of a notch on a component under cyclic stressing reduces the nominal stress of a smooth component by a factor $K_f$, instead of $K_t$. The $K_f$ is termed as the fatigue notch factor or fatigue strength reduction factor defined as follows:

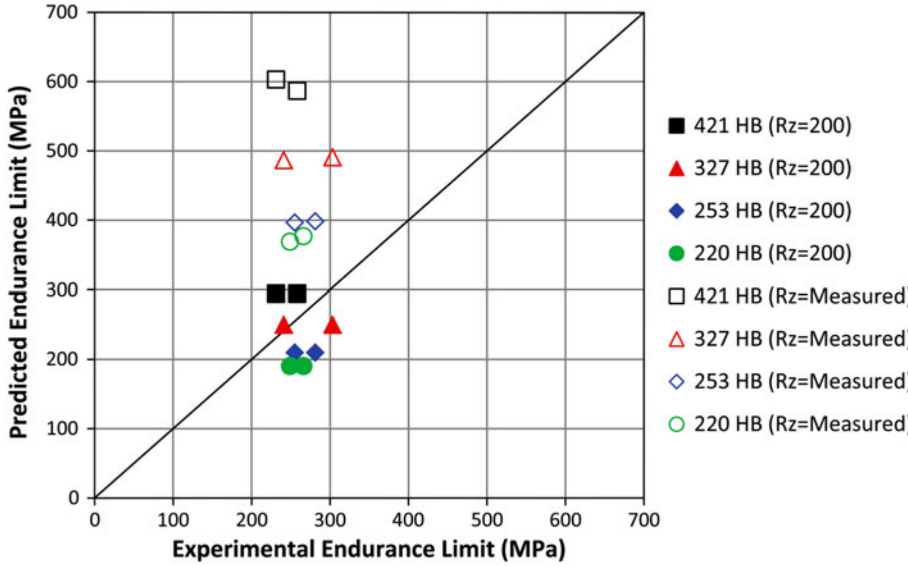



**Fig. 7** Predicted fatigue strength at $10^6$ cycles using measured and suggested values for $R_z$ versus experimental fatigue strength at $10^6$ cycles (experimental data taken from [18])

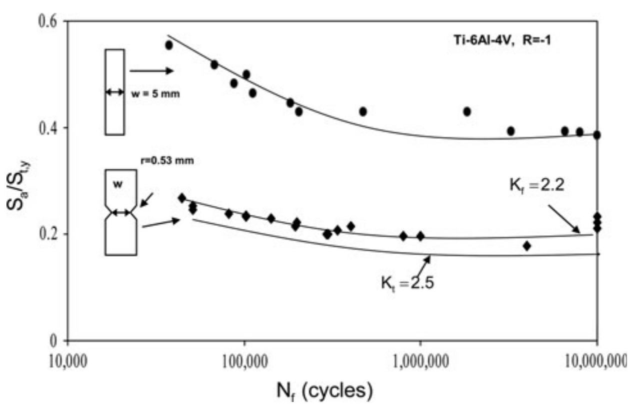



**Fig. 8** Effect of a notch on *S–N* behavior for Ti–6Al–4V in the HCF regime [16]

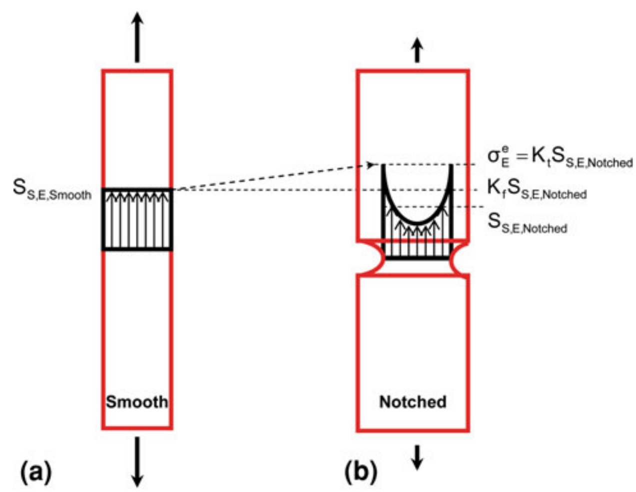



**Fig. 9** Identical crack initiation life for smooth and notched components [16]

$$K_f = \frac{S_{S,E,Smooth}}{S_{S,E,Notched}} \leq K_t. \quad \text{(Eq 54)}$$

Equation 54 can be interpreted as that when $K_f S_{S,E,Notched} = S_{S,E,Smooth}$, both the notched and the smooth components would have the same endurance cycle limit, as shown in Fig. 9.

The smaller $K_f$ than $K_t$ can be explained either by the local cyclic yielding behavior or by the stress field intensity theory [1, 27, 38]. The local yielding theory suggests the cyclic material yielding at a notch root reduces the peak pseudo-surface stress, while the stress field intensity theory postulates that the fatigue strength of a notched component depends on the "average stress" in a local damage zone, instead of the peak pseudo-surface stress at a notch root.

The stress field intensity theory is valid in the endurance cycle limit regime where the peak pseudo-surface stress is approximately equal to the true surface stress. According to the stress field intensity theory, the "average stress" is responsible for the crack initiation life, and associated with the stress distribution and the local damage zone at the notch. The "average stress" is defined as $K_f S_C$ as opposed to the peak pseudo-surface stress, $K_t S_C$, where $S_C$ is the nominal stress of a notched component.

Figure 10 schematically shows two notched components with the same peak pseudo-surface stress and steel material. Note that the subscripts 1 and 2 denote the notched components 1 and 2, respectively. For illustration, the damage zone of steel material is assumed to be the order of two grain sizes. As the notch radius decreases, the stress gradient becomes steeper, resulting in a lower "average stress" level. Consequently, the notched component with a smaller notch radius in Fig. 10(b) would have a lower $K_f$ value and longer fatigue initiation life than the component with a larger notch radius in Fig. 10(a).

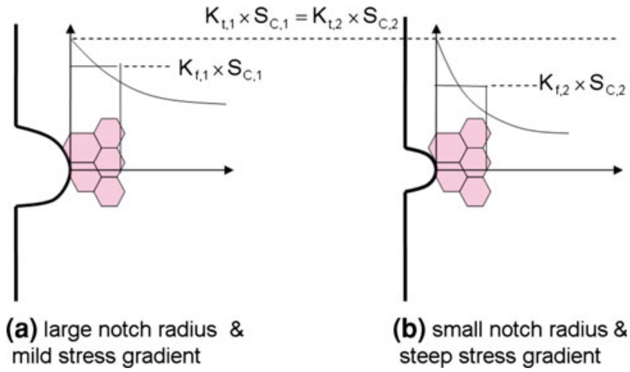



**Fig. 10** Effect of notch size and stress gradient on $K_f$ [16]

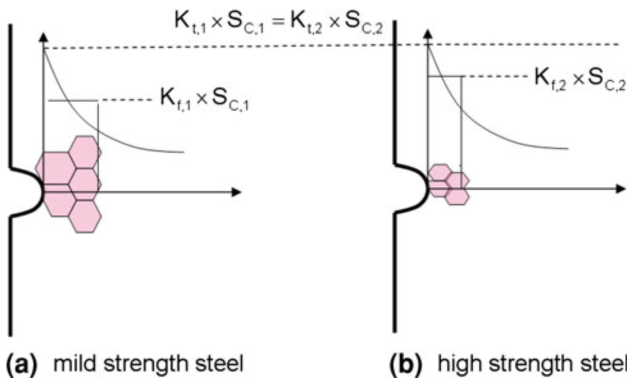



**Fig. 11** Effect of strength of materials on $K_f$ [16]

Figure 11 schematically illustrates another example of two notched components of identical geometry made of mild strength and high strength steels. Note that the subscripts 1 and 2 denote the notched components 1 and 2, respectively. Again, the damage zone of steel material is assumed to be the order of two grain sizes. As the high strength steel has smaller grain size than the mild strength steel, it suggests that the damage zone of high strength steel is smaller than that of the mild strength steel. Under the same peak pseudo-surface stress and distribution, the component made of mild strength steel in Fig. 11(a) would have a lower "average stress" in a larger damage zone, a lower $K_f$ value, and longer fatigue initiation life than the component made of high strength steel in Fig. 11(b).

Based on the stress field intensity theory, the fatigue notch factor is closely related to a notch root radius (or a stress gradient) and the strength of materials (or the grain size). Therefore, several empirical methods have been developed to determine the $K_t$–$K_f$ relationship based on any combination of the above two parameters. For example, a notch sensitivity factor ($q$) was introduced by Peterson [26] as follows:

$$q = \frac{K_f - 1}{K_t - 1}, \qquad \text{(Eq 55)}$$

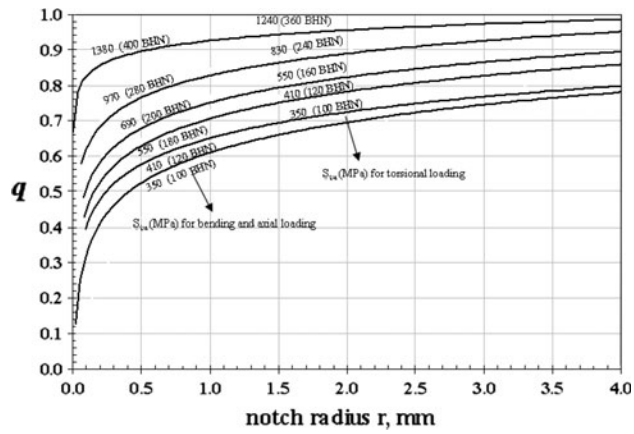



**Fig. 12** Peterson's notch sensitivity curves for steels [16]

where $q$ is a function of a notch root radius and the ultimate tensile strength of a material. Also, the $K_t/K_f$ ratio or the supporting factor ($n_K$) was developed

$$n_K = \frac{K_t}{K_f}, \qquad \text{(Eq 56)}$$

where $n_K$ depends either on a relative stress gradient and tensile yield strength [28] or on a notch root radius and ultimate tensile strength [11].

The three approaches will be discussed in the following sections, among which the one based on FKM-Guideline is recommended by the authors.

### *Notch Sensitivity Factor*

Based on Eq 55, the formula for $K_f$ can be written as follows:

$$K_f = 1 + (K_t - 1)q. \qquad \text{(Eq 57)}$$

When $q = 1$ or $K_t = K_f$, the material is considered to be fully notch sensitive. On the other hand, when $q = 0$ and $K_f = 1.0$, the material is considered not to be notch sensitive (the so-called notch blunting effect).

Peterson [26] assumed fatigue damage occurs when the stress at a critical distance ($a_P$) away from the notch root is equal to the fatigue strength of a smooth component. Based on the assumption that the stress near a notch reduces linearly, Peterson obtained the following empirical equation for $q$:

$$q = \frac{1}{1 + \frac{a_P}{r}}, \qquad \text{(Eq 58)}$$

where $r$ is the notch root radius and $a_P$ is Peterson's material constant related to the grain size (or $S_{t,u}$) and the loading mode. A plot by Peterson is provided in Fig. 12 to determine the notch sensitivity factor for high and mild strength steels.

Furthermore, Neuber [23] postulated that fatigue failure occurs if the average stress over a length from the notch root equals to the fatigue strength of a smooth component, and proposed the following empirical equation for $q$:

$$q = \frac{1}{1 + \sqrt{\frac{a_N}{r}}}, \quad \text{(Eq 59)}$$

where $a_N$ is Neuber's material constant related to the grain size or the ultimate tensile strength.

*Relative Stress Gradient*

Siebel and Stieler [28] introduced a new parameter ($\bar{G}$) with units of $mm^{-1}$, termed as the relative stress gradient, which is defined as follows:

$$\bar{G} = \frac{(G)_{x=0}}{\sigma^e_{max}} = \frac{1}{\sigma^e_{max}} \left( \frac{d\sigma^e(x)}{dx} \right)_{x=0}, \quad \text{(Eq 60)}$$

where $x$ is the distance from the notch root, $G$ is the stress gradient along $x$, $\sigma^e(x)$ is the calculated pseudo-stress distribution along $x$; $\sigma^e_{max}$ is the maximum pseudo-stress at $x = 0$, as illustrated in Fig. 13. By testing many smooth and notched components to the endurance cycle limit of $2 \times 10^7$ cycles, they generated a series of empirical curves relating the $K_t/K_f$ ratios to $\bar{G}$ values for various materials in terms of tensile yield strength ($S_{t,y}$ in MPa). These empirical curves, as illustrated in Fig. 14, can be expressed by the following generic formula:

$$n_K = \frac{K_t}{K_f} = 1 + \sqrt{a_{SS} \cdot \bar{G}}, \quad \text{(Eq 61)}$$

where $a_{SS}$ is the Siebel and Stieler material parameter.

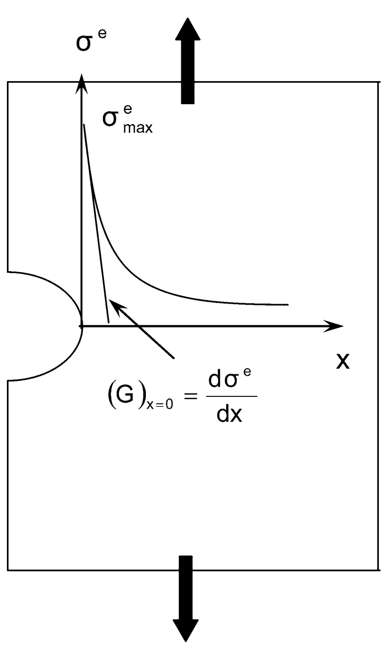



**Fig. 13** Pseudo-stress distribution and the stress gradient at a notch root [16]

*FKM-Guideline*

The fatigue notch factors for a notched shaft under axial, bending, shear, and torsional stress ($K_{ax,f}$, $K_{b,f}$, $K_{s,f}$, $K_{t,f}$) can be calculated from the corresponding elastic stress concentration factors ($K_{ax,t}$, $K_{b,t}$, $K_{s,t}$, $K_{t,t}$) and the $K_t/K_f$ ratios or the supporting factors ($n_{K,\sigma}(r)$, $n_{K,\sigma}(d)$, $n_{K,\tau}(r)$, $n_{K,\tau}(d)$) as follows:

$$K_{ax,f} = \frac{K_{ax,t}}{n_{K,\sigma}(r)} \quad \text{(Eq 62)}$$

$$K_{b,f} = \frac{K_{b,t}}{n_{K,\sigma}(r) \cdot n_{K,\sigma}(d)} \quad \text{(Eq 63)}$$

$$K_{s,f} = \frac{K_{s,t}}{n_{K,\tau}(r)} \quad \text{(Eq 64)}$$

$$K_{t,f} = \frac{K_{t,t}}{n_{K,\tau}(r) \cdot n_{K,\tau}(d)}, \quad \text{(Eq 65)}$$

where $r$ is the notch radius and $d$ is the net diameter or net width of a notched section. Similarly, the fatigue notch factors for a notched shell-shaped component under normal stresses in $x$ and $y$ directions and shear stress ($K_{x,f}$, $K_{y,f}$, $K_{\tau_{xy},f}$) can be calculated from the corresponding elastic stress concentration factors ($K_{x,t}$ $K_{y,t}$, $K_{\tau_{xy},t}$) and the $K_t/K_f$ ratios ($n_{K,\sigma,x}(r)$, $n_{K,\sigma,y}(r)$, $n_{K,\tau}(r)$) as follows:

$$K_{x,f} = \frac{K_{x,t}}{n_{K,\sigma,x}(r)} \quad \text{(Eq 66)}$$

$$K_{y,f} = \frac{K_{y,t}}{n_{K,\sigma,y}(r)} \quad \text{(Eq 67)}$$

$$K_{\tau_{xy},f} = \frac{K_{\tau_{xy},t}}{n_{K,\tau}(r)}. \quad \text{(Eq 68)}$$

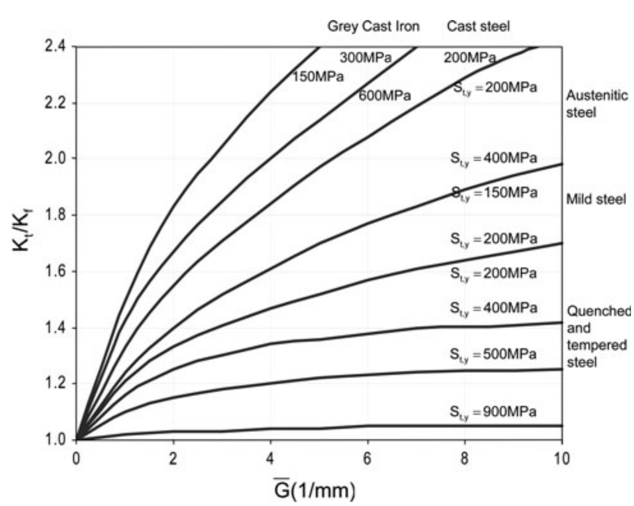



**Fig. 14** Relative stress gradient effect on $K_t$–$K_f$ ratios for various materials in terms of tensile yield strength [16]

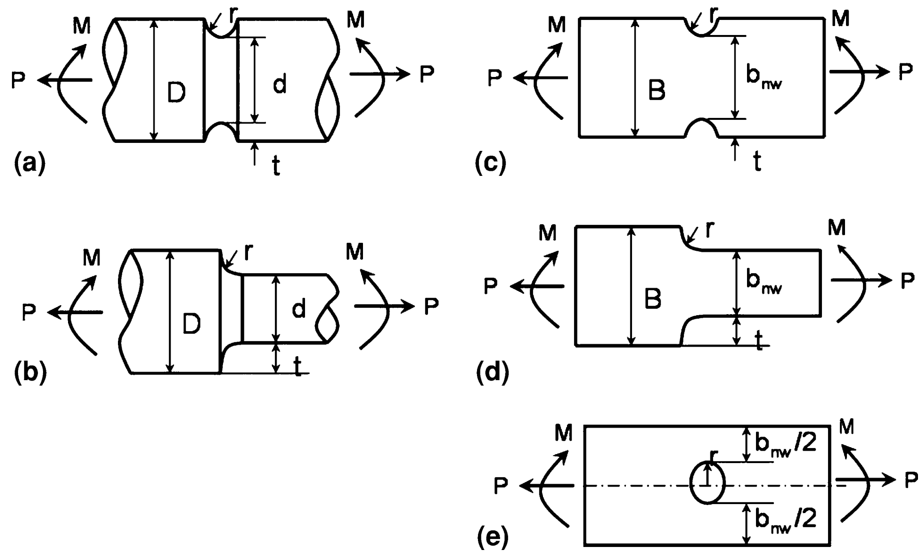



**Fig. 15** Definition of notched components in FKM-Guideline [16]

The $K_t/K_f$ ratios ($n_{K,\sigma}(r)$ and $n_{K,\sigma}(d)$) for normal stress are calculated from the relative normal stress gradients $\bar{G}_\sigma(r)$ and $\bar{G}_\sigma(d)$

$$n_{K,\sigma} = 1 + \bar{G}_\sigma \times 10^{-\left(a_G - 0.5 + S_{t,u}/b_G\right)} \quad \text{for } \bar{G}_\sigma \le 0.1\ \text{mm}^{-1} \qquad \text{(Eq 69)}$$

$$n_{K,\sigma} = 1 + \sqrt{\bar{G}_\sigma} \times 10^{-\left(a_G + S_{t,u}/b_G\right)} \quad \text{for } 0.1\ \text{mm}^{-1} < \bar{G}_\sigma \le 1\ \text{mm}^{-1} \qquad \text{(Eq 70)}$$

$$n_{K,\sigma} = 1 + \sqrt[4]{\bar{G}_\sigma} \times 10^{-\left(a_G + S_{t,u}/b_G\right)} \quad \text{for } 1\ \text{mm}^{-1} < \bar{G}_\sigma \le 100\ \text{mm}^{-1}. \qquad \text{(Eq 71)}$$

Likewise, the $K_t/K_f$ ratios for shear stress are calculated from the relative shear stress gradients $\bar{G}_\tau(r)$and $\bar{G}_\tau(d)$

$$n_{K,\tau} = 1 + \bar{G}_\tau \times 10^{-\left(a_G - 0.5 + C_\tau S_{t,u}/b_G\right)} \quad \text{for } \bar{G}_\tau \le 0.1\ \text{mm}^{-1} \qquad \text{(Eq 72)}$$

$$n_{K,\tau} = 1 + \sqrt{\bar{G}_\tau} \times 10^{-\left(a_G + C_\tau S_{t,u}/b_G\right)} \quad \text{for } 0.1\ \text{mm}^{-1} < \bar{G}_\tau \le 1\ \text{mm}^{-1} \qquad \text{(Eq 73)}$$

$$n_{K,\tau} = 1 + \sqrt[4]{\bar{G}_\tau} \times 10^{-\left(a_G + C_\tau S_{t,u}/b_G\right)} \quad \text{for } 1\ \text{mm}^{-1} < \bar{G}_\tau \le 100\ \text{mm}^{-1}, \qquad \text{(Eq 74)}$$

where $S_{t,u}$ is the ultimate tensile strength with a 97.5% reliability in units of MPa, $C_\tau$ is the shear stress correction factor, $a_G$ and $b_G$ are the material constants listed in Table 10.

The relative stress gradient of a notched component under a specific loading mode really depends on the diameter or net width of the component and its notch radius. The relative stress gradients for bending and torsion as a function of the net diameter ($d$) of a notched shaft or net width ($b_{nw}$) of a notched plate can be obtained

$$\bar{G}_{K,\sigma}(d) = \bar{G}_{K,\tau}(d) = \frac{2}{d} \qquad \text{(Eq 75)}$$

$$\bar{G}_{K,\sigma}(b_{nw}) = \bar{G}_{K,\tau}(b_{nw}) = \frac{2}{b_{nw}}. \qquad \text{(Eq 76)}$$

Also, the relative stress gradients ($\bar{G}_{K,\sigma}(r)$ and $\bar{G}_{K,\tau}(r)$) can be found in Table 11 for various notched geometries.

**Table 10** $a_G$ and $b_G$ for various materials (adopted from FKM-Guideline)

| Materials | $a_G$ | $b_G$, MPa |
|---|---|---|
| Stainless steel | 0.40 | 2400 |
| Steels except for stainless steel | 0.50 | 2700 |
| Steel castings | 0.25 | 2000 |
| Ductile irons | 0.05 | 3200 |
| Malleable cast iron | −0.05 | 3200 |
| Gray cast iron | −0.05 | 3200 |
| Wrought aluminum alloys | 0.05 | 850 |
| Cast aluminum alloys | −0.05 | 3200 |

**Table 11** Relative stress gradients $\bar{G}_{K,\sigma}(r)$ and $\bar{G}_{K,\tau}(r)$ for various notched geometries (adopted from FKM-Guideline)

| Notched components | $\bar{G}_{K,\sigma}(r)$, mm$^{-1}$ | $\bar{G}_{K,\tau}(r)$, mm$^{-1}$ |
|---|---|---|
| A groove shaft in Fig. 15(a) | $\frac{2}{r}(1+\varphi)$ | $\frac{1}{r}$ |
| A shoulder shaft in Fig. 15(b) | $\frac{2.3}{r}(1+\varphi)$ | $\frac{1.15}{r}$ |
| A groove plate in Fig. 15(c) | $\frac{2}{r}(1+\varphi)$ | … |
| A shoulder plate in Fig. 15(d) | $\frac{2.3}{r}(1+\varphi)$ | … |
| A central hole plate in Fig. 15(e) | $\frac{2.3}{r}$ | … |

Note that (1) $\varphi = 1/\left(4\sqrt{t/r}+2\right)$ for $t/d \le 0.25$ or $t/b_{nw} \le 0.25$;; (2) $\varphi = 0$ for $t/d > 0.25$ or $t/b_{nw} > 0.25$

To calculate the fatigue notch factor the following steps should be followed. First, calculate the elastic stress concentration factor. Second, determine the relative stress gradient. If the component has a complicated geometry, FEA software is needed to determine the relative stress gradient. For well-known geometries, the equations listed in Table 11 are used to calculate the relative stress gradient based on the notch radius. If the component (having a well-known geometry listed in Table 11) is subjected to bending or torsion loading, the relative stress gradient based on net diameter or width also needs to be calculated using Eq 75 or 76. Third, calculate the supporting factor using the appropriate equation based on the relative stress gradient. Note, if a stress gradient based on the net diameter or width was calculated, then a supporting factor will also need to be calculated based on the net diameter or width. Finally, the relationship between the supporting factor and the elastic stress concentration factor is used to determine the fatigue notch factor (see section 7 for examples).

FKM-Guideline also specifies that the resulting fatigue notch factors for superimposed notches (e.g., $K_{1,\mathrm{f}}$ and $K_{2,\mathrm{f}}$) can be estimated as

$$K_\mathrm{f} = 1 + \left(K_{1,\mathrm{f}} - 1\right) + \left(K_{2,\mathrm{f}} - 1\right). \qquad \text{(Eq 77)}$$

Superposition does not need to be considered if the distance between notches is equal to $2r_{\max}$ or above, where $r_{\max}$ is the larger one of both notch radii.

## Constant Amplitude Stress-Life Curve for a Notched Component Under Fully Reversed Loading

Based on the definition of stress (namely nominal stress or pseudo-stress), the procedure to generate a constant amplitude stress-life curve for a notched component under fully reversed loading is discussed.

### Constant Amplitude Nominal Stress-Life Curve

This section presents the FKM method to construct the synthetic *S–N* curve for a notched component, based on a reference point and a specified slope factor ($k$). The endurance limit ($S_\mathrm{E}$) at an endurance cycle limit $N_\mathrm{E} = 10^6$ cycles) is the reference point. Thus, the *S–N* equation can be obtained as follows:

$$NS^k = N_\mathrm{E} S_\mathrm{E}^k = \text{Constant}. \qquad \text{(Eq 78)}$$

The specified slope factor ($k$) depends on the type of material and stress. Below are the specifications from FKM-Guideline:

- For surface non-hardened components made of steels and cast irons, except austenitic steel, the component constant amplitude *S–N* curves based on normal stress and shear stress are illustrated in Fig. 16(a) and (b), respectively. The endurance limit value ($S_{\sigma,\mathrm{E}}$ or $S_{\tau,\mathrm{E}}$) at $10^6$ cycles is the fatigue limit at which the fatigue initiation life becomes infinite or when fatigue initiation failure does not occur. The specific slope factors for normal stress and shear stress ($k_{1,\sigma}$ and $k_{1,\tau}$) are defined as 5 and 8, respectively.
- For surface hardened components made of steels and cast irons, the component constant amplitude *S–N* curves for both normal and shear stresses have larger slope factors than the non-hardened components by a factor close to 3.0. As shown in Fig. 17, the specific slope factors for normal stress and shear stress ($k_{1,\sigma}$ and $k_{1,\tau}$) are defined as 15 and 25, respectively.
- For components made of aluminum alloys and austenitic steel, the component constant amplitude *S–N* curves based on normal stress and shear stress are illustrated in Fig. 18(a) and (b), respectively. The stress amplitude at $10^8$ cycles is defined as the fatigue limit ($S_{\sigma,\mathrm{FL}}$ or $S_{\tau,\mathrm{FL}}$). The *S–N* curve between $10^6$ and $10^8$ cycles are defined by the same reference point, but with a different slope factor such as 15 for normal stress or 25 for shear stress. The fatigue limit can be calculated using Eq 78.

### Constant Amplitude Pseudo-stress-Life Curve

The synthetic constant amplitude pseudo-stress-life curve for a notched component is preferable if the local stress is determined by a linear elastic finite element analysis. This section presents the method to convert the above nominal stress-life curve to the local pseudo-stress-life curve.

The following equations are valid:

$$\sigma_\mathrm{E}^\mathrm{e} = K_\mathrm{t} S_\mathrm{S,E,Notched} \qquad \text{(Eq 79)}$$

$$K_\mathrm{f} = \frac{S_\mathrm{S,E,Smooth}}{S_\mathrm{S,E,Notched}} \qquad \text{(Eq 80)}$$

$$n_K = \frac{K_\mathrm{t}}{K_\mathrm{f}}, \qquad \text{(Eq 81)}$$

where $\sigma_\mathrm{E}^\mathrm{e}$ is the pseudo-endurance limit, $n_K$ is the $K_\mathrm{t}/K_\mathrm{f}$ ratio or the supporting factor defined in section 3, $S_\mathrm{S,E,Notched}$ is the endurance limit of a notched component, calculated by the equations in section 3, $S_\mathrm{S,E,Smooth}$ is the endurance limit of a smooth component, calculated by the equations in section 3 with $K_\mathrm{f} = 1$.

The above equations lead to a new reference stress point at $10^6$ cycles as

$$\sigma_\mathrm{E}^\mathrm{e} = n_K S_\mathrm{S,E,Smooth}. \qquad \text{(Eq 82)}$$

With the assumption of the identical slope factor specified by FKM-Guideline, the synthetic constant amplitude pseudo-stress-life curve can then be

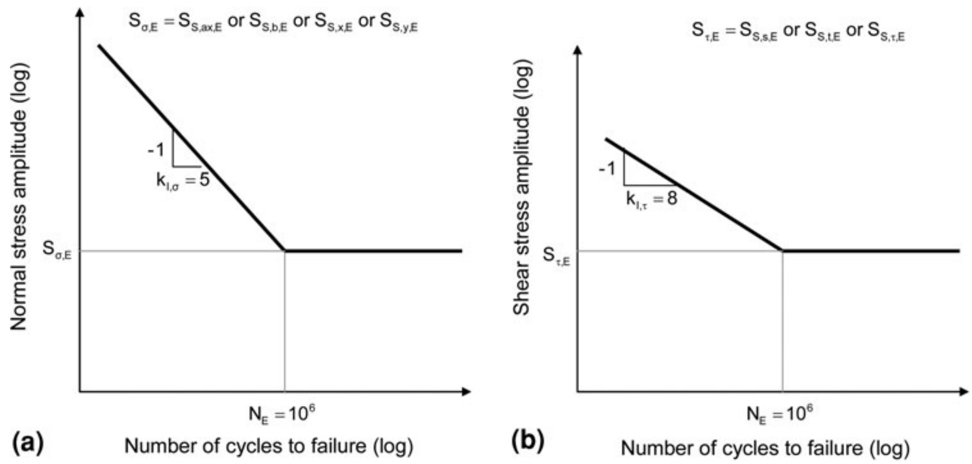



**Fig. 16** Synthetic component constant amplitude *S–N* curve for surface non-hardened components made of steels and cast irons [16]

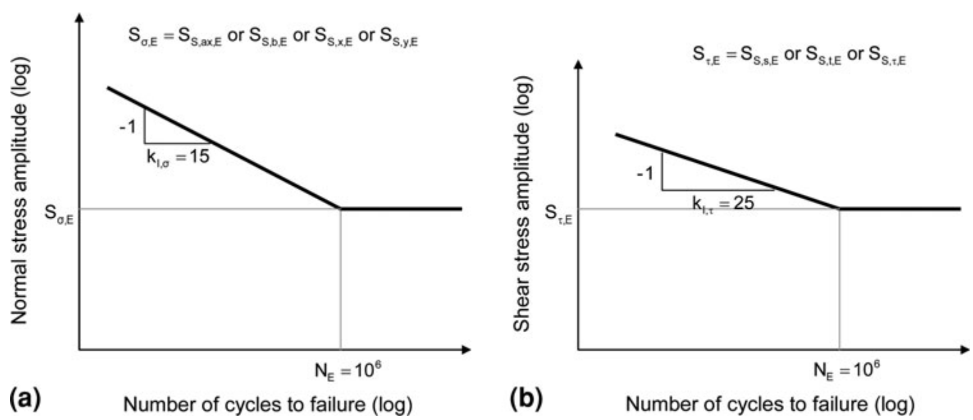



**Fig. 17** Synthetic component constant amplitude *S–N* curve for surface hardened components made of steels and cast irons [16]

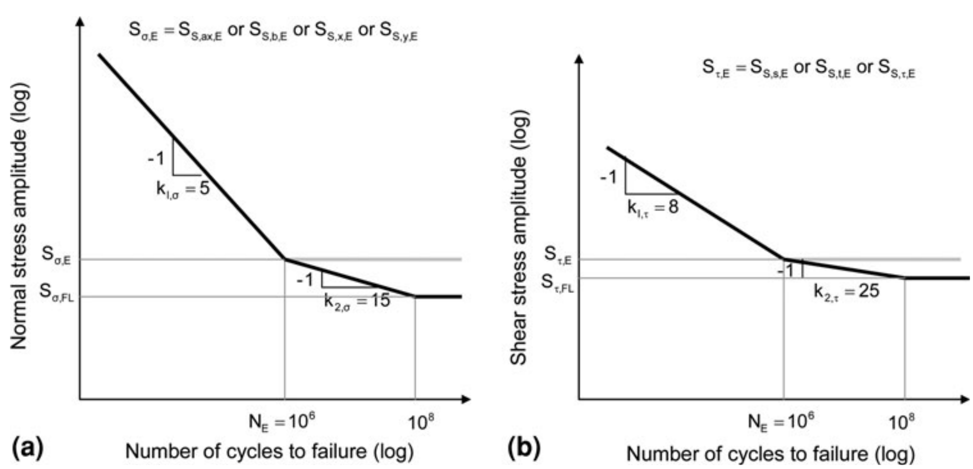



**Fig. 18** Synthetic component constant amplitude *S–N* curve for components made of aluminum alloys and austenitic steel [16]

determined. Figure 19 shows the concept of defining the pseudo-endurance limit with respect to the endurance limit of a smooth component as well as the comparison between the constant amplitude *S–N* and $\sigma_E^e - N$ curves for a notched component made of steels.

## Stress-Life Curve for a Component Under Variable Amplitude Loading

For a component subjected to variable amplitude loading over time, a rainflow cycle counting technique is typically

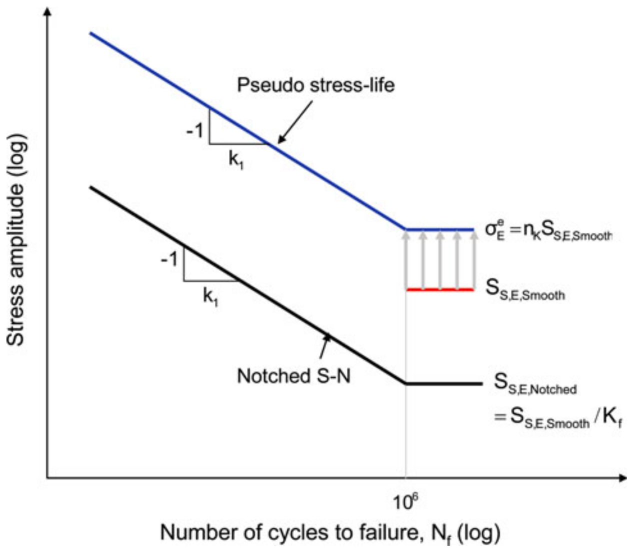



**Fig. 19** Synthetic constant amplitude $S$–$N$ and $\sigma^{e}$–$N$ curves for a notched component made of steel [16]

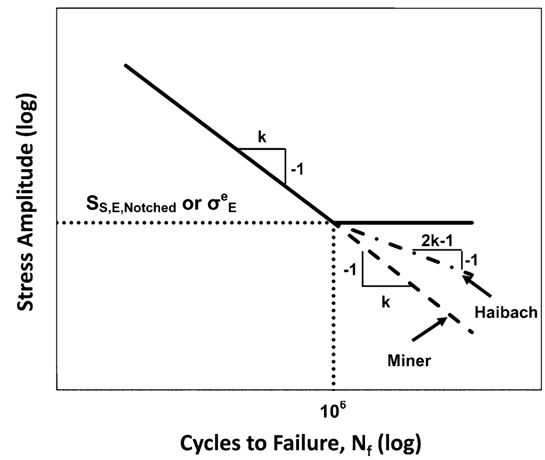



**Fig. 20** Constant amplitude $S$–$N$ curve for a component made of steels and subjected to variable amplitude loading [16]

used to convert a complicated time-varying stress history to a series of discrete simple constant amplitude stress events which consist of a mean stress level and a number of stress cycles ($n_{\mathrm{i}}$). The fatigue life ($N_{\mathrm{f,i}}$) corresponding to the number of cycles to failure at the specific stress event can be estimated from the component constant amplitude $S$–$N$ curve. In this case, the fatigue damage is defined as the cycle ratio ($=n_{\mathrm{i}}/N_{\mathrm{f,i}}$). The Palmgren–Miner [20, 25] linear damage rule is adopted to calculate the accumulated damage, which assumes fatigue damage occurs when the sum of the cycle ratios at each constant amplitude stress event reaches a critical damage value ($D_{\mathrm{PM}}$). Fatigue failure can be represented mathematically as

$$\sum \frac{n_{\mathrm{i}}}{N_{\mathrm{f,i}}} \geq D_{\mathrm{PM}}. \qquad \text{(Eq 83)}$$

Palmgren and Miner found the critical damage value of 1.0 in their studies. But since their work was conducted, it has been shown [13, 15, 37] that the critical damage value is a random variable varying from 0.15 to 1.06. For mechanical designs, FKM-Guideline [11] recommends $D_{\mathrm{PM}} = 0.3$ for steel, steel castings, and aluminum alloys, while $D_{\mathrm{PM}} = 1.0$ for ductile irons, gray cast irons, and malleable cast irons. For electronic equipment design, Steinberg [32] suggests $D_{\mathrm{PM}} = 0.7$.

The component constant amplitude $S$–$N$ curve is supposed to be used for estimating the fatigue life of a component at a given constant amplitude stress event. But when the Palmgren–Miner linear damage rule is applied to a component in variable amplitude loading, the stress cycles with amplitudes below the fatigue limit could become damaging if some of the subsequent stress amplitudes exceed the original fatigue limit. It is believed that the increase in crack driving force due to the periodic overloads will overcome the original grain barrier strength and help the crack to propagate until failure. Therefore, there is a need to modify the fatigue limit for a component subjected to variable amplitude-loading history because the fatigue limit obtained from constant amplitude loading might be negatively influenced by periodic overloads. Two methods such as the Miner rule and the Miner–Haibach model [10], as shown in Fig. 20, were proposed to include the effect of periodic overloads on the stress cycle behavior below the original fatigue limit. The Miner rule extends the $S$–$N$ curve with the same slope factor k to approach zero stress amplitude, while the Miner–Haibach model extends the original $S$–$N$ curve below the fatigue limit to the zero stress amplitude with a flatter slope factor $2k - 1$. Stanzl et al. [31] concluded that a good agreement is found for measured and calculated results according to the Miner–Haibach model.

## Mean Stress Effect

From the perspective of applied cyclic stresses, the fatigue damage of a component strongly correlates with the applied stress amplitude or applied stress range, and is secondarily influenced by the mean stress. The mean stress effect should be seriously considered in fatigue analyses. In the HCF regime, normal mean stresses have a significant effect on fatigue behavior of components. As the opening of microcracks accelerates the rate of crack propagation and the closing of microcracks retards the growth of cracks, tensile mean normal stresses are detrimental and compressive mean normal stresses are beneficial in terms of fatigue strength. There is very little or no effect of mean stress on fatigue strength in the LCF regime where the large amount of plastic deformation significantly reduces any beneficial or detrimental effect of the mean stress.

The mean normal stress effect can be represented by the mean stress ($S_{\sigma,\mathrm{m}}$) or the stress ratio ($R$). Both are defined as follows:

$$S_{\sigma,\mathrm{m}} = \frac{\left(S_{\sigma,\max} + S_{\sigma,\min}\right)}{2} \qquad \text{(Eq 84)}$$

$$R = \frac{S_{\sigma,\min}}{S_{\sigma,\max}}, \qquad \text{(Eq 85)}$$

where $S_{\sigma,\max}$ and $S_{\sigma,\min}$ are the maximum and minimum normal stresses in a stress cycle, respectively. For example, it can be found that $S_{\sigma,\mathrm{m}} = 0$ and $R = -1$ under fully reversed stress conditions.

The early models to account for the mean stress effect, such as Gerber [8], Goodman [9], Haigh [12], Soderberg [30], and Morrow [22], were usually plotted against empirical data in constant life plots of stress amplitude ($S_{\sigma,\mathrm{a}}$) versus mean stress ($S_{\sigma,\mathrm{m}}$). In German, these constant life plots are termed as Haigh's diagram, while commonly referred to as Goodman's diagram in North America. As schematically illustrated in Fig. 21, Haigh's diagram can be determined from a family of constant amplitude $S_{\sigma,\mathrm{a}}$–$N$ curves (Wohler curves) with various mean stress values ($0,\ S_{\sigma,\mathrm{m1}},\ S_{\sigma,\mathrm{m2}},$ and $S_{\sigma,\mathrm{m3}}$). The equivalent fully reversed stress amplitude ($S_{\sigma,\mathrm{ar}}$) is the generic interception point of the $S_{\sigma,\mathrm{a}}$ axis, which is used to determine the fatigue life ($N_\mathrm{i}$) from a corresponding component $S$–$N$ curve. According to Goodman's and Morrow's models as illustrated in Fig. 21(c), the ultimate tensile strength ($S_{\mathrm{t,u}}$) and the fatigue strength coefficient ($S'_{\sigma,\mathrm{f}}$) are the physical limits to $S_{\sigma,\mathrm{m}}$ and the interception of the $S_{\sigma,\mathrm{m}}$ axis, respectively. Alternatively, Haibach in FKM-Guideline introduces the mean stress sensitivity factor ($M_\sigma$) to define the Haigh diagram, which is the absolute value of the slope of the constant life plot. The mean stress sensitivity factor depends on the type of material and loading condition.

Even though there are numerous models developed to account for the mean stress effect on fatigue strength and lives, four commonly used formulas are chosen for discussion: Goodman's, Morrow's [22], Smith–Watson–Topper's (SWT in brevity) [29], and Walker's [35]. The differences among the four models can be observed from the following expressions of the equivalent fully reserved stress amplitude in the case of moderate mean stress values

1. Goodman's

$$S_{\sigma,\mathrm{ar}} = \frac{S_{\sigma,\mathrm{a}}}{1 - \frac{S_{\sigma,\mathrm{m}}}{S_{\mathrm{t,u}}}}. \qquad \text{(Eq 86)}$$

2. Morrow's

$$S_{\sigma,\mathrm{ar}} = \frac{S_{\sigma,\mathrm{a}}}{1 - \frac{S_{\sigma,\mathrm{m}}}{S'_{\sigma,\mathrm{f}}}}. \qquad \text{(Eq 87)}$$

3. SWT's

$$S_{\sigma,\mathrm{ar}} = \sqrt{S_{\sigma,\max} S_{\sigma,\mathrm{a}}} = \sqrt{\left(S_{\sigma,\mathrm{a}} + S_{\sigma,\mathrm{m}}\right) S_{\sigma,\mathrm{a}}}. \qquad \text{(Eq 88)}$$

4. Walker's

$$S_{\sigma,\mathrm{ar}} = S_{\sigma,\max}^{1-\gamma_\mathrm{W}} S_{\sigma,\mathrm{a}}^{\gamma_\mathrm{W}} = \left(S_{\sigma,\mathrm{a}} + S_{\sigma,\mathrm{m}}\right)^{1-\gamma_\mathrm{W}} S_{\sigma,\mathrm{a}}^{\gamma_\mathrm{W}}, \qquad \text{(Eq 89)}$$

where $\gamma_\mathrm{W}$ is a mean stress fitting parameter. The SWT and the Walker equations predict that fatigue crack will not initiate if the maximum normal stress in a cycle is less than or equal to zero, meaning $S_{\sigma,\max} \leq 0$.

The following conclusions are extracted from the extensive studies by Dowling et al. [7] on experimental fatigue data for steels, aluminum alloys, and a titanium

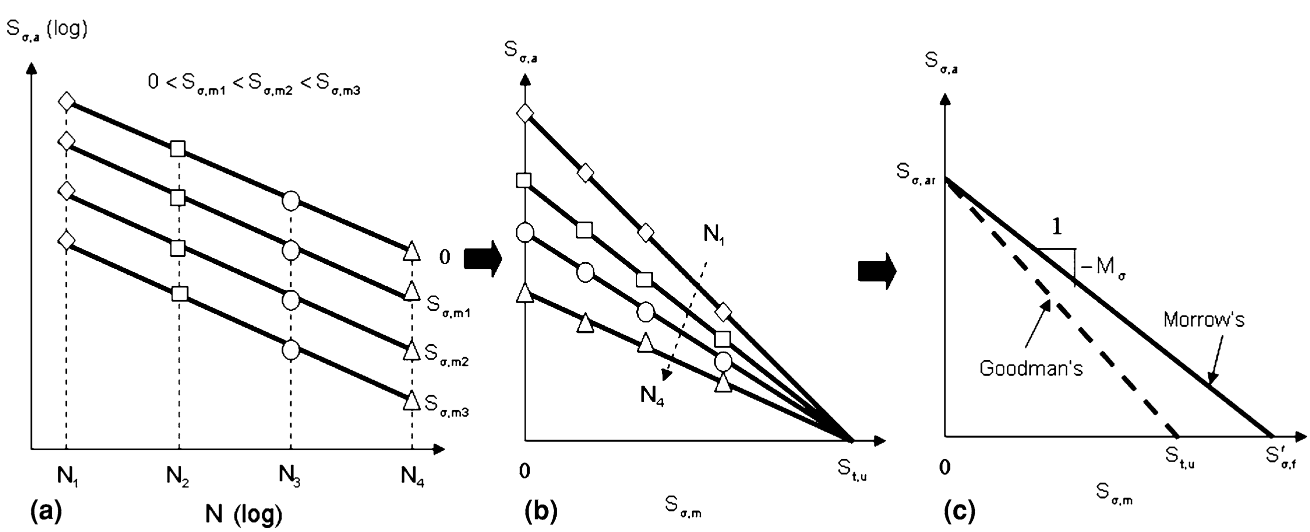



**Fig. 21** Construction of constant life plots on normal stress in $S_{\sigma,\mathrm{a}}$ and $S_{\sigma,\mathrm{m}}$ coordinates [16]

alloy where the $R$ ratio ranges from $-2$ to 0.45. Goodman's model for life predictions is highly inaccurate and should not be used. Walker's model gives superior results if an additional mean stress fitting parameter ($\gamma_W$) is provided. Otherwise, both Morrow's and SWT's models yield reasonable life estimates for steels. For aluminum alloys, the SWT model is recommended. Dowling et al. concluded that the SWT method provides good results in most cases and is a good choice for general use.

If there are no experimental data available for materials or the R ratio is beyond the range of the previous studies, FKM-Guideline [11] is recommended for use. According to FKM-Guideline, Haigh's diagram based on a normal stress can be classified as four regimes as shown in Fig. 22 and summarized below:

- Regime I is for the stress ratio $R > 1$ where the maximum and minimum stresses are under compression.
- Regime II is applicable to the case of $-\infty \le R \le 0$ where $R = -\infty$ is when the max stress is equal to 0; $R = -1$, the fully reversed stress; $R = 0$, the alternating tension stress with a zero minimum stress.
- Regime III is for $0 < R < 0.5$ where the maximum and minimum stresses are under tension.
- Regime IV is for $R \ge 0.5$, the regime of high alternating tension stress.

FKM-Guideline specifies various mean stress sensitivity factors due to normal stress for all four regimes and any material. The fully reversed fatigue strength $S_{\sigma,\text{ar}}$ in the classified regimes can be written as follows:

- Regimes I and IV

$$S_{\sigma,\text{ar}} = S_{\sigma,\text{a}}. \quad \text{(Eq 90)}$$

- Regime II

$$S_{\sigma,\text{ar}} = S_{\sigma,\text{a}} + M_\sigma S_{\sigma,\text{m}}. \quad \text{(Eq 91)}$$

- Regime III

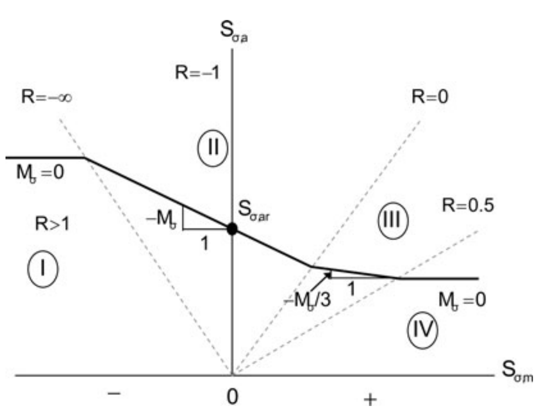



**Fig. 22** Haigh's diagram for mean stress effect on normal stress [16]

$$S_{\sigma,\text{ar}} = (1 + M_\sigma)\frac{S_{\sigma,\text{a}} + (M_\sigma/3)S_{\sigma,\text{m}}}{1 + M_\sigma/3}. \quad \text{(Eq 92)}$$

Note Eq 90 implies that the alternating stress has a greater influence on fatigue behavior than mean stress in regions 1 and 4 where mean stresses are very high.

Haigh's diagram based on shear stress can be classified as three regimes as shown in Fig. 23 because the negative mean stress in shear is always regarded to be positive and treated the same as the positive mean stress. For the case of ambient or elevated temperatures, the mean stress sensitivity factors for normal and shear stresses can be obtained as follows:

$$M_\sigma = a_\text{M} S_{\text{t,u}} + b_\text{M} \quad \text{(Eq 93)}$$

$$M_\tau = C_\tau M_\sigma, \quad \text{(Eq 94)}$$

where $a_\text{M}$ and $b_\text{M}$ are the material parameters listed in Table 12, $S_{\text{t,u}}$ and $C_\tau$ are the ultimate tensile strength in MPa and the shear stress correction factor, respectively.

For mechanical designs, Wilson and Haigh [36] introduced the line of constant tensile yield strength as an additional design constraint for materials, termed as the safe design region for the consideration of both the fatigue strength and the tensile yield strength. The line of constant tensile yield strength was constructed by the line connecting the two tensile yield strength values in $S_{\sigma,\text{a}}$ and $S_{\sigma,\text{m}}$ axes. Any combination of

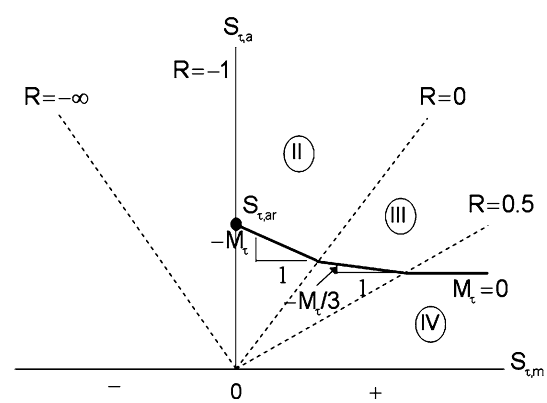



**Fig. 23** Haigh's diagram for mean stress effect on shear stress [16]

**Table 12** $a_\text{M}$ and $b_\text{M}$ for various materials (adopted from FKM-Guideline)

| Materials | $a_\text{M}$ | $b_\text{M}$ |
|---|---|---|
| Steel | 0.00035 | −0.1 |
| Steel casting | 0.00035 | 0.05 |
| Ductile irons | 0.00035 | 0.08 |
| Malleable cast iron | 0.00035 | 0.13 |
| Gray cast iron | 0 | 0.5 |
| Wrought aluminum alloys | 0.001 | −0.04 |
| Cast aluminum alloys | 0.001 | 0.2 |

$S_{\sigma,a}$ and $S_{\sigma,m}$ falling inside of the enclosed area is considered a safe design which will meet both the yield strength and the fatigue strength criteria. Figure 24 shows the isothermal temperature effect on the safe design regions for the fatigue strength and tensile yield strength at both ambient and elevated temperatures. The temperature correction factors for tensile yield strength and fatigue strength follow the guidelines in sections 2 and 3, respectively.

## Examples and Case Studies

This section contains example problems as well as a few case studies. Two sets of solutions are provided with each example, one set using the methods described in the FKM-Guideline, and the other using common North American analysis techniques.

Example 1 [15]

A grooved shaft made of high-grade alloy steel is heat-treated to 405 HB and finished with a careful grinding operation. The geometry of the shaft is shown in Fig. 25, with the dimensions of $D = 28.0$ mm, $d = 25.5$ mm, and

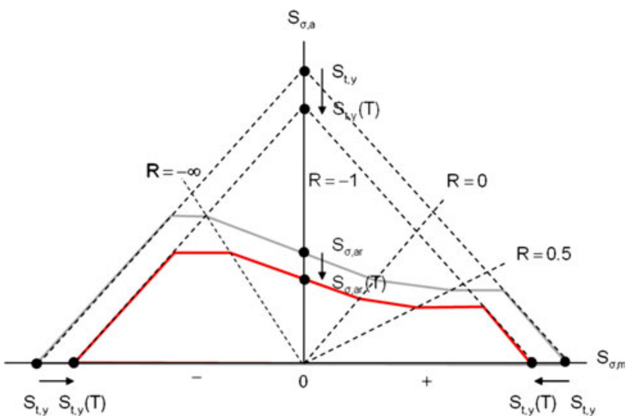



**Fig. 24** Safe design regions for fatigue strength at both ambient and elevated temperatures [16]

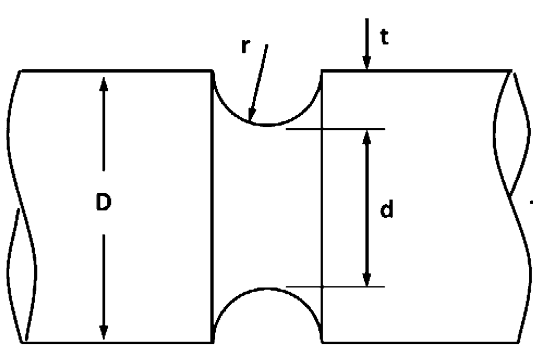



**Fig. 25** Geometry of grooved shaft in Example 1

the notch radius of $r = 1.28$ mm. The elastic stress concentration factors due to bending, axial, and torsional loading are calculated as $K_{t,bending} = 2.26$, $K_{t,axial} = 2.50$, and $K_{t,torsion} = 1.64$. Estimate the endurance limit (at $10^6$ cycles) of the notched shaft with a reliability of 97.5% for the following loading conditions:

- Fully reversed bending loading.
- Fully reversed axial loading.
- Fully reversed torsional loading.

Solution (North American)

Using the North American analysis techniques, the endurance limit can be estimated by the following equation:

$$S_e = \frac{S_{b,E} K_L K_s K_d C_R}{K_f},$$

where $S_{b,E}$ is the material fatigue strength, $K_L$ is the loading factor, $K_s$ is the surface effect factor (effect of roughness and surface treatment), $K_d$ is the size effect factor, and $C_R$ is the reliability factor. The parameter $S_{b,E}$ is defined as follows:

$$S_{b,E} = 0.5 S_{t,u,std} \quad \text{for } S_{t,u,std} < 1400\,\text{MPa}$$

$$S_{b,E} = 700\,\text{MPa} \quad \text{for } S_{t,u,std} \geq 1400\,\text{MPa},$$

where $S_{t,u,std}$ is in units of MPa. Therefore, for component with a Brinell hardness of 405, $S_{b,E} = 700$ MPa. For a reliability of 97.5%, the reliability correction factor is as follows:

$$C_R = 1 - \left|\Phi^{-1}(1 - R_r)\right| \text{COV}_S = 0.843.$$

The correction factor for surface effects resulting from a careful grinding procedure is as follows:

$$K_S = 1.58 (S_u)^{-0.085} = 0.85.$$

It should be noted that the surface effect factor ($K_S$) is based on data generated over 70 years ago [24]. The values for $S_{b,E}$, $C_R$, and $K_s$ will apply for all three loading conditions.

For *axial loading*, the factors for loading ($K_L$) and size effect ($K_d$) are constant and defined as follows:

$$K_L = 0.85$$

$$K_d = 1.$$

Peterson's formula for fatigue strength reduction at the endurance limit is used to calculate the fatigue notch factor as follows:

$$a = 0.0254 \left(\frac{2079}{S_{t,u,std}}\right)^{1.8} = 0.0254 \left(\frac{2079}{1400}\right)^{1.8} = 0.0518\,\text{mm}$$

$$q = \frac{1}{1 + \frac{a}{r}} = \frac{1}{1 + \frac{0.0518}{1.28}} = 0.961$$

$$K_f = 1 + \left(K_{t,axial} - 1\right) q = 1 + (2.50 - 1)0.961 = 2.45.$$

Therefore, the endurance limit at $10^6$ cycles for this component under axial loading is as follows:

$$S_{\mathrm{e,axial}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}} = \frac{700(0.85)(0.85)(1)(0.843)}{2.45} = 174\,\mathrm{MPa}.$$

For *bending loading*, the factor for loading is constant and is defined as follows:

$$K_{\mathrm{L}} = 1.0.$$

The size effect factor is given by the following equation:

$$K_{\mathrm{d}} = 1.189(d)^{-0.097} = 1.189(25.5)^{-0.097} = 0.868.$$

Similar to axial loading Peterson's formula for fatigue strength reduction at the endurance limit is used to calculate the fatigue notch factor for bending. Note that the fatigue notch factor is influenced by loading condition as well as other variables. The fatigue notch factor for bending is defined below

$$a = 0.0254\left(\frac{2079}{S_{\mathrm{t,u,std}}}\right)^{1.8} = 0.0254\left(\frac{2079}{1400}\right)^{1.8} = 0.0518$$
$$q = \frac{1}{1+\frac{a}{r}} = \frac{1}{1+\frac{0.0518}{1.28}} = 0.961$$
$$K_{\mathrm{f}} = 1 + \left(K_{\mathrm{t,bending}} - 1\right)q = 1 + (2.26 - 1)0.961 = 2.21.$$

Therefore, the endurance limit at $10^6$ cycles for this component under bending loading is as follows:

$$S_{\mathrm{e,bending}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}} = \frac{700(1)(0.85)(0.868)(0.843)}{2.21} = 197\,\mathrm{MPa}.$$

For *torsional loading*, the factor for loading is constant and is defined as follows:

$$K_{\mathrm{L}} = 0.58.$$

The size effect factor is given by the following equation:

$$K_{\mathrm{d}} = 1.189(d)^{-0.097}.$$

Peterson's formula for fatigue strength reduction for torsional loading is given below:

$$a = 0.01524\left(\frac{2079}{S_{\mathrm{t,u,std}}}\right)^{1.8} = 0.01524\left(\frac{2079}{1400}\right)^{1.8} = 0.0311$$
$$q = \frac{1}{1+\frac{a}{r}} = \frac{1}{1+\frac{0.0311}{1.28}} = 0.976$$
$$K_{\mathrm{f}} = 1 + \left(K_{\mathrm{t,torsion}} - 1\right)q = 1 + (1.64 - 1)0.976 = 1.62.$$

Therefore, the endurance limit at $10^6$ cycles for this component under torsional loading is as follows:

$$S_{\mathrm{e,torsion}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}} = \frac{700(0.58)(0.85)(0.868)(0.843)}{1.62} = 156\,\mathrm{MPa}.$$

Solution (FKM-Guideline)

Using the analysis techniques described in FKM-Guideline, the endurance limit can be estimated by the following equations (these equations and parameters were defined in section 3, but are repeated here for convenience):

$$S_{\mathrm{S,ax,E}} = \frac{C_{\mathrm{S}}S_{\mathrm{S},\sigma,\mathrm{E}}}{K_{\mathrm{ax,f}} + \frac{1}{C_{\sigma,\mathrm{R}}} - 1} \quad \text{for axial loading}$$
$$S_{\mathrm{S,b,E}} = \frac{C_{\mathrm{S}}S_{\mathrm{S},\sigma,\mathrm{E}}}{K_{\mathrm{b,f}} + \frac{1}{C_{\sigma,\mathrm{R}}} - 1} \quad \text{for bending loading}$$
$$S_{\mathrm{S,t,E}} = \frac{C_{\mathrm{S}}S_{\mathrm{S},\tau,\mathrm{E}}}{K_{\mathrm{t,f}} + \frac{1}{C_{\tau,\mathrm{R}}} - 1} \quad \text{for torsional loading,}$$

where $C_{\mathrm{S}}$ is the surface treatment factor, $S_{\mathrm{S},\sigma,\mathrm{E}}$ and $S_{\mathrm{S},\tau,\mathrm{E}}$ are the material fatigue strength for normal and shear stresses, respectively, $K_{\mathrm{ax,f}}$, $K_{\mathrm{b,f}}$, and $K_{\mathrm{t,f}}$ are the fatigue notch factors for axial, bending, and torsional loading, respectively, $C_{\sigma,\mathrm{R}}$ and $C_{\tau,\mathrm{R}}$ are the roughness correction factors for normal and shear stresses, respectively. The surface treatment factor is equal to 1 for all loading conditions because there was no surface treatment applied to the component. The size effect correction factor is the same for all three loading conditions and is given by the following equation:

$$C_{\mathrm{D}} = \frac{1 - 0.7686(a_{\mathrm{d}})\log\left(\frac{d_{\mathrm{eff}}}{7.5}\right)}{1 - 0.7686(a_{\mathrm{d}})\log\left(\frac{d_{\mathrm{eff,min}}}{7.5}\right)} = \frac{1 - 0.7686(0.3)\log\left(\frac{25.5}{7.5}\right)}{1 - 0.7686(0.3)\log\left(\frac{16}{7.5}\right)} = 0.949,$$

where $d_{\mathrm{eff}}$ is the net diameter at the notched section, $a_{\mathrm{d}}$ and $d_{\mathrm{eff,min}}$ are material constants defined by the FKM-Guideline.

For *axial loading*, the material fatigue strength ($S_{\mathrm{S},\sigma,\mathrm{E}}$) can be estimated using the following equation:

$$S_{\mathrm{S},\sigma,\mathrm{E}} = C_{\sigma,\mathrm{E}}C_{\mathrm{D}}C_{\mathrm{R}}S_{\mathrm{t,u,std}} = 0.45(0.949)(0.843)(1400) = 504\ \mathrm{MPa},$$

where $C_{\sigma,\mathrm{E}}$ is the endurance limit modification factor based on material type and is equal to 0.45 for the shaft in this example. The roughness correction factor is calculated based on a roughness value ($R_z$) of 12 µm and constants ($a_{\mathrm{R}}$ and $S_{\mathrm{t,u,min}}$) from Table 9. The roughness value was chosen to stay on the conservative side since it is the lower limit of roughness values suggested by FKM-Guidelines for a ground surface. The roughness correction factor is defined as

$$C_{\sigma,\mathrm{R}} = 1 - a_{\mathrm{R}}\log(R_Z)\log\left(\frac{2C_{\mathrm{R}}C_{\mathrm{D}}S_{\mathrm{t,u,std}}}{S_{\mathrm{t,u,min}}}\right)$$

$$C_{\sigma,\mathrm{R}} = 1 - (0.22)\log(12)\log\left(\frac{2(0.843)(0.949)(1400)}{400}\right) = 0.823.$$

To calculate the fatigue notch factor ($K_{\mathrm{f}}$), the first step is to calculate the stress concentration factor. Based on the analysis provided in the FKM-Guideline

$$K_{\mathrm{ax,t}} = 1 + \frac{1}{\sqrt{0.22(r/t) + 2.74(r/d)(1+2(r/d))^2}}$$

$$K_{\mathrm{ax,t}} = 1 + \frac{1}{\sqrt{0.22\left(1.28/1.25\right) + 2.74\left(1.28/25.5\right)\left(1+2\left(1.28/25.5\right)\right)^2}} = 2.598.$$

Next, the relative stress gradient needs to be calculated. The relative stress gradient for axial loading is given by

$$\bar{G}_{K,\sigma}(r) = \frac{2}{r}(1+\varphi) = \frac{2}{1.28}(1+\varphi) = 1.825,$$

where

$$\varphi = 1/\left(4\sqrt{t/r}+2\right) = 1/\left(4\sqrt{1.25/1.28}+2\right) = 0.168.$$

Next, the supporting factor or $K_{\mathrm{t}}/K_{\mathrm{f}}$ ratio ($n_{K,\sigma}(r)$) is calculated as

$$n_{K,\sigma}(r) = 1 + \sqrt[4]{\bar{G}_{K,\sigma}(r)} \times 10^{-\left(a_{\mathrm{G}} + \left(C_{\mathrm{R}}C_{\mathrm{D}}S_{\mathrm{t,u,std}}\right)/b_{\mathrm{G}}\right)}$$

$$n_{K,\sigma}(r) = 1 + \sqrt[4]{1.25} \times 10^{-(0.5+(0.843)(0.939)(1400)/2700)} = 1.142.$$

The constants, $a_{\mathrm{G}}$ and $b_{\mathrm{G}}$, in the above equation for $n_{K,\sigma}$ are based on the type of material. Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor ($K_{\mathrm{t}}$) and the supporting factor ($n_{K,\sigma}$) as

$$K_{\mathrm{ax,f}} = \frac{K_{\mathrm{ax,t}}}{n_{K,\sigma}(r)} = \frac{2.598}{1.142} = 2.275.$$

Therefore, the endurance limit at $10^6$ cycles for this component under axial loading is as follows:

$$S_{\mathrm{S,ax,E}} = \frac{C_{\mathrm{S}}S_{\mathrm{S},\sigma,\mathrm{E}}}{K_{\mathrm{ax,f}} + \frac{1}{C_{\sigma,\mathrm{R}}} - 1} = \frac{(1)(504)}{2.275 + \frac{1}{0.823} - 1} = 202\,\mathrm{MPa}.$$

For *bending loading*, the material fatigue strength ($S_{\mathrm{S},\sigma,\mathrm{E}}$) can be estimated using the following equation:

$$S_{\mathrm{S},\sigma,\mathrm{E}} = C_{\sigma,\mathrm{E}}C_{\mathrm{D}}C_{\mathrm{R}}S_{\mathrm{t,u,std}} = 0.45(0.949)(0.843)(1400) = 504.$$

The roughness correction factor is calculated based on a roughness value ($R_z$) of 12 μm and constants ($a_{\mathrm{R}}$ and $S_{\mathrm{t,u,min}}$) from Table 9. The roughness correction factor is defined as

$$C_{\sigma,\mathrm{R}} = 1 - a_R\log(R_Z)\log\left(\frac{2C_{\mathrm{R}}C_{\mathrm{D}}S_{\mathrm{t,u,std}}}{S_{\mathrm{t,u,min}}}\right)$$

$$C_{\sigma,\mathrm{R}} = 1 - (0.22)\log(12)\log\left(\frac{2(0.843)(0.949)(1400)}{400}\right) = 0.823.$$

To calculate the fatigue notch factor ($K_{\mathrm{f}}$), the first step is to calculate the stress concentration factor. Based on the analysis provided in the FKM-Guideline

$$K_{\mathrm{b,t}} = 1 + \frac{1}{\sqrt{0.2(r/t) + 5.5(r/d)(1+2(r/d))^2}}$$

$$K_{\mathrm{b,t}} = 1 + \frac{1}{\sqrt{0.2\left(1.28/1.25\right) + 5.5\left(1.28/25.5\right)\left(1+2\left(1.28/25.5\right)\right)^2}} = 2.362.$$

Next, the relative stress gradient needs to be calculated. For bending loading, it is also necessary to calculate a relative stress gradient based on the net diameter in addition to the relative stress gradient based on the notch radius. The relative stress gradient for bending loading based on notch radius ($G_{K,\sigma}(r)$) and net diameter ($G_{K,\sigma}(d)$) are given by

$$\bar{G}_{K,\sigma}(r) = \frac{2}{r}(1+\varphi) = \frac{2}{1.28}(1+\varphi) = 1.825$$
$$\bar{G}_{K,\sigma}(d) = \frac{2}{d} = \frac{2}{25.5} = 0.078,$$

where

$$\varphi = 1/\left(4\sqrt{t/r}+2\right) = 1/\left(4\sqrt{1.25/1.28}+2\right) = 0.168.$$

Next, the supporting factors based on notch radius ($n_{K,\sigma}(r)$) and net diameter ($n_{K,\sigma}(d)$) are calculated as

$$n_{K,\sigma}(r) = 1 + \sqrt[4]{\bar{G}_{K,\sigma}(r)} \times 10^{-\left(a_G + \left(C_R C_D S_{t,u,std}\right)/b_G\right)}$$
$$n_{K,\sigma}(r) = 1 + \sqrt[4]{1.25} \times 10^{-(0.5+(0.843)(0.939)(1400)/2700)} = 1.142$$
$$n_{K,\sigma}(d) = 1 + \bar{G}_{K,\sigma}(d) \times 10^{-\left(a_G - 0.5 + \left(C_R C_D S_{t,u,std}\right)/b_G\right)}$$
$$n_{K,\sigma}(d) = 1 + (0.078) \times 10^{-(0.5-0.5+(0.843)(0.939)(1400)/2700)} = 1.030.$$

$$S_{S,\tau,E} = C_\tau S_{S,\sigma,E} = C_\tau C_{\sigma,E} C_D C_R S_{t,u,std}$$

$$S_{S,\tau,E} = 0.577(0.45)(0.949)(0.843)(1400) = 291\,\text{MPa},$$

where $C_\tau$ is the shear stress correction factor based on von Mises criterion. The roughness correction factor is calculated based on a roughness value ($R_z$) of 12 µm and constants ($a_R$ and $S_{t,u,min}$) from Table 9. The roughness correction factor of torsion loading is different than that of the for the axial or bending loading and is defined as

$$C_{\tau,R} = 1 - C_\tau a_R \log(R_Z) \log\left(\frac{2C_R C_D S_{t,u,std}}{S_{t,u,min}}\right)$$

$$C_{\tau,R} = 1 - (0.577)(0.22)\log(12) \times \log\left(\frac{2(0.843)(0.949)(1400)}{400}\right) = 0.898.$$

To calculate the fatigue notch factor ($K_f$), the first step is to calculate the stress concentration factor. Based on the analysis provided in the FKM-Guideline

$$K_{t,t} = 1 + \frac{1}{\sqrt{0.7\left(r/t\right) + 20.6\left(r/d\right)\left(1+2\left(r/d\right)\right)^2}}$$
$$K_{t,t} = 1 + \frac{1}{\sqrt{0.7\left(1.28/1.25\right) + 20.6\left(1.28/25.5\right)\left(1+2\left(1.28/25.5\right)\right)^2}} = 1.713.$$

Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor ($K_t$) and the supporting factors as seen below

$$K_{b,f} = \frac{K_{b,t}}{n_{K,\sigma}(r)\, n_{K,\sigma}(d)} = \frac{2.362}{(1.142)(1.030)} = 2.008.$$

Therefore, the endurance limit at $10^6$ cycles for this component under bending loading is as follows:

$$S_{S,b,E} = \frac{C_S S_{S,\sigma,E}}{K_{b,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{(1)(504)}{2.008 + \frac{1}{0.823} - 1} = 227\,\text{MPa}$$

For *torsional loading*, the material fatigue strength in shear ($S_{S,\tau,E}$) can be estimated using the following equation:

Next, the relative stress gradient needs to be calculated. For torsional loading, it is also necessary to calculate a relative stress gradient based on the net diameter in addition to the relative stress gradient based on the notch radius. The relative stress gradient for torsion loading based on notch radius ($G_{K,\tau}(r)$) and net diameter ($G_{K,\tau}(d)$) are given by

$$\bar{G}_{K,\tau}(r) = \frac{1}{r} = \frac{1}{1.28} = 0.781$$
$$\bar{G}_{K,\tau}(d) = \frac{2}{d} = \frac{2}{25.5} = 0.078.$$

Next, the supporting factors based on notch radius ($n_{K,\tau}(r)$) and net diameter ($n_{K,\tau}(d)$) are calculated as

$$n_{K,\tau}(r) = 1 + \sqrt{\bar{G}_{K,\tau}(r)} \times 10^{-\left(a_G + C_{\tau,E} C_R C_D S_{t,u,std}/b_G\right)}$$
$$n_{K,\tau}(r) = 1 + \sqrt{0.781} \times 10^{-(0.5+(0.577)(0.843)(0.949)(1400)/2700)} = 1.161$$
$$n_{K,\tau}(d) = 1 + \bar{G}_{K,\tau}(d) \times 10^{-\left(a_G - 0.5 + \left(C_R C_D S_{t,u,std}\right)/b_G\right)}$$
$$n_{K,\tau}(d) = 1 + (0.078) \times 10^{-(0.5-0.5+(0.843)(0.949)(1400)/2700)} = 1.045.$$

Note the calculations of the supporting factors and the relative stress gradients are different for torsional loading compared to axial and bending loading. Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor and the supporting factors as seen below

$$K_{t,f} = \frac{K_{t,t}}{n_{K,\tau}(r)\, n_{K,\tau}(d)} = \frac{1.713}{(1.161)(1.045)} = 1.411.$$

Therefore, the endurance limit at $10^6$ cycles for this component under torsional loading is as follows:

$$S_{S,t,E} = \frac{C_S S_{S,\tau,E}}{K_{t,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{(1)(291)}{1.411 + \frac{1}{0.898} - 1} = 191\,\text{MPa}.$$

See Table 13 for summary of calculations.

**Table 13** Side-by-side comparison for Example 1

| Parameter | North American | FKM |
|---|---|---|
| Elastic stress concentration factor | | |
| $K_{ax,t}$ | 2.5 | 2.60 |
| $K_{b,t}$ | 2.26 | 2.36 |
| $K_{t,t}$ | 1.64 | 1.71 |
| Fatigue notch factor | | |
| $K_{ax,f}$ | 2.45 | 2.28 |
| $K_{b,f}$ | 2.21 | 2.01 |
| $K_{t,f}$ | 1.62 | 1.41 |
| Size effect factor | | |
| $K_D/C_D$ (tension) | 1 | 0.949 |
| $K_D/C_D$ (bending) | 0.868 | 0.949 |
| $K_D/C_D$ (torsion) | 0.868 | 0.949 |
| Reliability factor | | |
| $C_R$ | 0.843 | 0.843 |
| Surface roughness factor | | |
| $K_S/C_{\sigma,R}$ (tension) | 0.85 | 0.823 |
| $K_S/C_{\sigma,R}$ (bending) | 0.85 | 0.823 |
| $K_S/C_{\sigma,R}$ (torsion) | 0.85 | 0.898 |
| Endurance limit at $10^6$ cycles, MPa | | |
| $S_f$ (tension) | 174 | 202 |
| $S_f$ (bending) | 197 | 227 |
| $S_f$ (torsion) | 156 | 191 |

Example 2 [15]

A hot-rolled notched component made of SAE 1005 ($S_u$ = 321 MPa) consists of a plate with a center hole of $r$ = 2.0 mm. As shown in Fig. 26, the plate has a width of $B$ = 16 mm and a thickness of $t$ = 5 mm. The elastic stress concentration factor of $K_{ax,t}$ = 2.4 due to tension was calculated based on the geometric ratio. Assume that the surface was ground and polished with a mirror finish (i.e., $R_z = 0$). Fatigue tests with cyclic axial loading on the hot-rolled unnotched plates of the same material were conducted to generate a baseline S–N curve that is defined by the following equation:

$$S_a = S'_f (2N_f)^b = 886(2N_f)^{-0.14},$$

where $S'_f$ is the fatigue strength coefficient and $b$ is the slope. Based on Morrow's mean stress correction, determine the fatigue life of this notched plate if it is subjected to cyclic axial loading varying from $P_{max}$ = + 8 kN to $P_{min}$ = −6 kN.

The alternating stress ($S_a$) and mean stress ($S_m$) applied to the component are calculated based on the maximum and minimum loading

$$S_{max} = \frac{P_{max}}{A_{net}} = \frac{8000}{(16-4)(5)} = 133\,\text{MPa}$$
$$S_{min} = \frac{P_{min}}{A_{net}} = \frac{-6000}{(16-4)(5)} = -100\,\text{MPa}$$
$$S_a = \frac{(S_{max} - S_{min})}{2} = \frac{133 - (-100)}{2} = 117\,\text{MPa}$$
$$S_m = \frac{(S_{max} + S_{min})}{2} = \frac{133 + (-100)}{2} = 16.7\,\text{MPa}.$$

An equivalent fully reversed stress amplitude can be calculated based on Morrow's equation

$$S_{ar} = \frac{S_a}{1 - \frac{S_m}{S'_f}} = \frac{117}{1 - \frac{16.7}{886}} = 119.$$

This equivalent stress can be used to estimate the fatigue life based on the S–N curves for fully reversed loading (i.e., no mean stress)

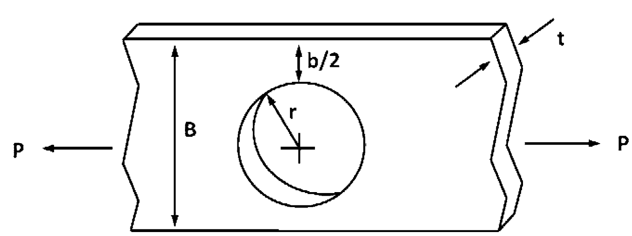


**Fig. 26** Geometry of notched plate in Example 2

Solution (North American)

The endurance limit (at $10^6$ cycles) for fully reversed loading is calculated based on the equation given for the *S–N* curve

$$S_f = 886\left(2\times10^6\right)^{-0.14} = 116\,\text{MPa}.$$

Next, the fatigue notch factor needs to be calculated using Peterson's equation based on the stress concentration factor ($K_{ax,t}$) and notch sensitivity ($q$)

$$K_{ax,f} = 1 + \left(K_{ax,t} - 1\right)q = 1 + (2.4-1)(0.68) = 1.952,$$

where $q$ was derived from an empirical notch sensitivity curve based on an ultimate strength of 321 MPa and a notch radius of 2 mm. Now the fatigue strength of the notched part under fully reversed loading (i.e., no mean stress) can be estimated

$$S_{f,notched} = \frac{S_f}{K_{ax,f}} = \frac{116}{1.952} = 59\,\text{MPa}.$$

Figure 27 shows the *S–N* curves for the notched part and the unnotched part under fully reversed loading based on Collin's approach.

The curves in Fig. 27 were generated by connecting a straight line from $S'_f$ at 1 reversal to the fatigue strengths of the notched and unnotched parts. Now the slope ($b'$) of the *S–N* curve for the notched component can be calculated

$$b' = \frac{\log(59) - \log(886)}{\log(2\times10^6) - \log(1)} = -0.186.$$

This results in the following stress-life equation for the notched component under fully reversed loading, which is valid between 1 and $2(10^6)$ reversals

$$S_a = 886(2N_f)^{-0.186}$$

Based on Morrow's equation

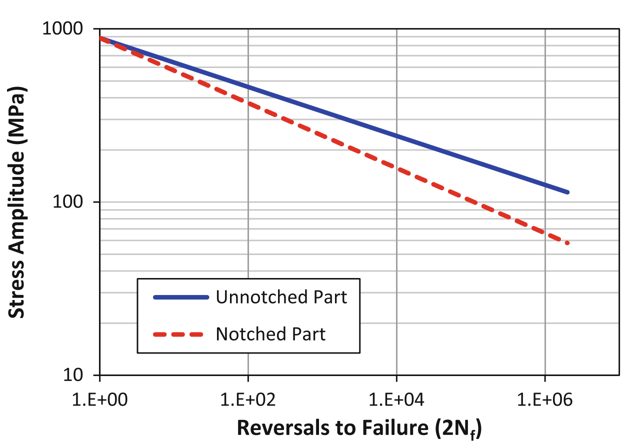



**Fig. 27** *S–N* curves for notched and unnotched components under fully reversed loading

$$S_a = \left(S'_f - S_m\right)(2N_f)^{b'}$$

therefore, the fatigue life of the notched component

$$2N_f = \left[\frac{S_a}{\left(S'_f - S_m\right)}\right]^{\frac{1}{b'}} = \left[\frac{117}{(886-16.7)}\right]^{\frac{1}{-0.186}} = 48,200\,\text{Reversals}.$$

This results in a fatigue life of 24,100 cycles.

Solution (FKM)

Using the analysis techniques described in FKM-Guideline, the endurance limit can be estimated by the following equation:

$$S_{S,ax,E} = \frac{C_S S_{S,\sigma,E}}{K_{ax,f} + \frac{1}{C_{\sigma,R}} - 1} \quad \text{for axial loading,}$$

where $C_S$ is the surface treatment factor, $S_{S,\sigma,E}$ is the material fatigue strength for normal stress, $K_{ax,f}$ is the fatigue notch factor, $C_{\sigma,R}$ is the roughness correction factor for normal stress. The surface treatment factor ($C_S$) is equal to 1 because there was no surface treatment applied to the component. The roughness correction factor ($C_{\sigma,R}$) is equal to one since the surface was ground and polished to a mirror finish.

For axial loading, the material fatigue strength ($S_{S,\sigma,E}$) can be estimated using the following equation:

$$S_{S,\sigma,E} = C_{\sigma,E} C_D C_R S_{t,u,std} = 0.45(1)(0.843)(321) = 122\,\text{MPa},$$

where $C_{\sigma,E}$ is the endurance limit modification factor based on material type and is equal to 0.45 for the component in this example. The size effect correction factor ($C_D$) is assumed to be equal to 1. To calculate the fatigue notch factor ($K_f$), the first step is to calculate the stress concentration factor. The elastic stress concentration factor for a circular hole in a plate is as follows:

$$K_{ax,t} = 3 - 3.13\left(\frac{2r}{B}\right) + 3.66\left(\frac{2r}{B}\right)^2 - 1.53\left(\frac{2r}{B}\right)^3$$

$$K_{ax,t} = 3 - 3.13\left(\frac{2(2)}{16}\right) + 3.66\left(\frac{2(2)}{16}\right)^2 - 1.53\left(\frac{2(2)}{16}\right)^3 = 2.422.$$

Next, the relative stress gradient needs to be calculated. The relative stress gradient for axial loading of a plate with a center hole is given by

$$\bar{G}_{K,\sigma}(r) = \frac{2.3}{r} = \frac{2.3}{2} = 1.15.$$

Next, the supporting factor or $K_t/K_f$ ratio ($n_{K,\sigma}(r)$) is calculated

$$n_{K,\sigma}(r) = 1 + \sqrt[4]{\bar{G}_{K,\sigma}(r)} \times 10^{-\left(a_G + \left(C_R C_D S_{t,u,std}\right)/b_G\right)}$$
$$n_{K,\sigma}(r) = 1 + \sqrt[4]{1.15} \times 10^{-(0.5+(0.843)(1)(321)/2700)}$$
$$= 1.260,$$

where $a_G$ and $b_G$, are material constants. Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor ($K_t$) and the supporting factor ($n_{K,\sigma}$)

$$K_{ax,f} = \frac{K_{ax,t}}{n_{K,\sigma}(r)} = \frac{2.422}{1.260} = 1.922.$$

Therefore, the endurance limit at $10^6$ cycles for this notched component under fully reversed axial loading is as follows:

$$S_{S,ax,E} = \frac{C_S S_{S,\sigma,E}}{K_{ax,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{(1)(122)}{1.922 + \frac{1}{1} - 1} = 63\,\text{MPa}.$$

FKM-Guidelines define slope factors ($k$) which are the negative inverse of the $S$–$N$ slope ($b$). For the steel component described in this example, $k = 5$. This results in the following stress-life equation for the notched component under fully reversed loading, which is valid from $\approx 10^4$ to $10^6$ cycles

$$S_a = 1147(2N_f)^{-1/5}.$$

Based on Morrow's equation

$$2N_f = \left[\frac{S_a}{(S'_f - S_m)}\right]^{-k} = \left[\frac{117}{(1147 - 16.7)}\right]^{-5}$$
$$= 84,200\ \text{Reversals}.$$

Therefore, the component should last 42,100 cycles.

Figure 28 shows a plot of the $S$–$N$ curves for fully reversed loading of the notched component described in this example. The figure includes a horizontal line at the equivalent fully reversed stress amplitude ($S_{ar} = 119$ MPa). This line intersects the $S$–$N$ curves at the cycles to failure determined in the two solution sets in this example. This figure illustrates how close the two solutions are to each other.

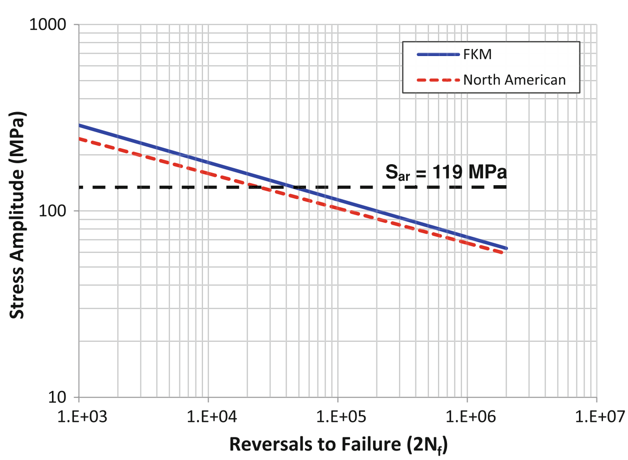



**Fig. 28** $S$–$N$ curves for fully reversed loading of a notched component

Example 3 [15]

An axle shaft was made of SAE 1050 steel with an ultimate strength of 690 MPa. The entire shaft was heat-treated to a Brinell hardness of 300 HB except for the shaft hub end. See Fig. 29 for geometry and dimensions of the hub end and spline end of the shaft. The pilot radius and the spline end were machined to meet the dimensional specification. The elastic stress concentration factors for the rotor pilot radius in bending and the spline end in torsion are 4.01 and 1.16, respectively. The gross axle weight rating (GAWR) is 15 kN and the static loaded radius of the tire (SLR) is 358 mm. The wheel base ($L$) is 3330 mm and the C.G. height ($H$) is 625 mm.

According to an axle manufacturer's design manual, the axle shaft should have the capability of resisting a centrifugal force of 0.9 G due to vehicle cornering for $10^6$ stress cycles. The shafts should also be designed for the maximum slip torque levels for 8000 cycles.

Vehicle cornering induces a bending moment and an axial force on the flange end of the axle shaft. The bending

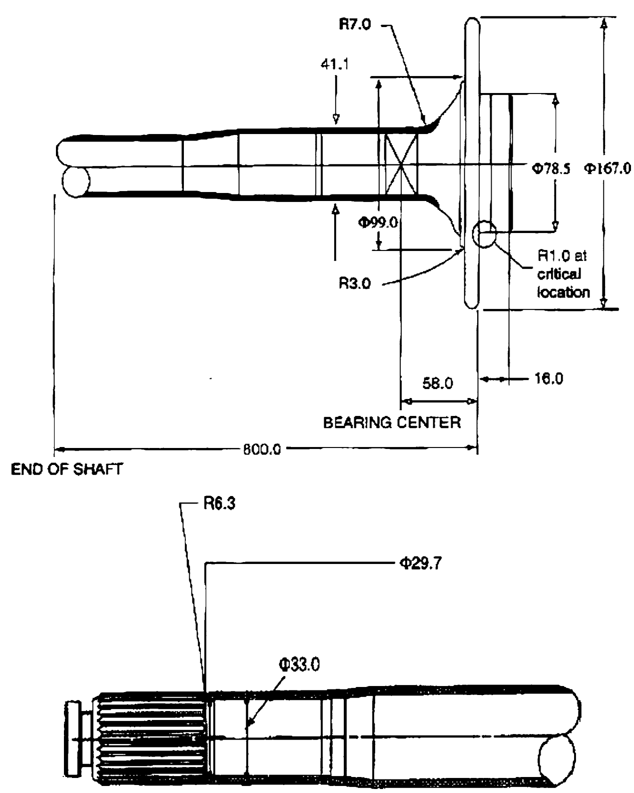



**Fig. 29** Axle shaft geometry and dimensions [15]

moment and axial forces due to cornering are calculated from the following equations:

$$M = 0.9\left(\frac{\mathrm{GAWR}}{2}\right)\ \mathrm{SLR} = 0.9\left(\frac{15{,}000}{2}\right)(0.358)$$
$$= 2417\,\mathrm{Nm}$$
$$P = -0.9\left(\frac{\mathrm{GAWR}}{2}\right) = -0.9\left(\frac{15{,}000}{2}\right) = -6750\ \mathrm{N}.$$

Because the shaft is rotating, the bending stress at the rotor will cycle from tension to compression (i.e., fully reversed bending stress). In addition, vehicle cornering induces a constant compressive axial load on the rotor pilot (i.e., zero load amplitude). Therefore, the axial load will cause a compressive mean stress and bending load will cause a fully reversed normal stress. The stress amplitude due to cyclic bending and the mean stress due to the compressive axial load are

$$S_{\mathrm{a}} = \frac{32M_{\mathrm{a}}}{\pi d^3} = \frac{32(2417)(1000)}{\pi\,(78.5)^3} = 50.9\,\mathrm{MPa}$$
$$S_{\mathrm{m}} = \frac{4P_{\mathrm{m}}}{\pi d^2} = \frac{4(-6750)}{\pi\,(78.5)^2} = -1.39\,\mathrm{MPa}.$$

Because of the compressive mean stress on the pilot radius, it is conservative to consider the fully reversed nominal bending stress (±50.9 MPa) acting alone in this fatigue analysis. As a result, the axial loading will be ignored in the following analysis.

The maximum slip torque for each axle shaft is calculated as

$$T_{\mathrm{r,max}} = \frac{\mu(\mathrm{GAWR})}{1-\frac{\mu \mathrm{H}}{\mathrm{L}}}(\mathrm{SLR})(0.5)$$
$$= \frac{(0.9)(15{,}000)}{1-\frac{(0.9)(0.625)}{3.33}}(0.358)(0.5) = 2908\,\mathrm{Nm},$$

where $\mu$ is the coefficient of friction for dry pavement and is equal to 0.9. The design criteria for the shaft in torsion requires that this torsion load is applied cyclically. The nominal shear stress amplitude is obtained

$$S_{\mathrm{a}} = \frac{16T_{\mathrm{a}}}{\pi d^3} = \frac{16(2908)(1000)}{\pi(29.7)^3} = 565\,\mathrm{MPa}.$$

Based on the given information, determine if the shaft will survive

1. Cyclic bending loading for $10^6$ cycles.
2. Cyclic torsional loading for 8000 cycles.

Solution (North American)

To determine whether the shaft subjected to alternating *bending stresses* of ±50.9 MPa can survive $10^6$ cycles, the endurance limit must be estimated. This value can be obtained as follows:

$$S_{\mathrm{e,bending}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}},$$

where $S_{\mathrm{b,E}}$ is the material fatigue strength, $K_{\mathrm{L}}$ is the loading factor, $K_{\mathrm{s}}$ is the surface effect factor (effect of roughness and surface treatment), $K_{\mathrm{d}}$ is the size effect factor, and $C_{\mathrm{R}}$ is the reliability factor. The parameter $S_{\mathrm{b,E}}$ is defined as follows:

$$S_{\mathrm{b,E}} = 0.5S_{\mathrm{t,u,std}} \quad \text{for } S_{\mathrm{t,u,std}} < 1400\,\mathrm{MPa}$$
$$S_{\mathrm{b,E}} = 700\,\mathrm{MPa} \quad \text{for } S_{\mathrm{t,u,std}} \geq 1400\,\mathrm{MPa},$$

where $S_{\mathrm{t,u,std}}$ is in units of MPa. Therefore, for component with an ultimate tensile strength of 690 MPa

$$S_{\mathrm{b,E}} = 0.5S_{\mathrm{t,u,std}} = 0.5(690) = 345\,\mathrm{MPa}.$$

The loading factor ($K_{\mathrm{L}}$) is equal to 1 for bending. Because no reliability requirement is given, it is assumed that a median value (i.e., a reliability of 50%) is adequate meaning $C_{\mathrm{R}} = 1$. The correction factor for surface effects resulting from a machining procedure is as follows:

$$K_{\mathrm{S}} = 4.51(S_{\mathrm{u}})^{-0.265} = 4.51(690)^{-0.265} = 0.798.$$

The size effect factor is given by the following equation:

$$K_{\mathrm{d}} = 1.189(d)^{-0.097} = 1.189(78.5)^{-0.097} = 0.779.$$

Next, the fatigue notch factor needs to be calculated using Peterson's equation based on the stress concentration factor ($K_{\mathrm{b,t}}$) and notch sensitivity ($q$), where $q$ is derived from an empirical equation for notch sensitivity as follows:

$$q = \frac{1}{1+\frac{a}{r}} = \frac{1}{1+\frac{0.254\left(\frac{2079}{S_{\mathrm{t,u,std}}}\right)^{1.8}}{r}} = \frac{1}{1+\frac{0.254\left(\frac{2079}{690}\right)^{1.8}}{1}} = 0.844$$

therefore,

$$K_{\mathrm{b,f}} = 1 + \left(K_{\mathrm{b,t}} - 1\right)q = 1 + (4.01-1)0.844 = 3.54.$$

Now the endurance limit of the notched part under fully reversed loading (i.e., no mean stress) can be estimated

$$S_{\mathrm{e,bending}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}} = \frac{345(1)(0.798)(0.779)(1)}{3.54}$$
$$= 56.9\,\mathrm{MPa}.$$

Because the alternating bending stress (50.9 MPa) is lower than the endurance limit (56.9 MPa), the shaft should survive the bending loading for $10^6$ cycles.

To determine whether the shaft subjected to alternating *torsional stresses* of ±565 MPa can survive 8000 cycles, first estimate the endurance limit as follows:

$$S_{\mathrm{e,torsion}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}}.$$

Because the shaft is heat-treated at the spline end, the ultimate strength is different compared to the ultimate strength of the base material. The material fatigue strength of the spline end of the shaft can be estimated as

$$S_{\mathrm{b,E}} = 0.5S_{\mathrm{t,u,std}} = 0.5(3.45(300)) = 518\,\mathrm{MPa}.$$

The loading factor for torsion is $C_{\mathrm{L}} = 0.58$. Because no reliability requirement is given, it is assumed that $C_{\mathrm{R}} = 1$. The correction factor for surface effects resulting from a machining procedure is as follows:

$$K_{\mathrm{S}} = 4.51(\mathrm{S_u})^{-0.265} = 4.51(1035)^{-0.265} = 0.717.$$

The size effect factor for the spline end is

$$K_{\mathrm{d}} = 1.189(d)^{-0.097} = 1.189(29.7)^{-0.097} = 0.856.$$

Next, the fatigue notch factor needs to be calculated using Peterson's equation based on the stress concentration factor ($K_{\mathrm{b,t}}$) and notch sensitivity ($q$), where $q$ is derived from an empirical equation for notch sensitivity as follows:

$$q = \frac{1}{1+\frac{a}{r}} = \frac{1}{1+\frac{0.254\left(\frac{2079}{S_{\mathrm{t,u,std}}}\right)^{1.8}}{r}} = \frac{1}{1+\frac{0.254\left(\frac{2079}{1035}\right)^{1.8}}{1}} = 0.992$$

therefore,

$$K_{\mathrm{t,f}} = 1 + \left(K_{\mathrm{t,t}} - 1\right)q = 1 + (1.16 - 1)0.992 = 1.16.$$

Now the endurance limit of the notched part under fully reversed torsion loading (i.e., no mean stress) can be estimated

$$S_{\mathrm{e,torsion}} = \frac{S_{\mathrm{b,E}}K_{\mathrm{L}}K_{\mathrm{s}}K_{\mathrm{d}}C_{\mathrm{R}}}{K_{\mathrm{f}}} = \frac{518(0.58)(0.717)(0.856)(1)}{1.16} = 159\,\mathrm{MPa}.$$

Using the Heywood model for *S–N* curves for notched components, the fatigue strength at $10^3$ cycles can be estimated

$$S_{1000} = \frac{0.9S_{\mathrm{US}}C_{\mathrm{R}}}{K_{\mathrm{f}}},$$

where $S_{\mathrm{US}}$ is the ultimate strength in shear. For steels

$$S_{\mathrm{US}} = 0.8S_{\mathrm{t,u,std}} = 0.8(1035) = 827\,\mathrm{MPa}.$$

The fatigue notch factor at $10^3$ cycles ($K_{\mathrm{f}}'$) can be found from the empirical relationship between $K_{\mathrm{f}}$ and $K_{\mathrm{f}}'$. For steel with $S_{\mathrm{t,u,std}} = 1035$ MPa

$$q_{1000} = \frac{K'_{\mathrm{t,f}} - 1}{K_{\mathrm{t,f}} - 1} = 0.384$$

therefore

$$K'_{\mathrm{t,f}} = 1 + \left(K_{\mathrm{t,f}} - 1\right)q_{1000} = 1 + (1.16 - 1)0.384 = 1.06.$$

Based on the Heywood model the fatigue strength at $10^3$ cycles is

$$S_{1000} = \frac{0.9S_{\mathrm{US}}C_{\mathrm{R}}}{K'_{\mathrm{f}}} = \frac{0.9(827)(1)}{(1.06)} = 702\,\mathrm{MPa}.$$

After both $S_{1000}$ and $S_{\mathrm{e,torsion}}$ have been determined, they can be used to determine the number of cycles the shaft will last under $S_{\mathrm{a}} = 565$ MPa. Based on fatigue strengths calculated at $10^6$ and $10^3$ cycles

$$\frac{\log(10^3) - \log(10^6)}{\log(702) - \log(153)} = \frac{\log(10^3) - \log(N_{\mathrm{f}})}{\log(702) - \log(565)}.$$

Solving for $N_{\mathrm{f}}$ gives 2,680 cycles which is <8000 cycle requirement. This means the spline area will require redesigning.

## Solution (FKM)

FKM-Guideline gives the following equations to estimate the endurance limit:

$$S_{\mathrm{S,b,E}} = \frac{C_{\mathrm{S}}S_{\mathrm{S},\sigma,\mathrm{E}}}{K_{\mathrm{b,f}} + \frac{1}{C_{\sigma,\mathrm{R}}} - 1} \quad \text{for bending loading}$$

$$S_{\mathrm{S,t,E}} = \frac{C_{\mathrm{S}}S_{\mathrm{S},\tau,\mathrm{E}}}{K_{\mathrm{t,f}} + \frac{1}{C_{\tau,\mathrm{R}}} - 1} \quad \text{for torsional loading,}$$

where $C_{\mathrm{S}}$ is the surface treatment factor, $S_{\mathrm{S},\sigma,\mathrm{E}}$ and $S_{\mathrm{S},\tau,\mathrm{E}}$ are the material fatigue strength for normal and shear stresses, respectively, $K_{\mathrm{b,f}}$ and $K_{\mathrm{t,f}}$ are the fatigue notch factors for bending and torsional loading, respectively, $C_{\sigma,\mathrm{R}}$ and $C_{\tau,\mathrm{R}}$ are the roughness correction factors for normal and shear stresses, respectively. The surface treatment factor ($C_{\mathrm{S}}$) is equal to 1 for all loading conditions because there was no surface treatment applied to the component.

For *bending loading*, the material fatigue strength ($S_{\mathrm{S},\sigma,\mathrm{E}}$) can be estimated using the following equation:

$$S_{\mathrm{S},\sigma,\mathrm{E}} = C_{\sigma,\mathrm{E}}C_{\mathrm{D}}C_{\mathrm{R}}S_{\mathrm{t,u,std}},$$

where $C_{\sigma,\mathrm{E}}$ is the endurance limit modification factor and is equal to 0.45 for the shaft in this example. The size effect factor for the rotor pilot end of the shaft is

$$C_{\mathrm{D}} = \frac{1 - 0.7686(a_{\mathrm{d}})\log\left(\frac{d_{\mathrm{eff}}}{7.5}\right)}{1 - 0.7686(a_{\mathrm{d}})\log\left(\frac{d_{\mathrm{eff,min}}}{7.5}\right)} = \frac{1 - 0.7686(0.3)\log\left(\frac{78.5}{7.5}\right)}{1 - 0.7686(0.3)\log\left(\frac{16}{7.5}\right)} = 0.828.$$

Because the analysis in FKM-Guideline is based on an ultimate strength with a reliability of 97.5%, $C_{\mathrm{R}} = 0.843$. Therefore, the estimated material fatigue strength is

$$S_{\mathrm{S},\sigma,\mathrm{E}} = C_{\sigma,\mathrm{E}}C_{\mathrm{D}}C_{\mathrm{R}}S_{\mathrm{t,u,std}} = 0.45(0.828)(0.843)(690) = 217\,\mathrm{MPa}.$$

The roughness correction factor is calculated based on a roughness value of $R_z = 100$ μm and constants ($a_{\mathrm{R}}$ and

$S_{t,u,min}$) from Table 9. The roughness value was chosen since it is the lower limit of roughness values suggested by FKM-Guidelines for a machined surface. The roughness correction factor is defined as

$$C_{\sigma,R} = 1 - a_R \log(R_Z) \log\left(\frac{2C_R C_D S_{t,u,std}}{S_{t,u,min}}\right)$$

$$C_{\sigma,R} = 1 - (0.22)\log(100)\log\left(\frac{2(0.843)(0.828)(690)}{400}\right) = 0.832.$$

To calculate the fatigue notch factor ($K_f$), the first step is to calculate the stress concentration factor. Based on the analysis provided in the FKM-Guideline

$$K_{b,t} = 1 + \frac{1}{\sqrt{0.62(r/t) + 11.6(r/d)(1 + 2(r/d))^2 + 0.2(r/t)^3 (d/D)}}$$

$$K_{b,t} = 1 + \frac{1}{\sqrt{0.62\left(1/10.25\right) + 11.6\left(1/78.5\right)\left(1 + 2\left(1/78.5\right)\right)^2 + 0.2\left(1/10.25\right)^3 \left(78.5/99\right)}}$$

$$K_{b,t} = 3.152.$$

Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor and the supporting factors as seen below

$$K_{b,f} = \frac{K_{b,t}}{n_{K,\sigma}(r)\, n_{K,\sigma}(d)} = \frac{3.152}{(1.263)(1.017)} = 2.455.$$

Therefore, the endurance limit at $10^6$ cycles for this component under bending loading is as follows:

$$S_{S,b,E} = \frac{C_S S_{S,\sigma,E}}{K_{b,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{(1)(217)}{2.455 + \frac{1}{0.832} - 1} = 82\,\text{MPa}.$$

Because the alternating bending stress (50.9 MPa) is lower than the endurance limit (82 MPa), the shaft should survive the bending loading for $10^6$ cycles.

Next, the relative stress gradient needs to be calculated. For bending loading, it is also necessary to calculate a relative stress gradient based on the net diameter in addition to the relative stress gradient based on the notch radius. The relative stress gradient for bending loading based on notch radius ($G_{K,\sigma}(r)$) and net diameter ($G_{K,\sigma}(d)$) are given by

$$\bar{G}_{K,\sigma}(r) = \frac{2.3}{r}(1 + \varphi) = \frac{2.3}{1}(1 + \varphi) = 2.455$$

$$\bar{G}_{K,\sigma}(d) = \frac{2}{d} = \frac{2}{78.5} = 0.025,$$

where

$$\varphi = 1/\left(4\sqrt{t/r} + 2\right) = 1/\left(4\sqrt{1.25/1.28} + 2\right) = 0.068.$$

Next, the supporting factors based on notch radius ($n_{K,\sigma}(r)$) and net diameter ($n_{K,\sigma}(d)$) are calculated as

$$n_{K,\sigma}(r) = 1 + \sqrt[4]{\bar{G}_{K,\sigma}(r)} \times 10^{-\left(a_G + \left(C_R C_D S_{t,u,std}\right)/b_G\right)}$$

$$n_{K,\sigma}(r) = 1 + \sqrt[4]{1.25} \times 10^{-(0.5 + (0.843)(0.828)(690)/2700)} = 1.263$$

$$n_{K,\sigma}(d) = 1 + \bar{G}_{K,\sigma}(d) \times 10^{-\left(a_G + \left(C_R C_D S_{t,u,std}\right)/b_G\right)}$$

$$n_{K,\sigma}(d) = 1 + (0.078) \times 10^{-(0.5 + (0.843)(0.828)(690)/2700)} = 1.017.$$

For *torsional loading*, the material fatigue strength in shear ($S_{S,\tau,E}$) can be estimated using the following equation:

$$S_{S,\tau,E} = C_\tau S_{S,\sigma,E} = C_\tau C_{\sigma,E} C_D C_R S_{t,u,std},$$

where $C_\tau$ is the shear stress correction factor based on von Mises criterion. The size effect factor for the spline end of the shaft is

$$C_D = \frac{1 - 0.7686(a_d)\log\left(\frac{d_{eff}}{7.5}\right)}{1 - 0.7686(a_d)\log\left(\frac{d_{eff,min}}{7.5}\right)} = \frac{1 - 0.7686(0.3)\log\left(\frac{29.7}{7.5}\right)}{1 - 0.7686(0.3)\log\left(\frac{16}{7.5}\right)} = 0.933.$$

Therefore, the material fatigue strength in shear is

$$S_{S,\tau,E} = 0.577(0.45)(0.933)(0.843)(3.45)(300) = 211\,\text{MPa}.$$

The roughness correction factor is calculated based on a roughness value ($R_z$) of 100 μm and constants ($a_R$ and $S_{t,u,min}$) from Table 9

$$C_{\tau,R} = 1 - C_{\tau} a_R \log(R_Z) \log\left(\frac{2C_{\tau} C_R C_D S_{t,u,std}}{S_{t,u,min}}\right)$$

$$C_{\tau,R} = 1 - (0.577)(0.22)\log(12) \times \log\left(\frac{2(0.577)(0.843)(0.933)(1035)}{400}\right) = 0.845.$$

To calculate the fatigue notch factor ($K_f$), the first step is to calculate the stress concentration factor. Based on the analysis provided in the FKM-Guideline

$$K_{t,t} = 1 + \frac{1}{\sqrt{3.4\left(r/t\right) + 38\left(r/d\right)\left(1 + 2\left(r/d\right)\right)^2 + \left(r/t\right)^2\left(d/D\right)}}$$

$$K_{t,t} = 1 + \frac{1}{\sqrt{3.4\left(6.3/1.65\right) + 38\left(6.3/29.7\right)\left(1 + 2\left(6.3/29.7\right)\right)^2 + \left(6.3/1.65\right)^2\left(29.7/33\right)}}$$

$$K_{t,t} = 1.153.$$

Next, the relative stress gradient needs to be calculated. For torsional loading, it is also necessary to calculate a relative stress gradient based on the net diameter in addition to the relative stress gradient based on the notch radius. The relative stress gradient for torsion loading based on notch radius ($G_{K,\tau}(r)$) and net diameter ($G_{K,\tau}(d)$) are given by

$$\bar{G}_{K,\tau}(r) = \frac{1.15}{\mathrm{r}} = \frac{1.15}{6.3} = 0.183$$

$$\bar{G}_{K,\tau}(d) = \frac{2}{\mathrm{d}} = \frac{2}{29.7} = 0.067.$$

Next, the supporting factors based on notch radius ($n_{K,\tau}(r)$) and net diameter ($n_{K,\tau}(d)$) are calculated as

$$n_{K,\tau}(r) = 1 + \sqrt{\bar{G}_{K,\tau}(r)} \times 10^{-\left(a_G + C_{\tau} C_R C_D S_{t,u,std}/b_G\right)}$$

$$n_{K,\tau}(r) = 1 + \sqrt{0.781} \times 10^{-(0.5+(0.577)(0.843)(0.933)(1035)/2700)} = 1.092$$

$$n_{K,\tau}(d) = 1 + \bar{G}_{K,\tau}(d) \times 10^{-\left(a_G + \left(C_{\tau} C_R C_D S_{t,u,std}\right)/b_G\right)}$$

$$n_{K,\tau}(d) = 1 + (0.078) \times 10^{-(0.5+(0.577)(0.843)(0.933)(1035)/2700)} = 1.045.$$

Note the calculations of the supporting factors and the relative stress gradients are different for torsional loading compared to axial and bending loading. Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor ($K_t$) and the supporting factors as

$$K_{t,f} = \frac{K_{t,t}}{n_{K,\tau}(r)\, n_{K,\tau}(d)} = \frac{1.153}{(1.092)(1.045)} = 1.012.$$

Therefore, the endurance limit at $10^6$ cycles for this component under torsional loading is as follows:

$$S_{S,t,E} = \frac{C_S S_{S,\tau,E}}{K_{t,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{(1)(211)}{1.012 + \frac{1}{0.845} - 1} = 177\,\text{MPa}.$$

The endurance limit ($S_{S,t,E}$) can now be used along with the slope factor ($k$) to determine the fatigue strength at 8000 cycles. The slope factor for the given material subjected to torsional loading is $k = 8$. The following equation can be used to estimate the fatigue strength at 8000 cycles:

$$S_{S,t,N_f} = \frac{S_{S,t,E}(N_f)^{-1/k}}{(10^6)^{-1/k}} = \frac{177(8,000)^{-1/8}}{(10^6)^{-1/8}} = 324\,\text{MPa}.$$

The requirement of 565 MPa is >324 MPa, meaning the spline end of the shaft will not last 8000 cycles of torsional loading. This means the spline area will require redesigning. Table 14 provides a side-by-side comparison of the different solutions.

**Table 14** Side-by-side comparison for Example 3

| Parameter | North American | FKM |
|---|---|---|
| Elastic stress concentration factor | | |
| $K_{b,t}$ | 4.01 | 3.152 |
| $K_{t,t}$ | 1.16 | 1.153 |
| Fatigue notch factor | | |
| $K_{b,f}$ | 3.54 | 2.455 |
| $K_{t,f}$ | 1.16 | 1.012 |
| Size effect factor | | |
| $K_D/C_D$ (bending) | 0.779 | 0.828 |
| $K_D/C_D$ (torsion) | 0.856 | 0.933 |
| Reliability factor | | |
| $C_R$ | 0.843 | 0.843 |
| Surface roughness factor | | |
| $K_S/C_{\sigma,R}$ (bending) | 0.798 | 0.823 |
| $K_S/C_{\sigma,R}$ (torsion) | 0.717 | 0.898 |
| Endurance limit (at $10^6$ cycles) | | |
| $S_f$ (bending) | 57 | 82 |
| $S_f$ (torsion) | 159 | 177 |

Example 4 [4]

A 1035 steel has a tensile strength of 70 ksi (483 MPa) and is to be used for a component that sees 450°F (232 °C) in service. The component is machined to a uniform diameter of 7.5 mm and then polished to a mirror finish. Fatigue testing showed that the mean material fatigue strength, $S_{S,\sigma,E}$, at room temperature is 39 ksi (269 MPa). Estimate the temperature modification factor ($K_{Temp}$) and the endurance limit of the part at $10^6$ cycles subjected to cyclic bending loading at 450°F.

Solution (North American)

The temperature modification factor for endurance limit of steel materials can be estimated as follows:

$$K_{Temp} = 0.975 + 0.432(10^{-3})T - 0.115(10^{-5})T^2 + 0.104(10^{-8})T^3 - 0.595(10^{-12})T^4,$$

where $T$ is in units of °F. This equation, which is based on ultimate strength, is the result of a fit to experimental data for 21 different carbon and alloy steels. Therefore, the following analysis assumes the temperature has the same effect on the endurance limit (at $10^6$ cycles) as the ultimate strength. At 450°F, the temperature modification factor is $K_{Temp} = 1.007$. Next, endurance limit of the component at $10^6$ cycles in a 450°F environment is calculated as follows:

$$S_e = K_{Temp}S_{S,\sigma,E} = 1.007(269) = 271\,\text{MPa}.$$

Solution (FKM)

For the component in this example the temperature correction factor is calculated as

$$C_{E,T} = 1 - 1.4 \times 10^{-3}(T - 100) = 1 - 1.4 \times 10^{-3}(232 - 100) = 0.815,$$

where $T$ is in units of °C. Note this factor is specifically for the endurance limit (at $10^6$ cycles) and a separate factor would be used for ultimate strength calculations. As the material fatigue strength is already known it does not need to be estimated from the ultimate tensile strength. However, the reliability factor still needs to be applied when estimating the component strength. Next, endurance limit of the component at $10^6$ cycles in a 450°F environment is calculated as follows:

$$S_{S,b,E} = \frac{C_S S_{S,\sigma,E}}{K_{b,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{C_S C_R C_{E,T} S_{S,\sigma,E}}{K_{b,f} + \frac{1}{C_{\sigma,R}} - 1}.$$

The fatigue notch factor is equal to 1 because the component is smooth. The roughness factor is equal to 1 because the component was polished to a mirror finish. The surface treatment factor also equals 1 because there was no surface treatment applied. As a result, the above equation reduces to

$$S_{S,b,E} = C_R C_{E,T} S_{S,\sigma,E} = (0.843)(0.815)(269) = 185\,\text{MPa}.$$

Note that the North American analysis results in increased fatigue strength due to elevated temperatures and the FKM results in reduced fatigue strength. A side-by-side comparison of the two correction factors $K_{Temp}$ and $C_{E,T}$ is shown in Fig. 30.

Example 5 [3]

A filleted component is made of 2024-T4 aluminum alloy ($E = 72$ GPa, $\nu = 0.33$, $S_u = 470$ MPa, $S_y = 330$ MPa, and $S_E = 190$ MPa at $10^6$ cycles). The dimensions of the component are $B = 56$ mm, $b = 50$ mm, $t = 3$ mm, and $r = 3$ mm (see Fig. 31 for geometry). The component has a thickness of 10 mm. The component is subjected to a tensile load ranging from minimum load of $P_{min} = 20$ kN to maximum load ($P_{max}$) to be determined. Assuming $q = 0.95$ and $K_t = 1.9$, determine the maximum load the component can survive for $10^6$ cycles.

In order to determine the maximum load the component can take for $10^6$ cycles, it is desired to work with stresses. The maximum and minimum stresses for the component are calculated as

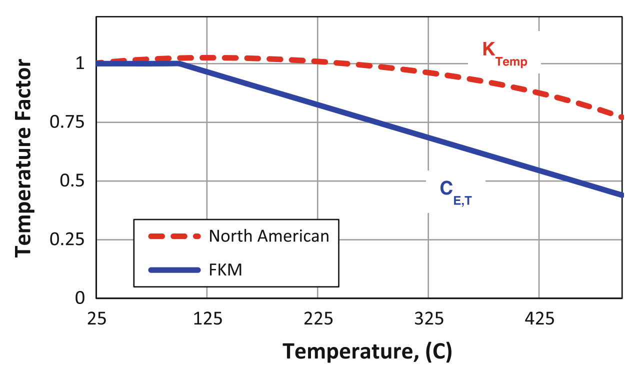



**Fig. 30** Side-by-side comparison of temperature correction factors

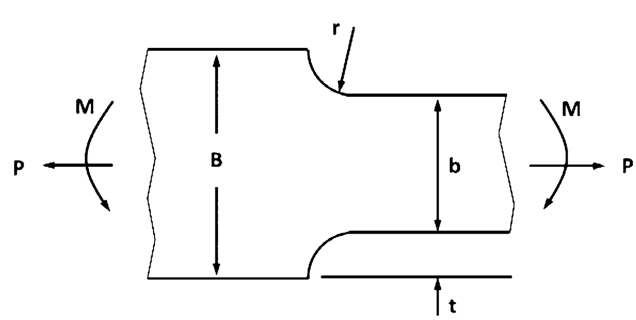



**Fig. 31** Geometry of notched component in Example 5

$$S_{\min} = \frac{P_{\min}}{A} = \frac{20,000}{0.05(0.01)} = 40\,\mathrm{MPa}$$

$$S_{\max} = \frac{P_{\max}}{A} = S_{\min} + 2S_{\sigma,\mathrm{a}},$$

where $S_{\min}$ is the minimum nominal stress, $S_{\max}$ is the maximum nominal stress, and $S_{\sigma,\mathrm{a}}$ is the nominal alternating stress.

Solution (North American)

First, the fatigue notch factor needs to be calculated based on $K_\mathrm{t}$ and $q$. The value for $K_\mathrm{t}$ was read from a curve based on the geometry and loading. Therefore, the fatigue notch factor is

$$K_\mathrm{f} = 1 + (K_\mathrm{t} - 1)q = 1 + (1.9 - 1)0.95 = 1.86.$$

The nominal fatigue strength of the component is then calculated as

$$S_\mathrm{e} = \frac{S_\mathrm{E}}{K_\mathrm{f}} = \frac{190}{1.86} = 102\,\mathrm{MPa},$$

where $S_\mathrm{E}$ is the material fatigue strength. The nominal mean stress ($S_{\sigma,\mathrm{m}}$) of the component can be written as

$$S_{\sigma,\mathrm{m}} = S_{\min} + S_{\sigma,\mathrm{a}}.$$

Gerber's relation is shown below written in terms of nominal stresses and can be used to determine a maximum nominal alternating stress ($S_{\sigma,\mathrm{a}}$) given a minimum stress and the nominal fatigue strength

$$\frac{S_{\sigma,\mathrm{a}}}{S_\mathrm{e}} + \left(\frac{S_{\sigma,\mathrm{m}}}{S_\mathrm{u}}\right)^2 = 1.$$

Substituting the above equation for mean stress gives

$$\frac{S_{\sigma,\mathrm{a}}}{S_\mathrm{e}} + \left(\frac{S_{\min} + S_{\sigma,\mathrm{a}}}{S_\mathrm{u}}\right)^2 = 1.$$

Solving the above equation for $S_{\sigma,\mathrm{a}}$ results in $S_{\sigma,\mathrm{a}} = 94$ MPa. Therefore, the maximum stress is given by

$$S_{\max} = S_{\min} + 2S_{\sigma,\mathrm{a}} = 40 + 2(94) = 228\,\mathrm{MPa}$$

therefore

$$P_{\max} = S_{\max}A = 228\left(10^6\right)(0.05)(0.01) = 114\,\mathrm{kN}.$$

Solution (FKM)

The material fatigue strength ($S_{\mathrm{S},\sigma,\mathrm{E}}$) can be estimated using the following equation:

$$S_{\mathrm{S},\sigma,\mathrm{E}} = C_{\sigma,\mathrm{E}} C_\mathrm{R} S_{\mathrm{t,u,std}} = 0.3(0.843)(470) = 119\,\mathrm{MPa},$$

where $C_{\sigma,\mathrm{E}}$ is the endurance limit modification factor based on material type and is equal to 0.3 for the component in this example. Next, the elastic stress concentration factor needs to be calculated as

$$K_{\mathrm{ax,t}} = 1 + \frac{1}{\sqrt{0.5\left(r/t\right) + 5\left(r/b\right)\left(1 + 2\left(r/b\right)\right)^2}}$$

$$K_{\mathrm{ax,t}} = 1 + \frac{1}{\sqrt{0.5\left(3/3\right) + 5\left(3/50\right)\left(1 + 2\left(3/50\right)\right)^2}} = 2.068.$$

Next, the relative stress gradient needs to be calculated. The relative stress gradient for axial loading of a shoulder filleted component is given by

$$\bar{G}_{K,\sigma}(r) = \frac{2.3}{r}(1 + \varphi) = \frac{2.3}{3}(1 + \varphi) = 0.894,$$

where

$$\varphi = 1/\left(4\sqrt{t/r} + 2\right) = 1/\left(4\sqrt{3/3} + 2\right) = 0.167.$$

Next, the supporting factor or $K_\mathrm{t}/K_\mathrm{f}$ ratio ($n_{K,\sigma}(r)$) is calculated as

$$n_{K,\sigma}(r) = 1 + \sqrt{\bar{G}_{K,\sigma}(r)} \times 10^{-\left(a_\mathrm{G} + \left(C_\mathrm{R} S_{\mathrm{t,u,std}}\right)/b_\mathrm{G}\right)}$$

$$n_{K,\sigma}(r) = 1 + \sqrt{0.894} \times 10^{-(0.05 + (0.843)(470)/850)} = 1.288.$$

The constants, $a_\mathrm{G}$ and $b_\mathrm{G}$, in the above equation for $n_{K,\sigma}$ are equal to 0.05 and 850, respectively. Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor and the supporting factor as seen below

$$K_{\mathrm{ax,f}} = \frac{K_{\mathrm{ax,t}}}{n_{K,\sigma}(r)} = \frac{2.068}{1.288} = 1.606.$$

With the correction factors $C_\mathrm{S}$ and $C_{\sigma,\mathrm{R}}$ equal to one, the nominal endurance limit at $10^6$ cycles for this component under axial loading is as follows:

$$S_{\mathrm{S,ax,E}} = \frac{C_\mathrm{S} S_{\mathrm{S},\sigma,\mathrm{E}}}{K_{\mathrm{ax,f}} + \frac{1}{C_{\sigma,\mathrm{R}}} - 1} = \frac{(1)(119)}{1.606 + \frac{1}{1} - 1} = 74\,\mathrm{MPa}.$$

The nominal mean stress ($S_{\sigma,\mathrm{m}}$) of the component can be written as

$$S_{\sigma,\mathrm{m}} = S_{\min} + S_{\sigma,\mathrm{a}}.$$

The nominal alternating stress ($S_{\sigma,\mathrm{a}}$) can be determined from the following equations:

$$S_{\sigma,\mathrm{ar}} = (1 + M_\sigma)\frac{S_{\sigma,\mathrm{a}} + (M_\sigma/3)S_{\sigma,\mathrm{m}}}{1 + M_\sigma/3} = (1 + M_\sigma)\frac{S_{\sigma,\mathrm{a}} + (M_\sigma/3)\left(S_{\min} + S_{\sigma,\mathrm{a}}\right)}{1 + M_\sigma/3}$$

and

$$M_\sigma = a_\mathrm{M} S_{\mathrm{t,u}} + b_\mathrm{M},$$

where $M_\sigma$ is the mean stress sensitivity factor and $a_M$ and $b_M$ are material constants. The constants for the aluminum material $a_M$ and $b_M$ are equal to 0.001 and -0.04, respectively. Therefore, mean stress sensitivity factor is

$$M_\sigma = a_M S_{t,u} + b_M = 0.001(0.843)(470) + (-0.04) \\ = 0.356.$$

When using this equation to solve for $S_{\sigma,a}$, $S_{S,ax,E}$ is substituted for $S_{\sigma,ar}$. The solution for $S_{\sigma,a}$ is as follows:

$$S_{\sigma,a} = \frac{S_{S,ax,E}}{(1 + M_\sigma)} - \frac{(M_\sigma/3)S_{min}}{\left(1 + {M_\sigma}/{3}\right)} = 50\,\text{MPa}.$$

The maximum stress the component can take is

$$S_{max} = S_{min} + 2S_{\sigma,a} = 40 + 2(50) = 140\,\text{MPa}$$

therefore

$$P_{max} = S_{max}A = 140\left(10^6\right)(0.05)(0.01) = 70\,\text{kN}.$$

The equation used to calculate the nominal alternating stress is only valid for a stress ratio of $0 < R < 0.5$. After calculating an alternating stress, the stress ratio should be check to see if it is within the acceptable range. The stress ratio based on the calculated nominal stress amplitude is

$$R = \frac{S_{min}}{S_{max}} = \frac{40}{140} = 0.29.$$

Therefore, the stress ratio is within the acceptable range and solution is valid.

Alternatively, calculations can be done using the mean material fatigue strength ($S_E$) given in the example. The estimated material fatigue strength ($S_{S,\sigma,E}$) based on the mean material fatigue strength is

$$S_{S,\sigma,E} = C_R S_E = (0.843)(190) = 160\,\text{MPa}.$$

Next, the elastic stress concentration factor needs to be calculated as

$$K_{ax,t} = 1 + \frac{1}{\sqrt{0.5\left({r}/{t}\right) + 5\left({r}/{b}\right)\left(1 + 2\left({r}/{b}\right)\right)^2}}$$

$$K_{ax,t} = 1 + \frac{1}{\sqrt{0.5\left({3}/{3}\right) + 5\left({3}/{50}\right)\left(1 + 2\left({3}/{50}\right)\right)^2}} \\ = 2.068.$$

Next, the relative stress gradient needs to be calculated. The relative stress gradient for axial loading is given by

$$\bar{G}_{K,\sigma}(r) = \frac{2.3}{r}(1 + \varphi) = \frac{2.3}{3}(1 + \varphi) = 0.894,$$

where

$$\varphi = 1/\left(4\sqrt{t/r} + 2\right) = 1/\left(4\sqrt{3/3} + 2\right) = 0.167.$$

Next, the supporting factor or $K_t/K_f$ ratio ($n_{K,\sigma}(r)$) is calculated as

$$n_{K,\sigma}(r) = 1 + \sqrt{\bar{G}_{K,\sigma}(r)} \times 10^{-\left(a_G + \left(C_R S_{t,u,std}\right)/b_G\right)}$$
$$n_{K,\sigma}(r) = 1 + \sqrt{0.894} \times 10^{-(0.05+(0.843)(470)/850)} = 1.288.$$

The constants, $a_G$ and $b_G$, in the above equation for $n_{K,\sigma}$ are equal to 0.05 and 850, respectively. Finally, the fatigue notch factor is determined based on the relationship between the elastic stress concentration factor and the supporting factor as seen below

$$K_{ax,f} = \frac{K_{ax,t}}{n_{K,\sigma}(r)} = \frac{2.068}{1.288} = 1.606.$$

With the correction factors $C_S$ and $C_{\sigma,R}$ equal to one, the nominal endurance limit at $10^6$ cycles for this component under axial loading is as follows:

$$S_{S,ax,E} = \frac{C_S S_{S,\sigma,E}}{K_{ax,f} + \frac{1}{C_{\sigma,R}} - 1} = \frac{(1)(160)}{1.606 + \frac{1}{1} - 1} = 100\,\text{MPa}.$$

The nominal mean stress ($S_{\sigma,m}$) of the component can be written as

$$S_{\sigma,m} = S_{min} + S_{\sigma,a}.$$

The nominal alternating stress ($S_{\sigma,a}$) can be determined from the following equations:

$$S_{\sigma,ar} = (1 + M_\sigma)\frac{S_{\sigma,a} + (M_\sigma/3)S_{\sigma,m}}{1 + M_\sigma/3} \\ = (1 + M_\sigma)\frac{S_{\sigma,a} + (M_\sigma/3)\left(S_{min} + S_{\sigma,a}\right)}{1 + M_\sigma/3}$$

and

$$M_\sigma = a_M S_{t,u} + b_M,$$

where $M_\sigma$ is the mean stress sensitivity factor and $a_M$ and $b_M$ are material constants. The constants for the aluminum material $a_M$ and $b_M$ are equal to 0.001 and −0.04, respectively. Therefore, mean stress sensitivity factor is

$$M_\sigma = a_M S_{t,u} + b_M = 0.001(0.843)(470) + (-0.04) \\ = 0.356.$$

When using this equation to solve for $S_{\sigma,a}$, $S_{S,ax,E}$ is substituted for $S_{\sigma,ar}$. The solution for $S_{\sigma,a}$ is as follows:

$$S_{\sigma,a} = \frac{S_{S,ax,E}}{(1 + M_\sigma)} - \frac{(M_\sigma/3)S_{min}}{\left(1 + {M_\sigma}/{3}\right)} = 69\,\text{MPa}.$$

The maximum stress the component can take is

$$S_{max} = S_{min} + 2S_{\sigma,a} = 40 + 2(69) = 178\,\text{MPa}$$

and

$$P_{max} = S_{max}A = 178\left(10^6\right)(0.05)(0.01) = 89\,\text{kN}.$$

Again, the equation used to calculate the nominal alternating stress is only valid for a stress ratio of $0 < R < 0.5$. The stress ratio based on the calculated nominal stress amplitude is

$$R = \frac{S_{min}}{S_{max}} = \frac{40}{178} = 0.22.$$

Therefore, the stress ratio is within the acceptable range and solution is valid. Using the mean material fatigue strength instead of the mean ultimate tensile strength, results in only a 19 kN difference. Table 15 provides a side-by-side comparison of the different solutions.

Case Study 1 (Based on Data Given in [18]

Rolled 10B40 steel rods were forged by two manufacturers to a diameter of 8 mm. The forged steel rods were heat-treated to four hardness levels, 421, 327, 253, and 220 HB. Manufacturer A represents a low-quality forging process and Manufacturer B represents a high-quality forging process. Reverse cantilever bending fatigue tests were performed on the forged steel rods. The rods were tested with an as-forged surface. Results from the fatigue testing are shown in Table 16. The endurance limits for the forged components at the different hardness levels were predicted using traditional North American analysis techniques as well as techniques described in the FKM-Guideline. Results of predictions are also shown in Table 16.

**Table 15** Side-by-side comparison for Example 5

| Parameter | North American | FKM based on $S_u$ | FKM based on $S_E$ |
|---|---|---|---|
| Elastic stress concentration factor | | | |
| $K_{ax,t}$ | 1.9 | 2.068 | 2.068 |
| Fatigue notch factor | | | |
| $K_{ax,f}$ | 1.86 | 1.606 | 1.606 |
| Reliability factor | | | |
| $C_R$ | 1 | 0.843 | 0.843 |
| Maximum stress, MPa | | | |
| $S_{max}$ | 228 | 140 | 178 |
| Maximum load, kN | | | |
| $P_{max}$ | 114 | 70 | 89 |
| Material endurance limit at $10^6$ cycles, MPa | | | |
| $S_E$ | 190 | 119 | 160 |
| Component endurance limit at $10^6$ Cycles, MPa | | | |
| $S_{S,ax,E}$ | 102 | 74 | 100 |

Endurance Limit Prediction (North American)

The endurance limit was predicted using traditional analysis techniques as follows:

$$S_{e,bending} = \frac{3.45\,\mathrm{HB}C_R}{2}(K_S),$$

where $C_R = 0.843$ based on a reliability of 97.5%, HB is the Brinell hardness number, and $K_S$ is the surface correction factor. The surface correction factor ($K_s$) was taken from Fig. 32. The various curves in this figure represent different qualities of surface finish produced by different types of manufacturing processes.

Endurance Limit Prediction (FKM)

The endurance limit was predicted using FKM analysis techniques as follows:

$$S_{S,b,E} = \frac{C_S C_{\sigma,E} 3.45\,\mathrm{HB}C_R}{K_{b,f} + \frac{1}{C_{\sigma,R}} - 1},$$

where $C_S = 1$ because there was no surface treatment, $C_{\sigma,E} = 0.40$ is for the steel material used, $C_R = 0.843$ is based on a reliability of 97.5%, $K_{b,f} = 1$ because the component was unnotched, and $C_{\sigma,R}$ was determined from the following equation:

**Table 16** Summary of experimental and estimated endurance limits for forged steel components (experimental data taken from [18])

| Manufacturer | Hardness, HB | Endurance limit at $10^6$ cycles, MPa | | | Surface correction factor | |
|---|---|---|---|---|---|---|
| | | Experimental | North American | FKM | North American ($K_S$) | FKM ($C_{\sigma,R}$) |
| A | 421 | 258 | 141 | 295 | 0.230 | 0.602 |
| | 327 | 241 | 142 | 250 | 0.298 | 0.657 |
| | 253 | 255 | 127 | 210 | 0.345 | 0.714 |
| | 220 | 249 | 120 | 191 | 0.375 | 0.744 |
| B | 421 | 231 | 141 | 295 | 0.230 | 0.602 |
| | 327 | 303 | 142 | 250 | 0.298 | 0.657 |
| | 253 | 281 | 127 | 210 | 0.345 | 0.714 |
| | 220 | 266 | 120 | 191 | 0.375 | 0.744 |

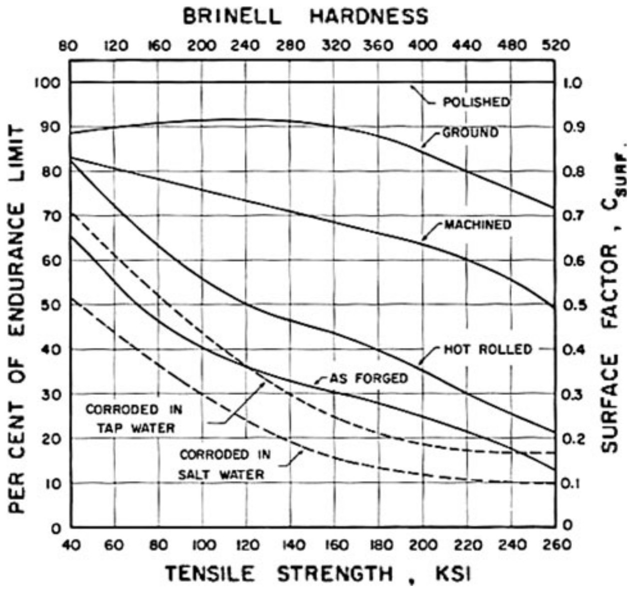



**Fig. 32** Surface correction factors for various surface conditions [17]

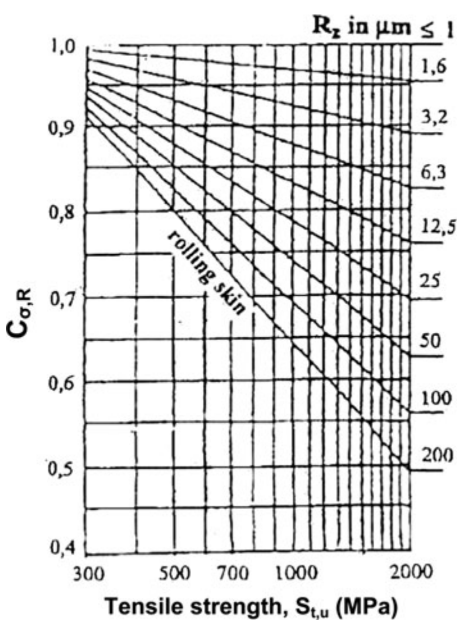



**Fig. 33** Roughness correction factor versus ultimate tensile strength for various roughness levels [11]

$$C_{\sigma,\mathrm{R}} = 1 - a_{\mathrm{R}}\log(R_Z)\log\left(\frac{2(3.45)(\mathrm{HB})C_{\mathrm{R}}}{S_{\mathrm{t,u,min}}}\right).$$

When calculating the roughness factor ($C_{\sigma,\mathrm{R}}$), a roughness value of $R_z = 200$ µm was used as specified by FKM-Guidelines. The constants $a_{\mathrm{R}}$ and $S_{\mathrm{t,u,min}}$ for the steel material used are 0.22 and 400 MPa, respectively. Figure 33 shows a plot of the equation for roughness correction factor ($C_{\sigma,\mathrm{R}}$) versus tensile strength for steel components. The different lines on this figure represent different values of $R_z$.

The endurance limits predicted using both analysis techniques are plotted versus experimental cycles to failure in Fig. 34. This figure also includes a 45° line representing perfect agreement with experimental data. It can be seen in Fig. 34 that the predictions based on FKM analysis have better agreement with the experimental data. One explanation for the large difference between the two sets of predictions is that the surface correction factors ($K_{\mathrm{S}}$) from traditional North American analysis are based on data published in the 1930s and 1940s. Improvements in forging and steel making technologies have made the correction factors ($K_{\mathrm{S}}$) for forged surfaces overly conservative [18].

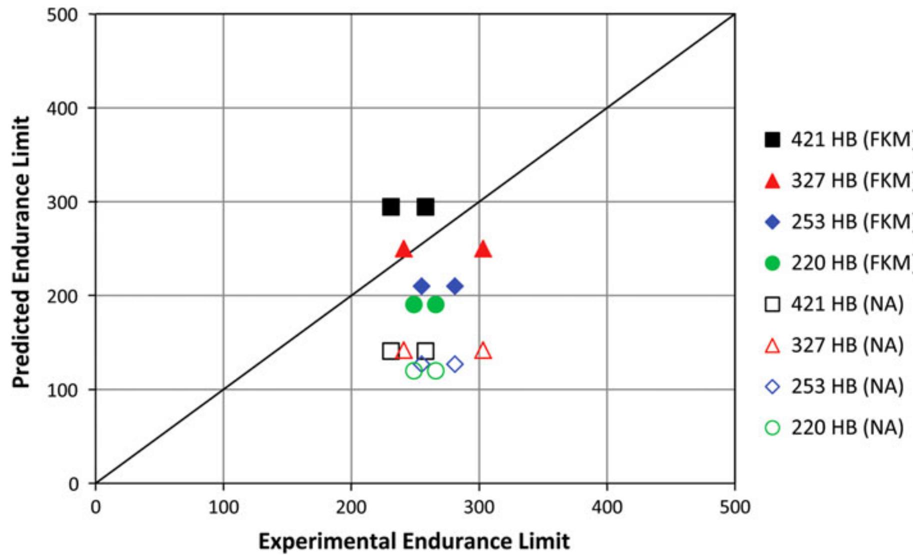
## Case Study 2 [5]

The mean stress sensitivity factor model developed by Schütz in 1967 is considered the most comprehensive method to account for the mean stress effect on HCF life and strength, which is applicable to various materials (such as steels, steel castings, ductile irons, malleable cast irons, gray cast iron, wrought aluminum alloys, and cast aluminum alloys) and to a broad range of mean stress or stress ratio. Even though the mean stress sensitivity factor method has been frequently used in European-based fatigue commercial software and engineering design codes such as FKM-Guideline, it has not received a great deal of attention worldwide. Although the FKM-Guideline provides extensive experimental test validation, very little evidence from independent sources have illustrated the validity of this method. The following study examines the Schütz mean stress sensitivity factor model by comparing to the Walker mean stress correction equation with the best fitting parameters published for steels and aluminum alloys.

The following comparisons of some of the available mean stress correction models are based on experimental data [7]. The experimental data points (seen in Figs. 35–37) were normalized in the vertical axis with the $\sigma_{\mathrm{ar}}$ calculated based on the life of that sample. In the horizontal axis, each mean stress value was normalized with respect to $S_{\mathrm{ut}}$. This produces curves that are independent of fatigue life. These figures were purposed as a snapshot of all the methods with respect to experimental data. Figure 35 is an example of a steel alloy. Figure 36 is an example of a chrome molly steel. Figure 37 is an example of an aluminum alloy. These examples, like most available data, have small mean stress ranges.

The experimental data illustrated in Fig. 35 are sensitive to compressive and tensile mean stresses. The Goodman line is the conservative bound in the tensile region. The SWT is also conservative in the tensile region and follows the Goodman line for small tensile stresses. The Morrow line passes through the middle of the data in the tensile and

**Fig. 34** Predicted endurance limit versus experimental endurance limit for forged steel components (experimental data taken from [18])



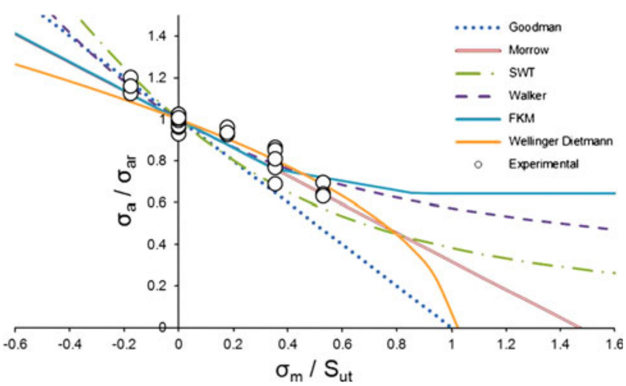




**Fig. 35** AISI 4340 Steel [5]

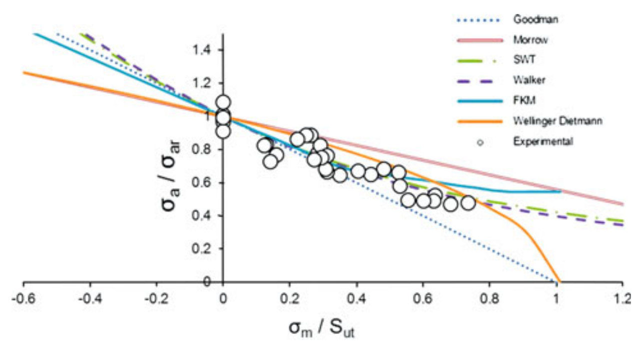



**Fig. 37** 2014-T6 Aluminum [5]

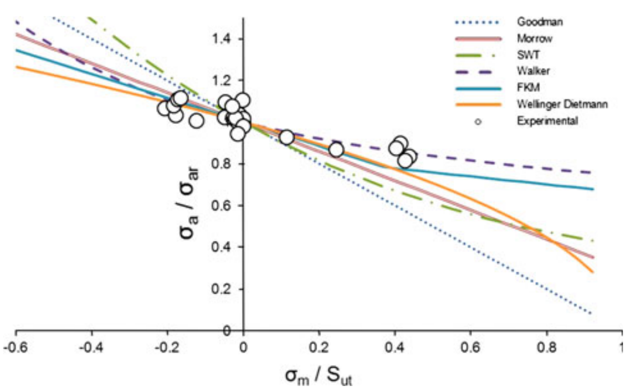



**Fig. 36** 50CrMo4 Steel [5]

compressive regions. For the stress range which contains data for this material, the FKM, Walker and Wellinger Dietmann lines are about the same.

The experimental data illustrated in Fig. 36 are insensitive to mean stress. Most of the methods over correct for positive mean stresses. The Walker, FKM (the Schütz mean stress sensitivity factor model), and Wellinger Dietmann have the most agreement with experimental data. For this material and tensile mean stresses, the Walker equation produces the least conservative equally reversed stress estimates. This means that the equally reversed stress amplitude is punished the least for an increase in mean stress.

The experimental data in Fig. 37 are bounded on the conservative side by the Goodman line and on the non-conservative side by the Morrow line. The SWT, Morrow, and FKM (the Schütz mean stress sensitivity factor model) perform similarly by creating good predictions for small to moderate mean stresses.

From this study, it was concluded that the FKM method can produce similar results as the Walker equation with the best fitting parameters published for steels and aluminum alloys. The Walker relationship was used as the method of comparison because it has been proven to correlate to experimental data well.

## Summary

According to FKM-Guideline, the empirical procedures to generate the synthetic nominal $S$–$N$ and the pseudo-$\sigma^{e}$–N curves have been introduced. These procedures are not scientifically based but are simple and useful engineering tools for generating the synthetic component stress-life curves for various materials.

If the test data are not available, the ultimate strength of a real component at a specific temperature with a survival rate can be estimated based on an estimated strength value of a standard smooth, polished round test specimens of 7.5 mm diameter with the correction factors for temperature, reliability, size, load, and stress. This estimation process has been presented in section 2.

The endurance limit of a real component at a specific temperature with a survival rate can be estimated based on an estimated ultimate tensile strength value of a standard smooth, polished round test specimens of 7.5 mm diameter with the correction factors for temperature, reliability, size, endurance limit in normal or shear stress, surface treatment, roughness, and fatigue notch factor. This estimation process has been described in section 3.

The stress field intensity theory has been used to explain why the fatigue notch factor or fatigue strength reduction factor is less than the elastic stress concentration factor. There are three popular approaches (notch sensitivity factor by Peterson and Neuber, relative stress gradient by Siebel and Stieler, and FKM-Guideline by Haibach) to estimate the fatigue notch factor, among which FKM-Guideline is recommended by the authors.

The constant amplitude nominal stress-life approach and the local pseudo-stress-life approach for a notched component under fully reversed loading have been introduced in section 4. Either one can be derived on a reference point, the endurance limit at an endurance cycle limit, and a suggested slope factor by FKM-Guideline.

The application of the Palmgren–Miner linear damage rule to a component subjected to variable amplitude loading over time has been discussed in section 5. It has been shown that the critical damage value is a random variable varying from 0.15 to 1.06. So for mechanical designs, FKM-Guideline recommends $D_{PM} = 0.3$ for steels, steel castings, and aluminum alloys, while $D_{PM} = 1.0$ for ductile irons, gray cast irons, and malleable cast irons. Also for electronic equipment designs, Steinberg suggests $D_{PM} = 0.7$.

The mean stress effect on the fatigue strength and lives of a component has been addressed in section 6. Even though there are numerous models developed to account for the mean stress effect, five commonly used formulas such as Goodman's, Morrow's, Smith–Watson–Topper's, Walker's, and the one by FKM-Guideline were discussed.

According to the extensive studies [7] on experimental fatigue data for steels, aluminum alloys and a titanium alloy where the $R$ ratio ranges from $-2$ to 0.45, Goodman's model for life predictions is highly inaccurate and should not be used. Walker's model gives the superior results if an additional mean stress fitting parameter $(\gamma_W)$ is provided. Both Morrow's and SWT's models yield reasonable life estimates for steels. SWT's model is recommended for aluminum alloys. If there are no experimental data available for materials or the $R$ ratio is beyond the range of the previous studies, FKM-Guideline is recommended for use.

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